Physics of Accretion Disks: A Primer for GLAST Science

Jim Chiang
GSSC

Outline

- Motivation and Observational Evidence for Disks
- Thin Disk Structure
- Viscous Shear and Angular Momentum Transport
- Viscosity Mechanisms — MRI
- Disk Temperature Profile and SED
- Advection-Dominated Flows

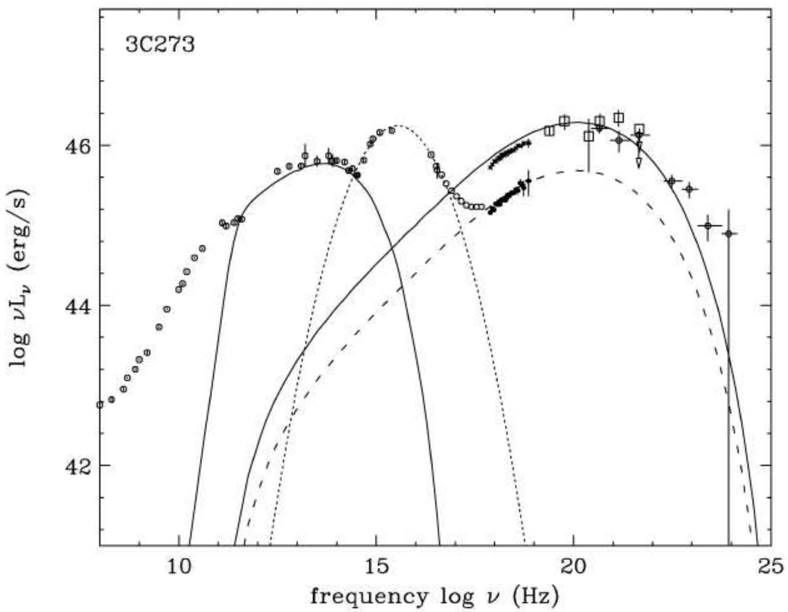
Motivation

There are no direct observation signatures of accretion disks in the LAT band, so why do we care?

- Accretion ultimately powers all emission from certain classes of compact objects:
  - Jets in AGNs, GRBs, XRBs/micro-quasars
  - UV continuum in AGNs (the big blue bump)
  - soft X-ray continuum in XRBs
  - hard X-ray ($\lesssim 1$ MeV) from thermal Comptonization component in XRBs and Seyfert galaxies.
- Similar physics plays a role in galactic dynamics and planet formation.

Observational Evidence of Accretion Disks

- UV and soft X-ray spectral energy distributions of AGNs and XRBs
- Double peaked emission lines in CVs and AGNs
- Hard X-ray SED of AGNs and XRBs
- Polarization
- Power density spectra of XRBs and AGNs



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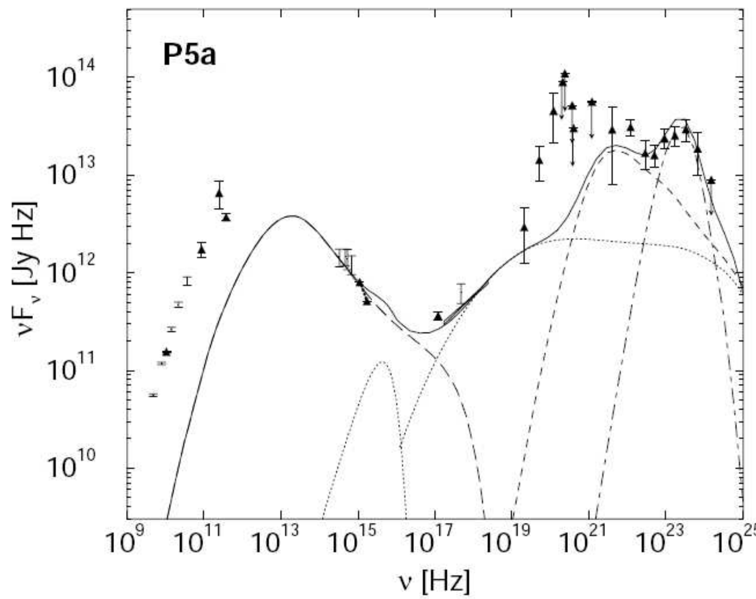

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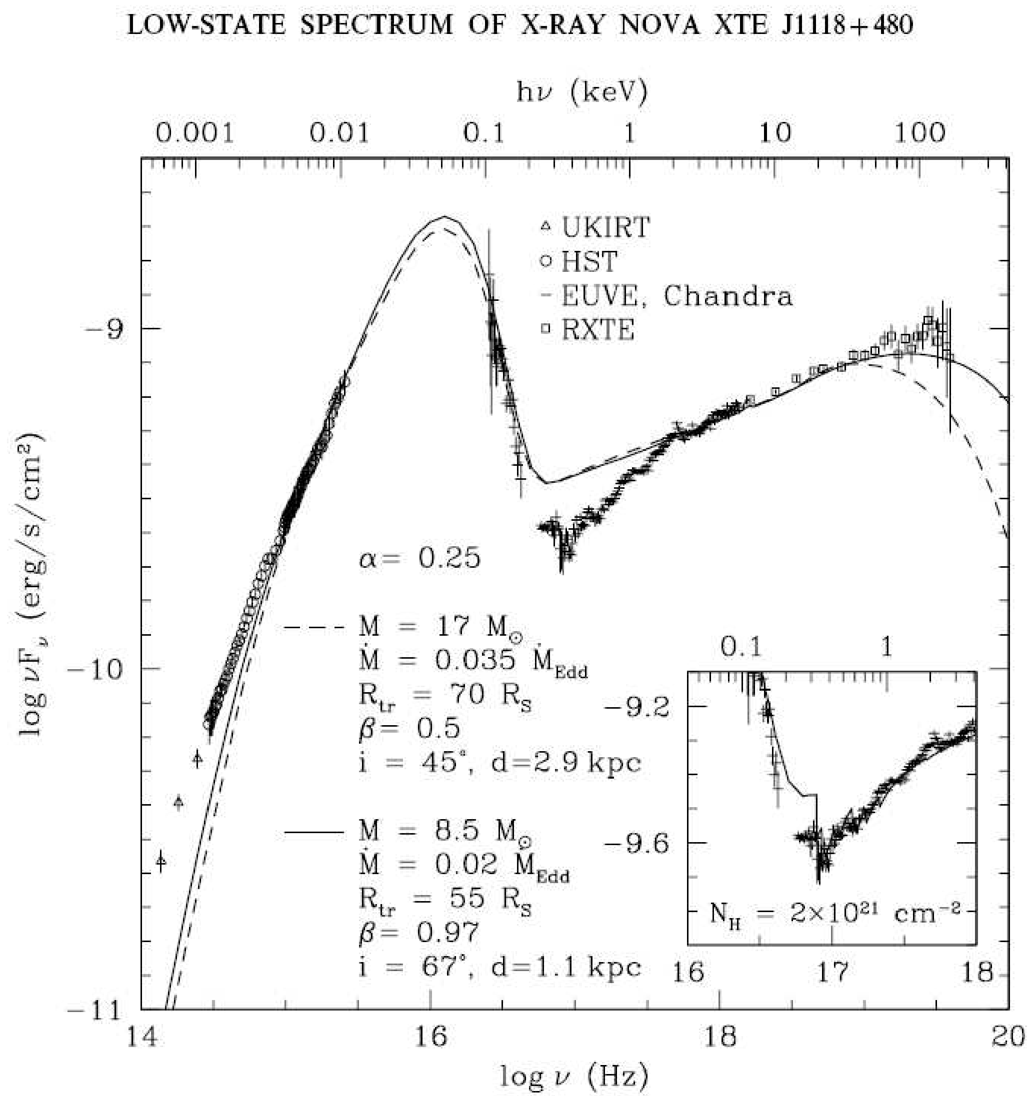

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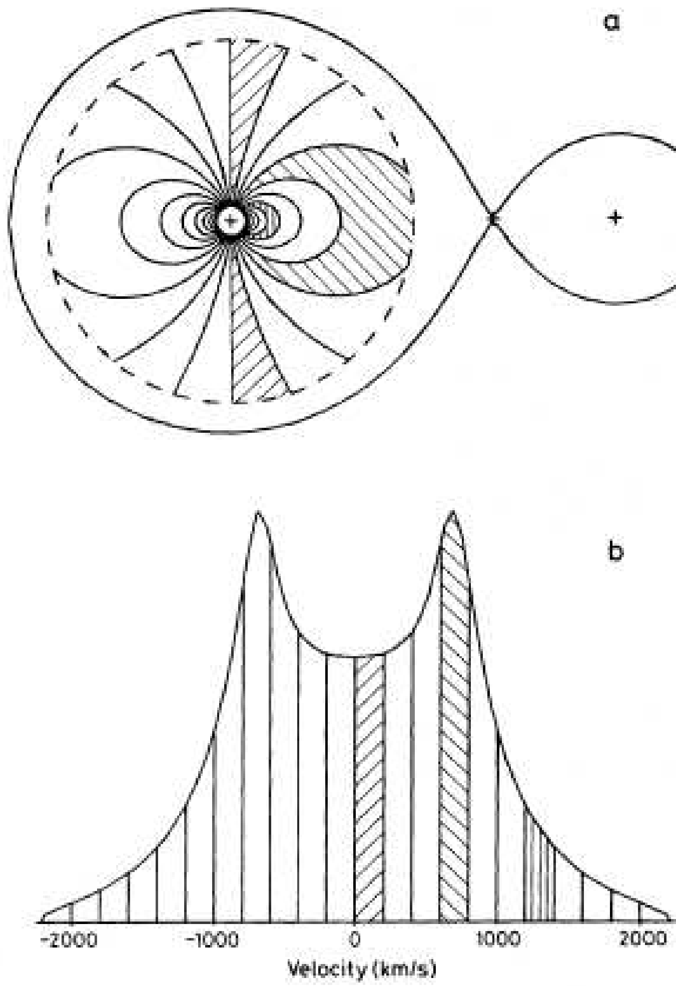

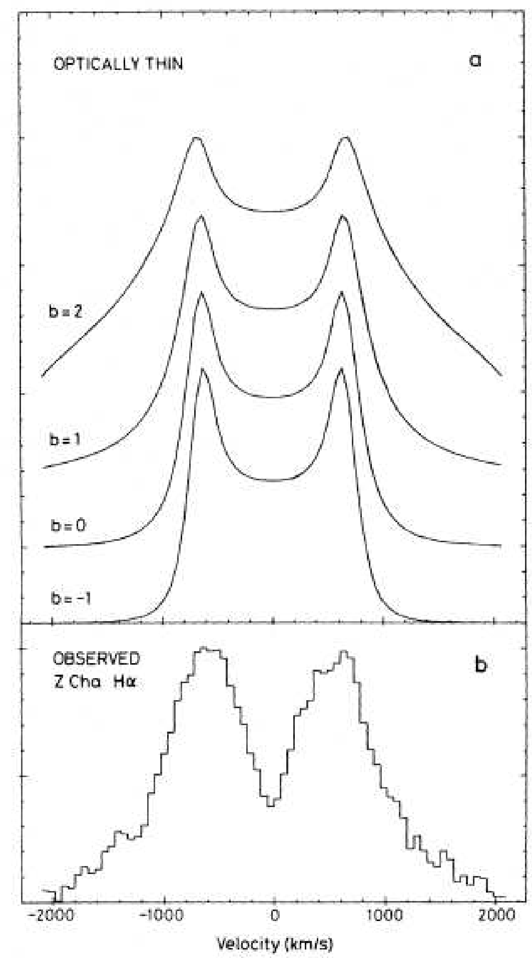

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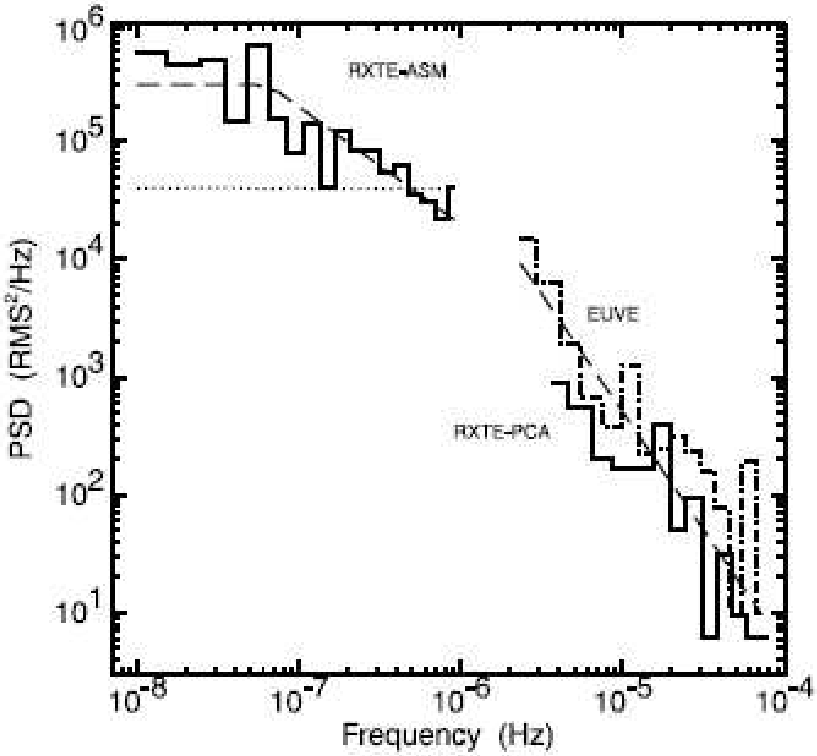

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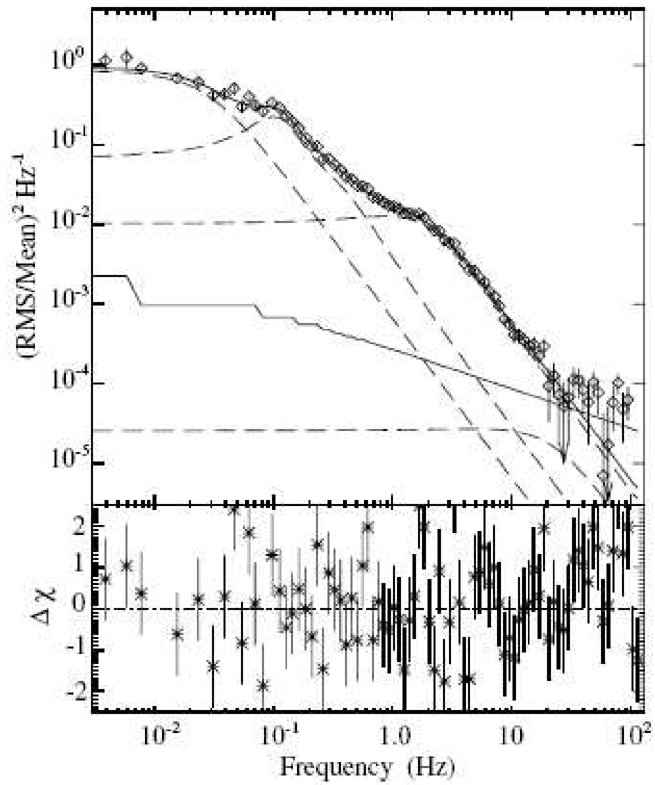

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Thin Disks

Assume the disk is thin, i.e., $h \ll r$. From vertical hydrostatic equilibrium,

$$\frac{1}{\rho}\frac{\partial p}{\partial z} = g_z \tag{1}$$

$$g_z = \frac{z}{r}\frac{GM}{r^2} \sim \frac{h}{r}\frac{GM}{r^2} \tag{2}$$

$$\frac{1}{\rho}\frac{\partial p}{\partial z} \sim \frac{p}{\rho}\frac{1}{h} \sim \frac{c_s^2}{h} \tag{3}$$

For a Keplerian disk, where the radial pressure gradients and the self-gravity of the disk material are negligible, the disk is centrifugally supported against gravity from the central object with mass M:

$$\frac{v_\phi^2}{r} = \frac{GM}{r^2} \tag{4}$$

$$\Rightarrow \frac{c_s}{v_\phi} \sim \frac{h}{r} \sim \frac{1}{\mathcal{M}} \ll 1 \tag{5}$$

Here c_s is the thermal/sound speed of the gas and thus $\mathcal{M}$ is the Mach number of the azimuthal component of disk velocity. Since $c_s \ll v_\phi$, thin disks are *cold* in this sense.

Viscous Shear and Angular Momentum Transport

A Keplerian velocity profile,

$$v_\phi \propto r^{-1/2} \Rightarrow \Omega \propto r^{-3/2}, \tag{6}$$

means that material at smaller radii rotates faster than at larger radii, resulting in a shear stress between adjacent radial layers. The viscous torque is just the integral of the shear stress:

$$\begin{aligned} \mathcal{T} &= 2\pi r^2 \int_{-\infty}^{+\infty} \Pi_{r\phi} dz \qquad (7) \\ &\approx 4\pi r^2 h \Pi_{r\phi} \qquad (8) \end{aligned}$$

The stress is related to the shear strain by the viscosity coefficient, μ,

$$\begin{aligned} \Pi_{r\phi} &= \mu \frac{\partial v_\phi}{\partial r} = -\frac{\mu}{2}\frac{v_\phi}{r} \qquad (9) \\ &= -\frac{\rho\nu}{2}\frac{v_\phi}{r} \qquad (10) \end{aligned}$$

ν ($\sim lv$) is the kinematic viscosity;

$$\mathcal{T} = \pi(2h\rho)\nu r v_\phi = \pi\Sigma\nu r v_\phi \tag{11}$$

$\Sigma = \int \rho dz \approx 2h\rho$ is the disk surface density.

Angular Momentum Transport

In steady state, the viscous torque must balance the angular momentum lost as material flows inwards. The equation of continuity relates the mass accretion rate to the mass inflow velocity:

$$\dot{M} = 2\pi r \Sigma v_r \tag{12}$$

The specific angular momentum is $j = rv_\phi$; hence, the viscous torque must satisfy $\mathcal{T} = j\dot{M}$:

$$\begin{aligned} \mathcal{T} = \pi\Sigma\nu r v_\phi &= 2\pi\Sigma r^2 v_\phi v_r &(13)\\ \Rightarrow \nu &= 2rv_r &(14) \end{aligned}$$

The accretion time scale is then

$$t_{\rm acc} \sim \frac{r}{v_r} \sim \frac{r^2}{\nu} \tag{15}$$

Viscosity Mechanisms

"Microscopic" Viscosity: We'd like to evaluate the accretion time scale for viscosity due to particle interactions, for which we have

$$\nu_{\rm pp} \sim l_{\rm mfp} c_s. \tag{16}$$

In AGNs or XRBs, the disks are ionized, so this sets a lower limit on the thermal speed:

$$c_s \gtrsim (1\,{\rm Ry}/m_p)^{1/2} \tag{17}$$

$$\sim 4\times 10^6\,{\rm cm\,s^{-1}} \tag{18}$$

For a plasma, the mean free path can be estimated by considering "strong encounters" between protons to get an effective cross-sectional radius:

$$\frac{e^2}{r_{\rm eff}} \sim kT \tag{19}$$

$$\Rightarrow l_{\rm mfp} \sim (n\pi r_{\rm eff}^2)^{-1} \sim \frac{(kT)^2}{\pi e^4 n} \tag{20}$$

In order for the disks to be thin, cooling must be efficient; and so they must be optically thick. Therefore, we have as a firm lower limit to the surface density

$$\Sigma > 1/\sigma_T \sim 10^{24}{\rm cm}^{-2}, \tag{21}$$

For XRBs, the Schwarzschild radius is of order

$$r_s \sim 10\,GM_\odot/c^2 \sim 10^6{\rm cm}. \tag{22}$$

Evaluating at $r = 100r_s = 10^8$ cm and using $h \approx rc_s/v_\phi \sim 10^5$ cm, we obtain a lower limit on the density,

$$n \sim \Sigma/h > 10^{19}\mathrm{cm}^{-3} \tag{23}$$

The mean free path and accretion time scale for particle viscosity are then

$$l_{\mathrm{mfp}} > 10^{-4}\,\mathrm{cm} \tag{24}$$

$$\Rightarrow t_{\mathrm{acc,pp}} \sim \frac{(100\,r_s)^2}{\nu_{pp}} \gtrsim 3 \times 10^{13}\mathrm{s} \sim 10^6\mathrm{yr} \tag{25}$$

We see state changes in XRBs such as Cygnus X-1 that occur on time scales of weeks and are probably related to changes in the accretion rate, so a stronger viscosity mechanism must be operating in those disks.

For turbulent viscosity, the maximum eddy size is h and the maximum velocity is c_s. This yields

$$t_{\mathrm{acc,turb}} \sim \frac{(100\,r_s)^2}{(10^5\,\mathrm{cm})(4 \times 10^6\,\mathrm{cm\,s^{-1}})} \sim 3 \times 10^4\mathrm{s}. \tag{26}$$

A key question in accretion disk research is (still): What can drive such turbulence?

Alpha Prescription: The shear stress associated with turbulent motion should be of the form

$$\Pi_{r\phi} = \rho\langle\delta v_r \delta v_\phi\rangle \tag{27}$$

where $\langle\delta v_r \delta v_\phi\rangle < c_s$ for sub-sonic turbulence. Since ρc_s^2 is the thermal energy density of the disk gas, one often expresses the turbulent shear stress as a fraction, α, of the gas pressure:

$$\Pi_{r\phi} = \alpha\rho c_s^2 = \alpha p. \tag{28}$$

where $\alpha \lesssim 1$. Similarly, for a fluctuating magnetic field,

$$\Pi_{r\phi} = \rho\langle\delta B_r \delta B_\phi\rangle \tag{29}$$

With a strongly chaotic field, magnetic field reversals will lead to reconnection that ultimately feeds the thermal energy of the gas, so that it is expected that magnetic shear stresses would also be some fraction of the gas pressure.

Although this is quite qualitative and there is no reason that a single α-parameter pertains everywhere within a disk, the alpha-prescription has allowed for thermal and viscous stability analyses of disk models to proceed, thereby providing a theoretical framework for understanding the time-dependent properties of accretion disk systems.

Magneto-Rotation Instability (MRI) The hope is that the turbulence that contributes to disk viscosity may arise from (magneto)hydrodynamic instabilities.

- Rayleigh criterion for rotational instability. The radial component of acceleration in a disk is

$$a_r\hat{r} = \left[-\frac{GM}{r^2} - \frac{1}{\rho}\frac{\partial p}{\partial r} + r\Omega^2\right]\hat{r} \tag{30}$$

  which is equal to zero in equilibrium. Now consider an outward radial displacement of a ring of material from r_1 to r_2. The specific angular momentum of the displaced material, $j_1 = r_1^2\Omega_1$, will be unchanged at its new location, and the net acceleration on the ring will be

$$a_{r2}\hat{r} = \left[-\frac{GM}{r_2^2} - \frac{1}{\rho}\frac{\partial p}{\partial r}\bigg|_{r_2} + \frac{j_1^2}{r_2^3}\right]\hat{r} \tag{31}$$

$$= \frac{j_1^2 - j_2^2}{r_2^3}\hat{r} \tag{32}$$

  If $j_2^2 < j_1^2$, the acceleration is outwards, in the direction of the original displacement. So the onset of instability requires

$$\frac{\partial j^2}{\partial r} = \frac{\partial}{\partial r}\left[(r^2\Omega)^2\right] < 0 \tag{33}$$

  Keplerian disks are Rayleigh stable:

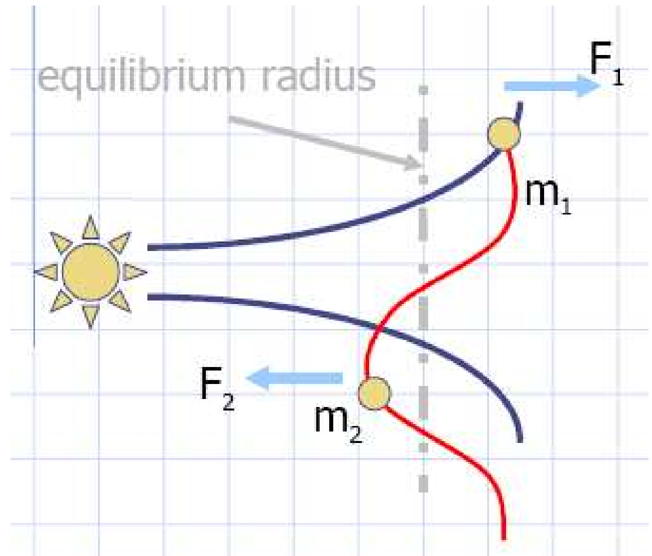
$$\Omega_k \propto r^{-3/2} \tag{34}$$

$$j = r^2\Omega_k \propto r^{1/2} \tag{35}$$

- Chandrasekhar instability criterion. Here the instability criterion is more difficult to derive, but is still rather simple to state:

$$\frac{\partial\Omega^2}{\partial r} < 0 \tag{36}$$

The basic idea is that in a disk with a (weak) poloidal magnetic field, neighboring fluid elements are tethered together such that relative azimuthal displacements tend to grow as a result of the magnetic tension linking the two fluid elements together.



Thin Disk Temperature Profile

For a disk annulus of width dr, the energy content of that ring of material is

$$d\varepsilon = 2\pi r dr \rho \frac{v_\phi^2}{2} \tag{37}$$

$$= dM \frac{v_\phi^2}{2} \tag{38}$$

for high Mach number flows. The local rate of change of this energy is

$$d\dot{\varepsilon} = \dot{M}\frac{v_\phi^2}{2} \tag{39}$$

Combining this with $v_\phi^2/2 = GM/r$ and mass continuity, $\dot{M} = 2\pi r \Sigma v_r$, we obtain the energy flux that must be lost at radius r:

$$F = \frac{d\dot{\varepsilon}}{dA} = \frac{1}{2\pi r}\frac{d}{dr}\left(\dot{M}\frac{GM}{2r}\right) \tag{40}$$

$$= \frac{GM\dot{M}}{2\pi r^3} \tag{41}$$

Assuming the disk emits locally as a black body, we have

$$2\sigma_B T_{\rm eff}^4 = \frac{GM\dot{M}}{2\pi r^3} \tag{42}$$

$$\Rightarrow \quad T_{\rm eff} = \left(\frac{GM\dot{M}}{4\pi\sigma_B}\right)^{1/4} r^{-3/4} \qquad \text{(incorrect)} \tag{43}$$

Multitemperature Disk SED

Disk luminosity:

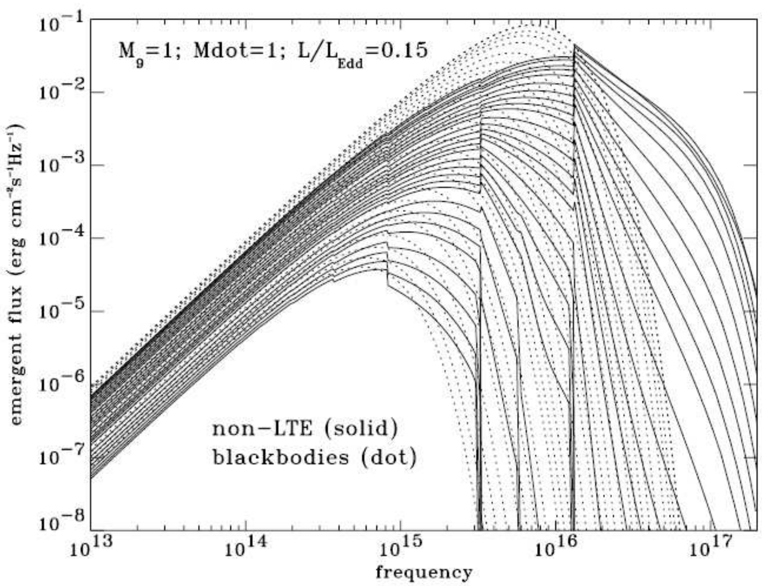
$$L_\nu = \int 2\pi r dr B_\nu(r) \tag{44}$$

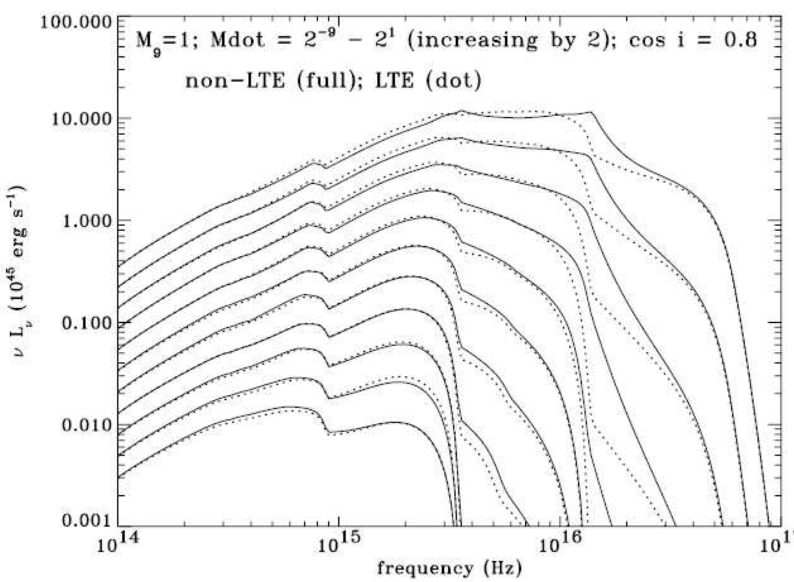
Approximate the Planck spectrum as a δ-function:

$$\begin{aligned} B_\nu &\sim \sigma_B T^4(r)\delta(\tilde{\nu}(r) - \nu) & (45)\\ &= \sigma_B T^4 \frac{\delta(\tilde{\nu}(r) - \nu)}{|d\tilde{\nu}/dr|} & (46)\end{aligned}$$

Use Wien displacement law:

$$\begin{aligned} \tilde{\nu}(r) &= cT/0.3 = (cT_0/0.3)r^{-3/4} & (47)\\ |d\tilde{\nu}/dr| &= \frac{3}{4}\frac{cT_0}{0.3}r^{-7/4} = \frac{3}{4}\frac{cT}{0.3}r^{-1} & (48)\\ \Rightarrow L_\nu &= \int 2\pi r dr \frac{\sigma_B T^4(r)}{(3/4)(cT/0.3)r^{-1}} & (49)\\ &= 0.3\frac{8\pi}{3c}\sigma_B r^2 T^3 \propto \nu^{-8/3}\nu^3 & (50)\\ &\propto \nu^{1/3} & (51)\\ \nu L_\nu &\propto \nu^{4/3} & (52)\end{aligned}$$





Hubeny, I., et al. 2000, ApJ, 533, 710.

Advection Dominated Accretion Flows (ADAFs)

Motivation:

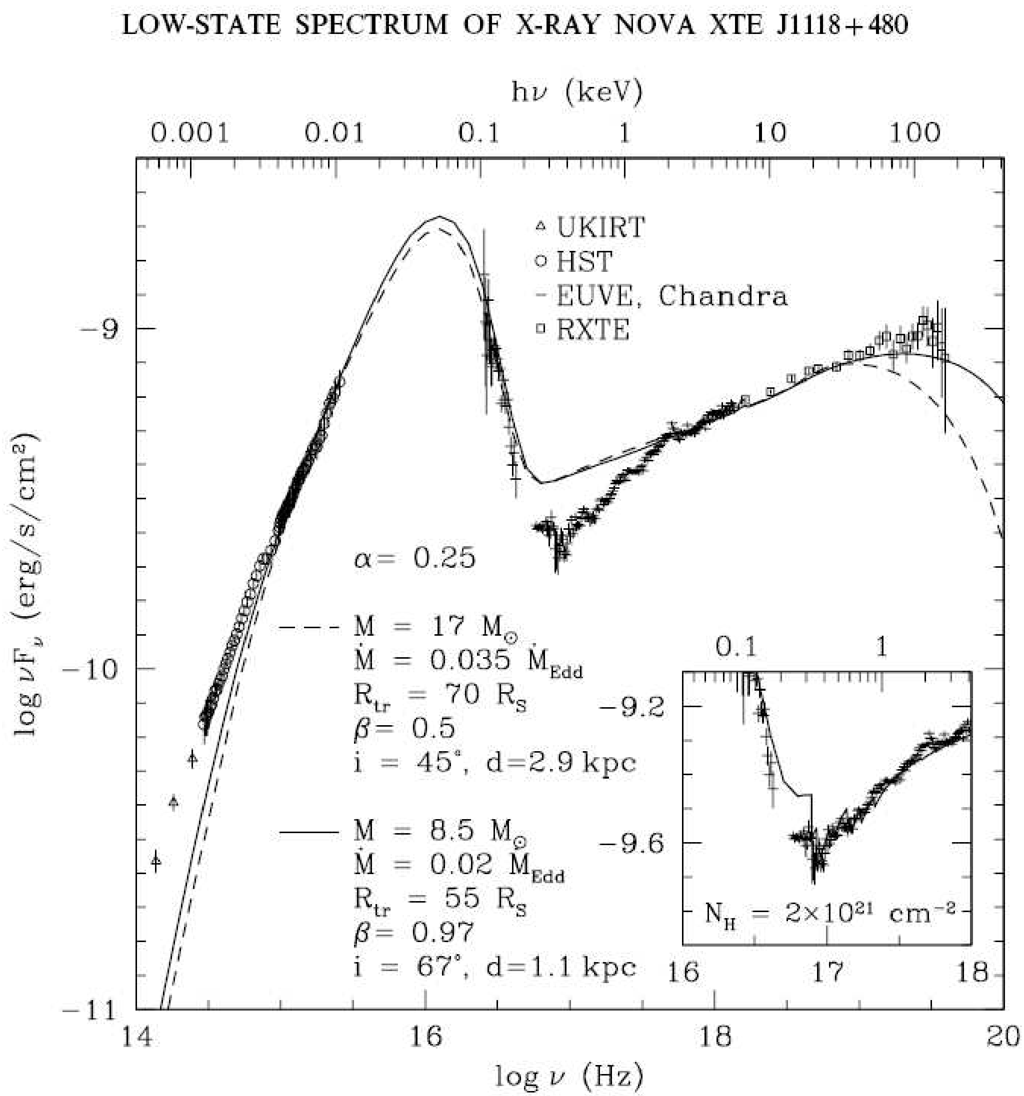
- The standard thin disk model is thermally unstable for optically thin emission:

$$\frac{dQ^-}{dT_c} < \frac{dQ^+}{dT_c} \quad \Rightarrow \quad \frac{d\log(\Lambda/\alpha)}{d\log T_c} < 3/2 \tag{53}$$

- Hard X-ray continua in XRBs and Seyferts require the *same* high temperature, optically thin gas ($T_e \sim 10^9$K). However, recall $T_{\rm eff} \propto (M\dot{M}/r^3)^{1/4}$.
- Some sources, e.g., the Galactic center, have anomalously low luminosities.

ADAFs seem to solve all of these problems (Narayan & Yi 1994, 1995):

- The bulk of the viscously dissipated energy is carried by the ions and advected with the accretion flow rather than radiated away.
- The flow comprises a two-temperature plasma: the ions have virial temperature $T_i \sim 10^{12}/(r/r_s)$K, and so the disk is thick, $h/r \sim c_s/v_\phi \sim 1$. The electrons have temperature 10^9K and cool via inverse Compton scattering.
- Since most of the accretion energy is advected rather than emitted, ADAFs around black holes can have much lower luminosities than the accretion rate would imply if there were a radiatively efficient thin disk.



Esin, A., et al. 2000, ApJ, 532, 1069.

Topics for Future Talks

- Numerical simulations of disks
- Hard X-ray SEDs and Thermal Comptonization
- More on Advection-dominated flows — ADIOS models, etc.
- Connections between Disks and Jets — Blandford-Payne, Blandford-Znajek mechanisms
- ...