Weak Decay Form Factors – Theory and Applications

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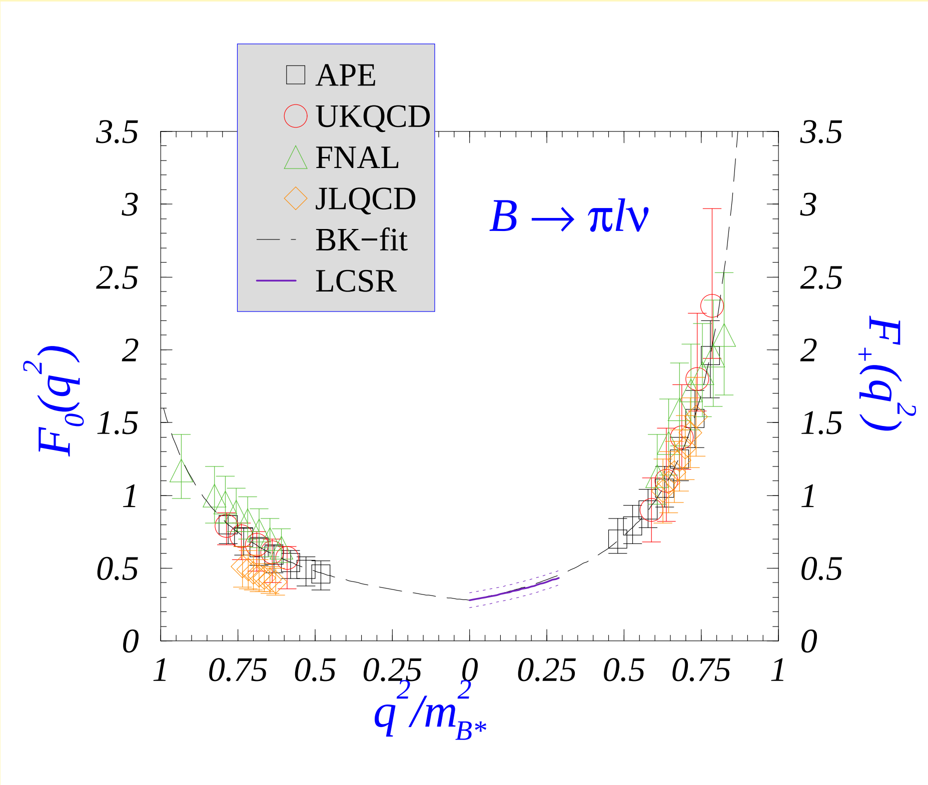


Setting the Stage for $B \longrightarrow \pi \ldots$

(. . . coming back to other decays later)

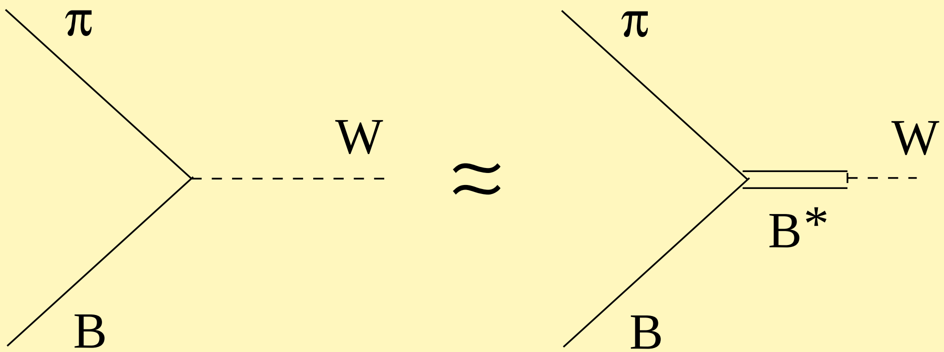
Definition of form factors:

$$\langle \pi | \bar{u}\gamma_\mu b | B \rangle = f_+(q^2)(p_{B\mu} + p_{\pi\mu}) + f_-(q^2) q_\mu \quad [q_\mu = p_{B\mu} - p_{\pi\mu}]$$

$B \to \pi e \nu$: f_- suppressed by m_e^2/m_B^2, $0 \le q^2 \le (m_B - m_\pi)^2$.

Naïve expectation: f_+ dominated by B^*-pole ($m_{B^*} = 5.32\,\mathrm{GeV}$):

$$f_+(q^2) \propto \frac{1}{m_{B^*}^2 - q^2}$$



Correct expression: $f_+(q^2) = \dfrac{c}{m_{B^*}^2 - q^2} + \displaystyle\int_{(m_B+m_\pi)^2}^{\infty} dt\, \frac{\rho(t)}{t - q^2}$

Need Form Factors for

- determination of $|V_{ub}|$ and $|V_{cb}|$ from semileptonic decays
- determination of CP-violating phases from nonleptonic decays in QCD factorisation (à la BBNS)

Factorization à la BBNS

Beneke/Buchalla/ Neubert/Sachrajda, PRL 83 (1999) 1914

Generic amplitude for heavy-to-light transitions:

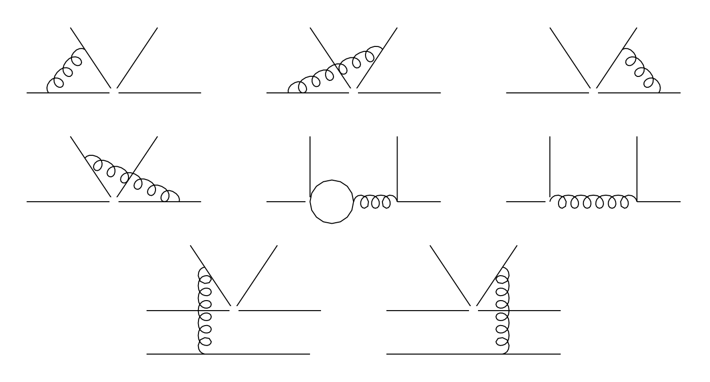
$$A(B \to \pi\pi) = f_+^{B\to\pi}(0) \int_0^1 dx\, T^I(x)\phi_\pi(x) + \int_0^1 d\xi dx dy\, T^{II}(\xi, x, y)\phi_B(\xi)\phi_\pi(x)\phi_\pi(y)$$

$$= A(B \to \pi\pi)_{\text{fact}} \times (1 + \mathcal{O}(\alpha_s) + \mathcal{O}(\Lambda_{\text{QCD}}/m_b))$$

$f_+^{B\to\pi}$: weak decay form factor

- shown to be valid at 1-loop in QCD
- naive factorization works up to (calculable) radiative corrections and (non calculable) power-suppressed terms

$T^{I,II}$: process-dependent hard scattering amplitudes



$\phi_{B,\pi}(x)$: universal light-cone distribution amplitudes

- describe collinear momentum-distribution of quarks in meson
- obtained from Bethe-Salpeter WFs by integration over transverse momenta
- well-studied for light mesons (e.g. π EM form factor)

Some Ideas How to Calculate

- Quark models: started 85 with relativistic harmonic oscillator + pole-dominance (Bauer/Stech/Wirbel)

- Lattice: calculations by several collaborations (APE, UKQCD, FNAL, JLQCD. . .)

- (local) QCD sum rules: nonperturbative terms ill-behaved for $m_b \to \infty$

- pQCD methods à la Brodsky-Lepage (to be cont'ed)

- QCD sum rules on the light-cone: hybrid of local QCD sum rules and pQCD (to be cont'ed)

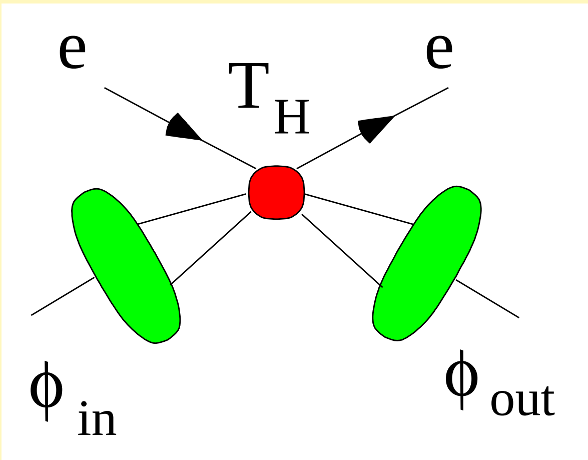
- very recent developments: factorisation of heavy-to-light form factors in soft-collinear effective theory (Bauer et al. hep-ph/0211069, Beneke/Feldmann hep-ph/0311335, Lange/Neubert hep-ph/0311345)

Hard & Soft pQCD: Brodsky/Lepage 1980

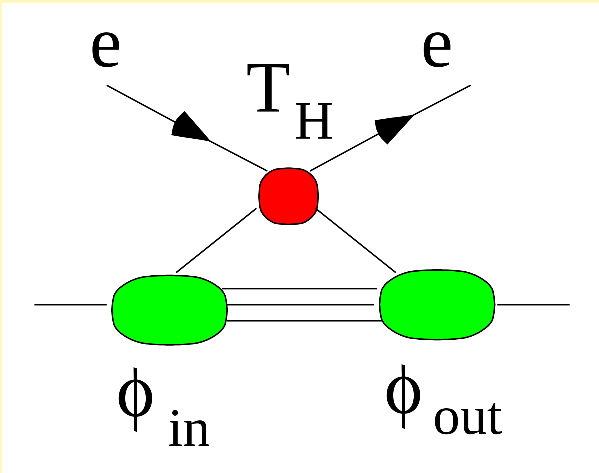
At large momentum transfer Q^2, exclusive QCD processes dominated by states with "valence" quark content; process amplitude factorizes:

$$M = \prod_j \phi_{\mathrm{out},j}(n_j) \otimes T_H(n_j, n_i) \otimes \prod_i \phi_{\mathrm{in},i}(n_i)$$

$\phi(u)$, $0 \leq u \leq 1$: probability amplitude for collinear quarks with momentum up and $(1-u)p$, resp., to form hadron with momentum p ($p^2 \ll Q^2$)



Purely hard process: dominant in "classical" applications of pQCD, e.g. EM π FF

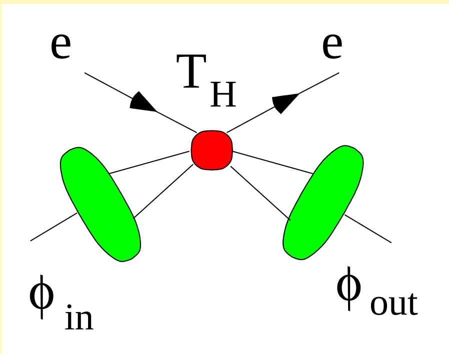


Soft (Feynman) mechanism: strongly asymmetric kinematical configuration of partons

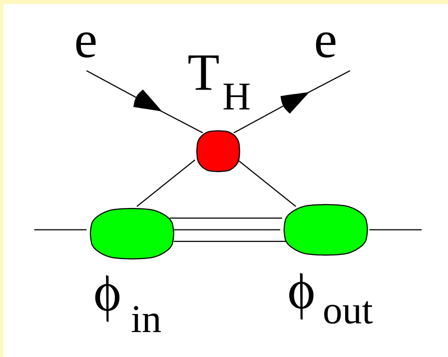
Hard & Soft pQCD: what about heavy meson decays?

- processes involving only light mesons: dominated by hard contributions (gluon exchange)
- heavy mesons: soft and hard processes of same order in $1/m_b$, although soft processes damped by Sudakov logs (Chernyak/Zhitnitsky 1990)
- naive calculation of $B \to \pi$ by hard-gluon exchange spoiled by soft divergences (Szczepaniak/Henley/Brodsky 1990)

$\rightsquigarrow$ need method to capture both hard-gluon-exchange and soft Feynman-mechanism!



Hard process



"Soft" or "Feynman" mechanism

Hard & Soft pQCD: State of the Art

$\leadsto$ SCET

- identify uncalculable nonperturbative/soft terms order by order in $1/m_b$ expansion
- determine soft terms from experiment and/or construct relations between form factors that are independent of soft terms (to given accuracy in $1/m_b$)
- discussion of meaningful separation between "hard" and "soft" ongoing (Beneke/Feldmann 11/03, Lange/Neubert 11/03)

$\leadsto$ QCD sum rules on the light-cone

- calculate both soft and hard terms using the same method, using the techniques of QCD sum rules
- obtain numerical predictions (and estimates of theoretical accuracy)

Enter the stage

QCD Sum Rules on the Light-Cone (LC)

$$i\int d^4y e^{iqy}\langle\pi(p)|T[\bar{u}\gamma_\mu b](y)[m_b\bar{b}i\gamma_5 d](0)|0\rangle \stackrel{\text{LCE}}{=} \sum_n T_H^{(n)}\otimes\phi_\pi^{(n)}$$

- $\phi_\pi^{(n)}$: π distribution amplitudes (DAs)
- n: twist
- $T_H^{(n)}$: perturbative amplitudes

$$= 2p_\mu\left(f_+(q^2)\,\frac{m_B^2 f_B}{m_B^2-p_B^2} + \text{higher poles and cuts}\right) + \text{terms contr. to other FF}$$

$\rightsquigarrow$ avoid B-meson DA as B described not as real particle, but via dispersion relation

$\rightsquigarrow$ LC-expansion starts at $O(1)$, not $O(\alpha_s)$ $\longrightarrow$ soft terms included

Features of LCSRs

- expansion effectively in $1/m_b \longrightarrow$ need to include higher-twist terms
- $\sum T_H^{(n)} \otimes \phi_\pi^{(n)}$ implies factorization – valid at higher twist?
  - calculate $O(\alpha_s)$, known for
    T2 (π (Khodjamirian et al. 97, Ball et al. 97), ρ (Ball/Braun 98))
    T3 (π (Ball/Zwicky 2001, 2004))

    $\longrightarrow$ factorization OK
- use standard SR techniques: Borel-transformation, continuum model

Advantages and Disadvantages of LCSRs

- numerical predictions based on QCD calculations

- accuracy can be improved by including perturbative QCD corrections and reducing uncertainties of hadronic input parameters

  (in particular m_b and the parameters determining the light-meson distribution amplitudes)

- (?) certain degree of model-dependence

  (separation between ground-state contribution to dispersion relation and contributions of higher states)

- / need additional nonperturbative input from light meson distribution amplitudes

Formal Definition of Twist-2 DAs

$$\langle 0|\bar{u}(z)\,[z,-z]\,\gamma_\mu\gamma_5 d(-z)|\pi^-(p)\rangle \,=\, i f_\pi p_\mu \int_0^1 du\, e^{i(2u-1)pz}\, \phi_\pi(u)$$

$p^2 = 0$, $z^2 = 0$ (light-cone)

Vector mesons: one distribution amplitude for longitudinally, one for transversely polarised mesons

N.B.: certain similarity to definition of DIS parton distribution functions. However:

PD functions	$\leftrightarrow$	probability
DAs	$\leftrightarrow$	amplitude

And Higher Twist. . .

Two-particle distribution amplitudes:

Generic form: $\langle 0 \mid \bar{\psi}(-z)\Gamma\psi(z) \mid M \rangle$

2 twist-3, 2 twist-4 for π

4 twist-3, 2 twist-4 for ρ

Three-particle distribution amplitudes:

Generic form: $\langle 0 \mid \bar{\psi}(-z)G_{\mu\nu}(uz)\Gamma\psi(z) \mid M \rangle$

1 twist-3, 4 twist-4 for π

3 twist-3, 10 twist-4 for ρ

NB: these DAs also enter QCD factorisation formulas. . .

Pandora's box?

Order Principles for DAs

Ball/Braun/Koike/Tanaka 98

Big advantage as compared to inclusive distribution functions: can exploit conformal symmetry of massless QCD (valid in LO) to derive

partial wave expansion of DAs

in terms of contributions of increasing conformal spin.

2nd important ingredient in analysis of DAs:

exact relations between non-local operators

from QCD equations of motion

Combine both: truncated conformal expansion of π DAs at next-to-leading conformal spin leaves 5 independent hadronic parameters for the 10 twist-2, 3 and 4 DAs of the π!

Conformal Expansion of Twist 2 DAs

$$\phi(u,\mu^2) = 6u(1-u)\left(1+\sum_{n=1}^{\infty} a_n(\mu^2) C_n^{3/2}(2u-1)\right)$$

- $6u(1-u)$: asymptotic DA, valid for $\mu \to \infty$, conformal spin $= 2$.
- $a_n(\mu)$: Gegenbauer moments; nonperturbative parameters, renormalize multiplicatively in LO QCD by virtue of conformal symmetry; $a_n(\mu) \to 0$ for $\mu \to \infty$
- $C_n^{3/2}$: Gegenbauer polynomials, orthogonal over asymptotic DA; conformal spin $2+n$ (analogues to spherical harmonics in "usual" partial wave expansion)

Gegenbauer polynomials rapidly varying functions
$\longrightarrow$ truncation justified for convolution with *smooth* functions.
Often just asymptotic distribution amplitude used.
Probably not appropriate for $B \to K, K^*$ transitions: $a_1 \propto (m_s - m_q)$ encodes SU(3) breaking!

Numerics of DAs

Nice expansion – but where's the numerical input to come from?

- in favour of asymptotic DA:
  - EXP: $e^+e^- \to e^+e^-\pi^0$ measured at CLEO
  - EXP+TH: π EM FF (Braun/Khodjamirian/Maul 2000)

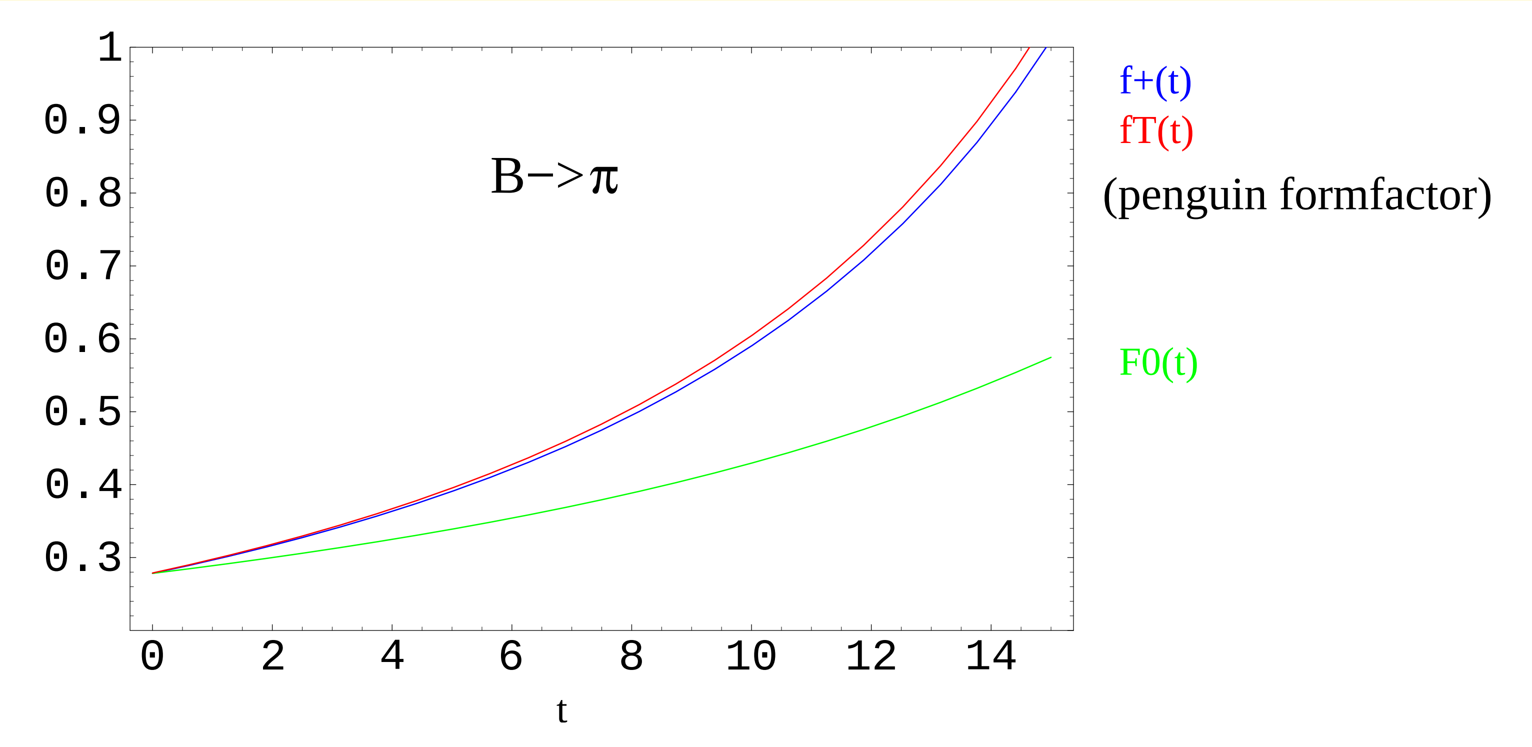
- TH: local QCD SRs: rather large values:
  $a_2(1\,\mathrm{GeV}) = 0.44$, $a_4(1\,\mathrm{GeV}) = 0.25$.

- lattice: old data for 2nd moment (from 87 to 91);
  one recent retry: UKQCD 99. What about another go???

And now, Ladies and Gentlemen:

New (& Preliminary) Results!

Ball/Zwicky 2004



- LCSRs only valid for $E_\pi \gg \Lambda_{QCD}$, i.e. $t = m_B^2 - 2m_B E_\pi < m_B^2$.
  Choose $E_\pi^{min} \approx 1.2\,\text{GeV}$.

What about q^2-Dependence?

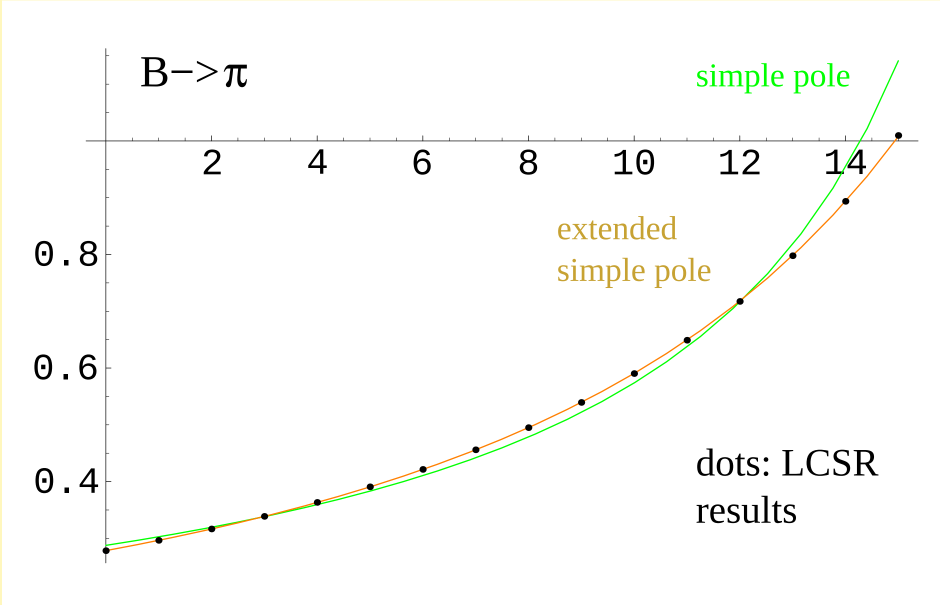
- poles + cuts: exact! $f_+(q^2) = \frac{c}{m_{B^*}^2 - q^2} + \int\limits_{(m_B+m_\pi)^2}^{\infty} dt\, \frac{\rho(t)}{t-q^2}$

  (first term: single-pole approximation)

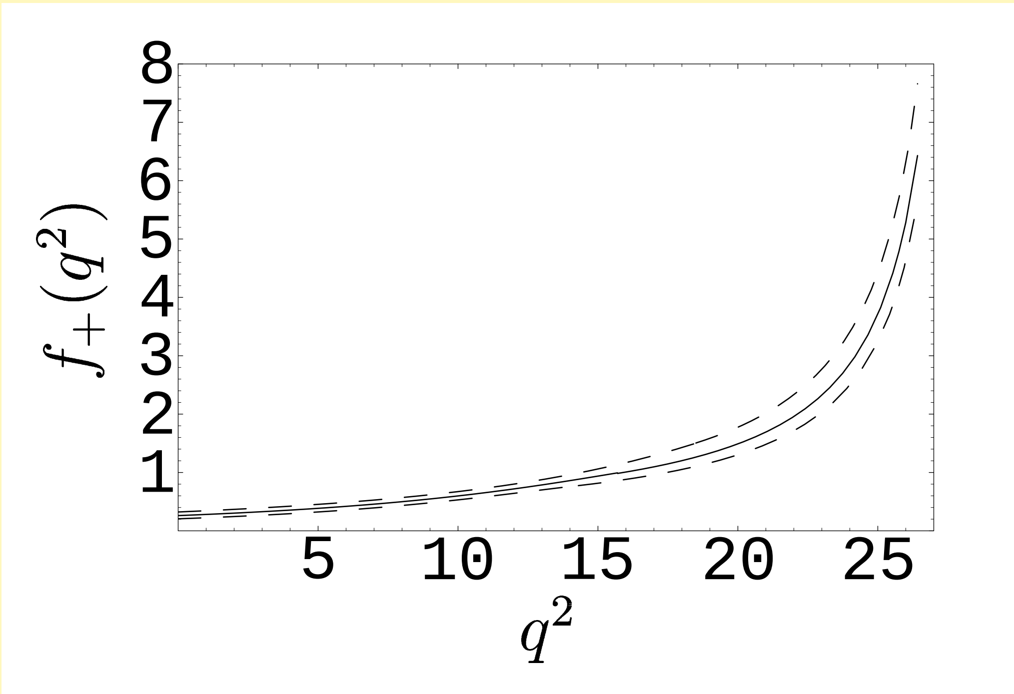
- Becirevic + Kaidalov (99): approximate $\int$ by a pole (???)

- LCSR: $$F(q^2) = \frac{F(0)}{1 - a_F\, \frac{q^2}{m_B^2} + b_F \left(\frac{q^2}{m_B^2}\right)^2}$$

  - motivation: simple extension of single pole
  - works extremely well numerically!



For $t > 15\,\mathrm{GeV}^2$, fit to simple pole at $m(B^*) = 5.34$ GeV:

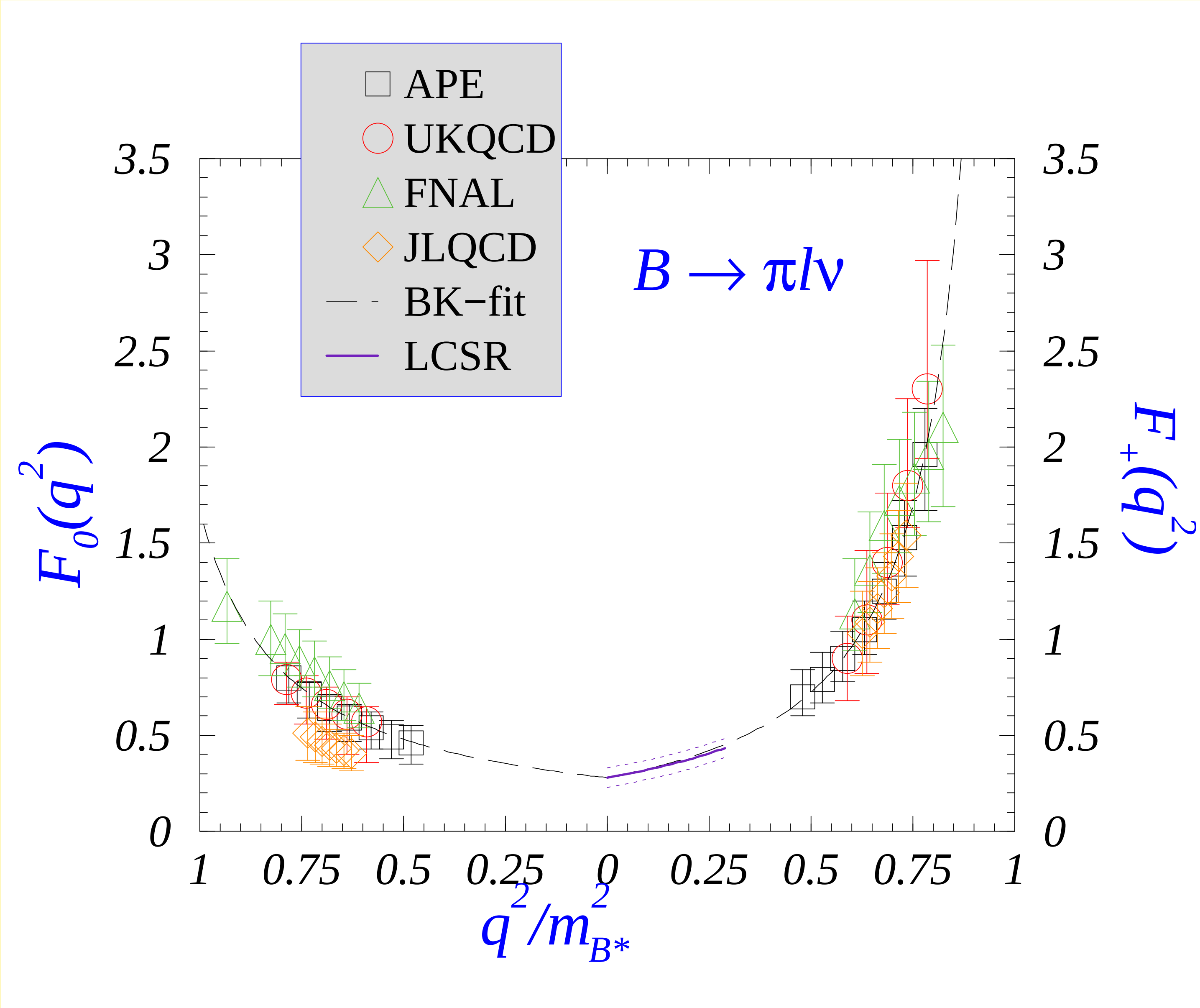


$$f_+(q^2) = \frac{g}{m_{B^*}^2 - q^2}$$

for $q^2 > 15\,\mathrm{GeV}^2$.

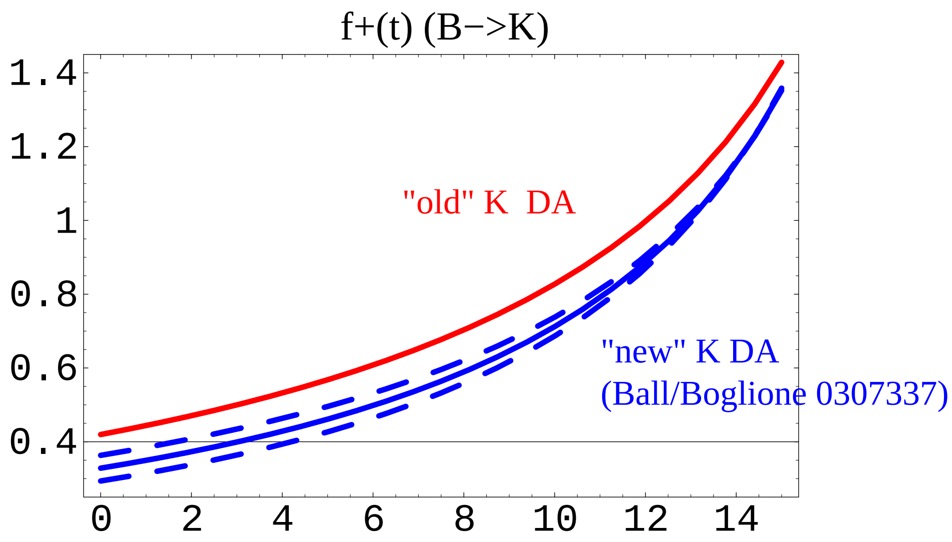
Ball/Zwicky 2001

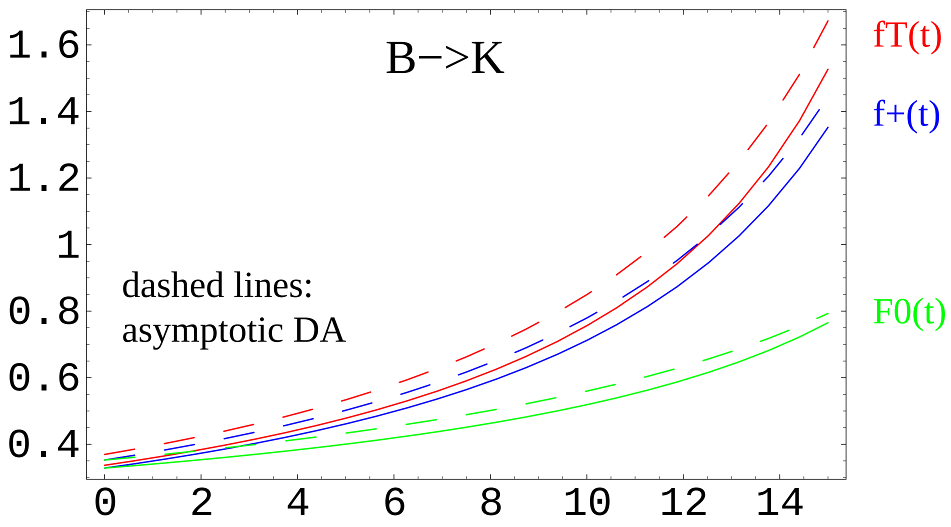
Compare to lattice: Becirevic 2002



$B \to K$

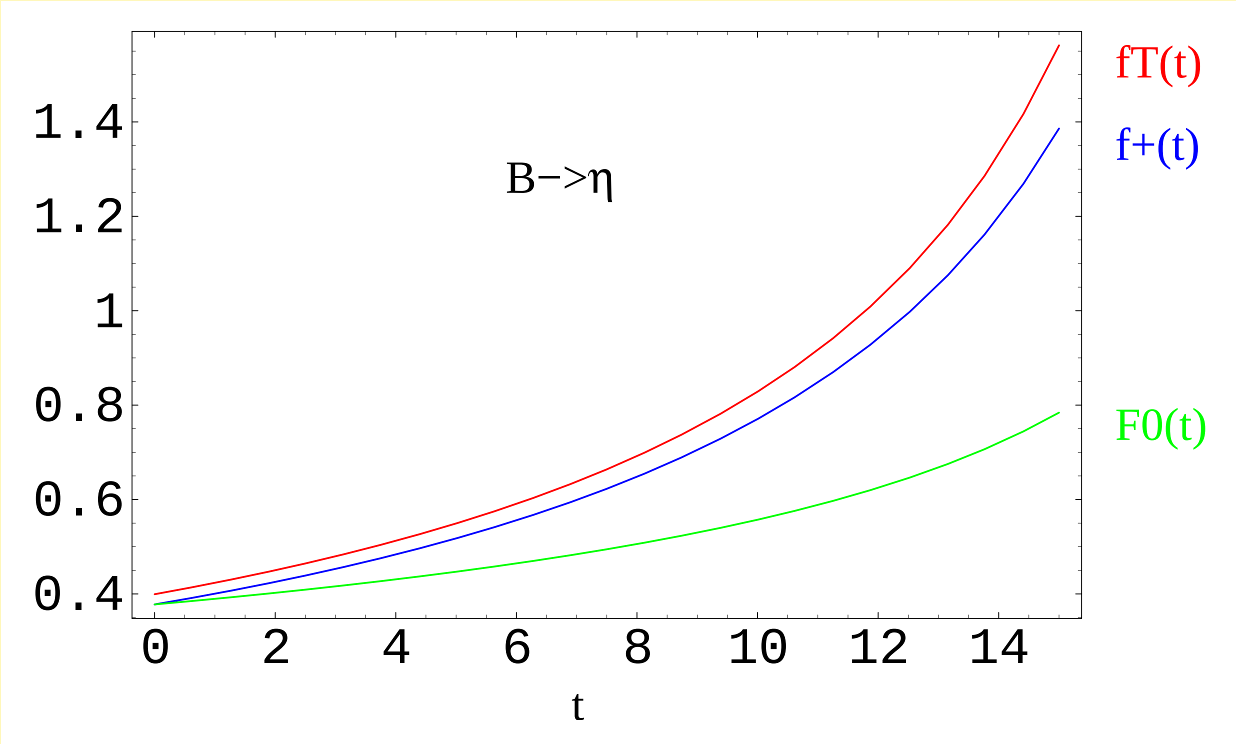
Recent redetermination of a_1 and a_2 for K: $a_1 < 0$! (in contrast to previous results): $f_+^{B\to K}$ becomes smaller!





Ball/Zwicky 2004

$B \to \eta$

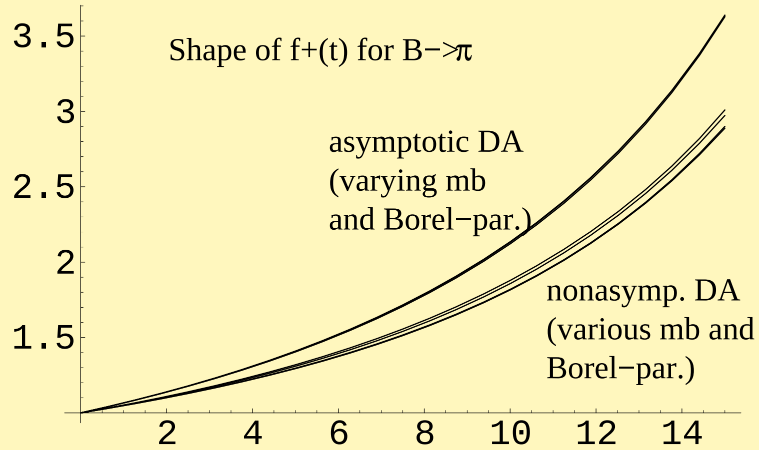


Ball/Zwicky 2004

Only flavour octet included.

And what about uncertainties?

- systematic uncertainties from various approximations involved in sum rule calculation:
  - Borel-transformation/ dep. on Borel-parameter
  - continuum model/subtraction



$\rightsquigarrow$ estimate at 20% (?)

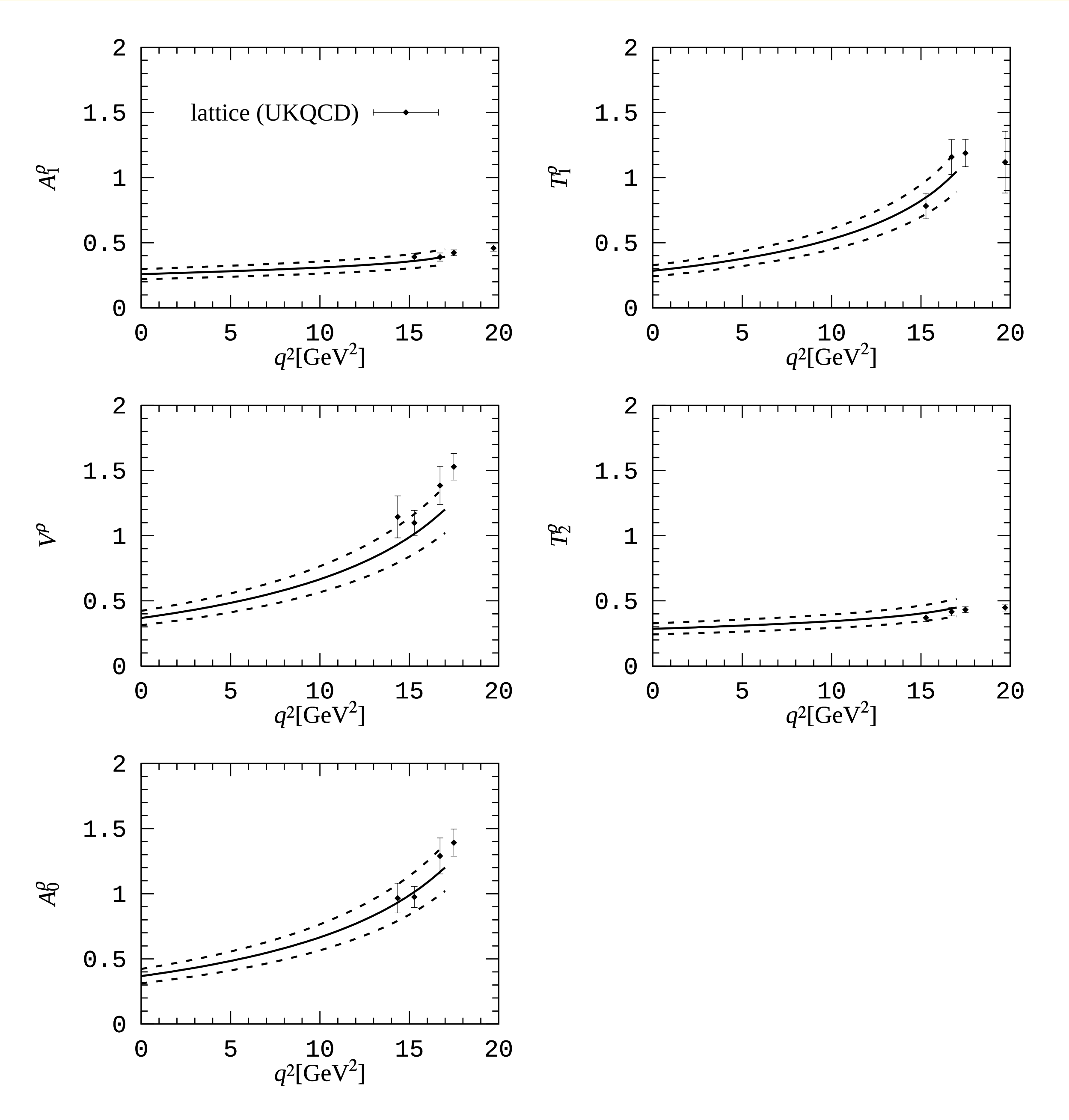
Uncertainties for shape expected to be smaller!

- uncertainties from input-parameters
  - π, ρ, K, K^* DAs: exp. or SRs or lattice?
  - m_b

- adds up to a total of what?

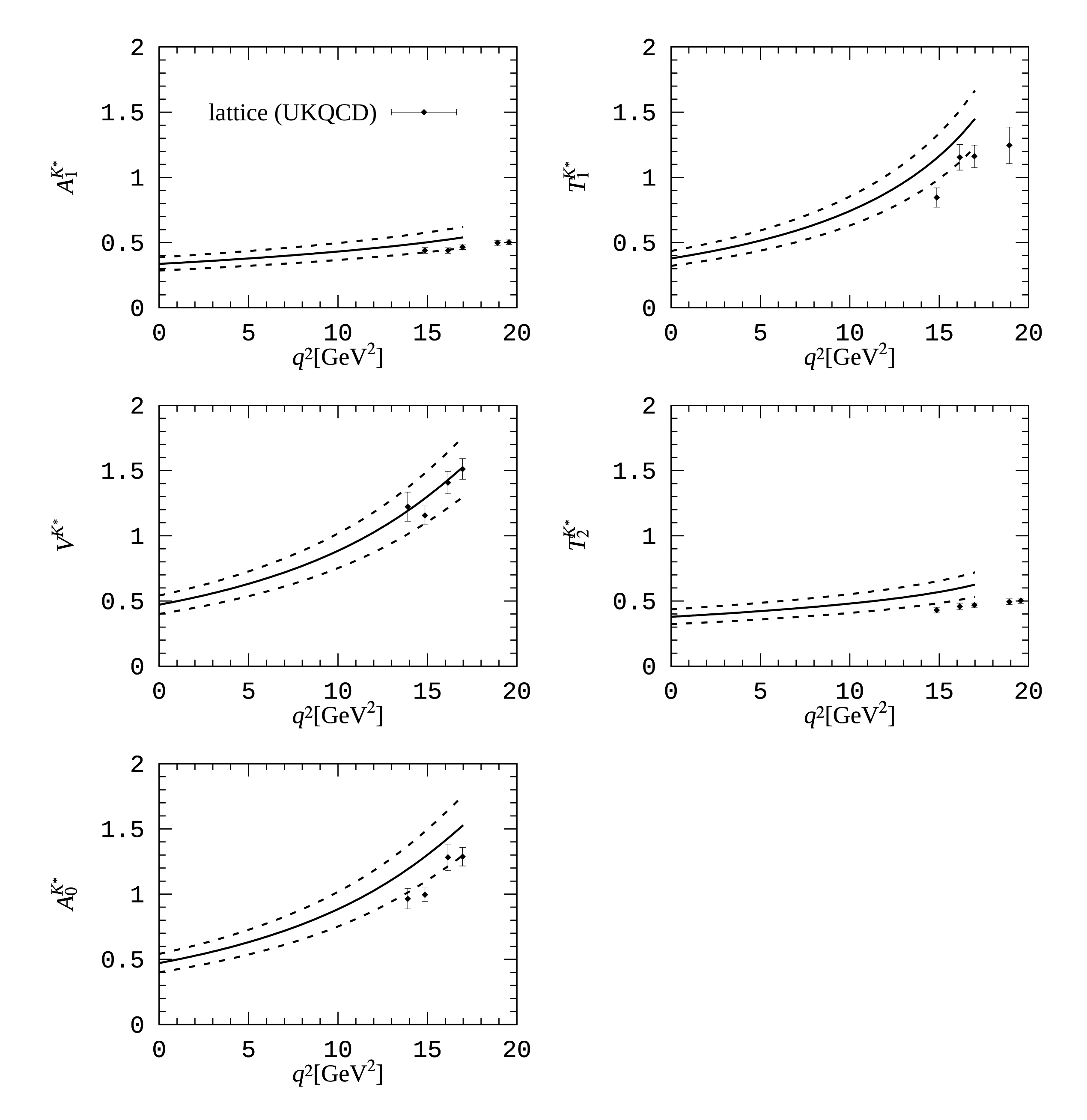
$B \to \rho$

Ball/Braun 98
(to be updated in 2004)



Ditto $B \to K^*$

Ball/Braun 98
(to be updated in 2004)



Summary

- npQCD devilishly complicated, no edging away from QCD-infested measurements (in particular for exclusive decays)
- no single & simple solution, try different approaches (LCSRs, SCET, lattice. . .);
  gain confidence if results point into one & the same direction
- QCD SRs on the light-cone appear well suited to describe heavy-to-light transitions for small to moderate momentum transfer
- test predictions for shape; challenge/confirm sympathy with lattice