Outline:
1) Pulsars: observational overview
2) Pulsar electrodynamics: theory
3) Application to observations: minding the gaps
4) Formation of magnetospheres: experimental theory
5) Conclusions
The Discovery...

Jocelyn Bell & Tony Hewish discover a periodic extra-terrestrial signal of 1.337 s at position:
RA 19:19:36
DEC +21:47:16
Pulsars are neutron stars, born in supernova explosions.
**Extreme gravity: almost black holes**

**Extreme matter:**

- 10x nuclear density, nucleon degenerate (neutrons+protons+electrons+?)
- $B \sim B_{\text{QED}} = 4.4 \times 10^{13}$ Gauss
- Voltage drops $\sim 10^{16}$ volts
- $F_{\text{EM}} = 10^9 F_g$
- Superfluid and superconductor inside

**Extreme speeds: relativistic plasma physics in action**

**Probes of turbulent interstellar medium**

**Precision tools**
Pulsars: cosmic lighthouses

- Sweeping dipole magnetic field
- Emit from radio to gamma ray
- Spin periods -- from 1.5 ms to 8 sec
- Individual pulses quite different, but average profile is very stable (geometry)
Pulsars: radio pulse shape

- Hollow cone model
- Hollow cone+core component
Pulsars: spectrum

Broad-band spectra

- Power peaked in γ-rays
- No pulsed emission detected above 20 GeV
- High-energy turnover
- Increase in hardness with age
Pulsar emission

- Spindown: wind+wave
- Equator-pole potential difference ($10^{15}$V for Crab)
- Charge extraction from the surface (E field >> gravity)
- Corotating zone
- Expect relativistic motion
- Pair formation -- pair-dominated plasma?
- Instability -- collective radiation

$$\phi_0 = \Omega B a^2 / c$$
All pulsars lose rotational energy and slow down

**Spindown age:**

\[ \tau = \frac{P}{2\dot{P}} \]

**Surface magnetic field**

\[ B = 3.2 \times 10^{19} \sqrt{P\dot{P}} \text{ G} \]

**Typical value** $10^{12}$G
Most of the spindown energy of a pulsar is sent out in the form of a wind. Pulsar wind nebulae provide the box calorimeters for pulsar spindown.

**Properties of pulsar winds:**
- Highly relativistic ($\gamma \sim 10^6$)
- Kinetic energy dominated at the nebula ($\sigma \sim 10^{-3}$)
- Pole-equator asymmetry and collimation

*How do they do this?*
Unipolar Induction: rotating magnetized conductors

- Alfven (1939), aka Faraday wheel
- Rule of thumb: \( V \sim \Omega \Phi; \) \( P \sim V^2 / Z_0 \)
- Crab Pulsar
  - \( B \sim 10^{12} \, \text{G}, \quad \Omega \sim 200 \, \text{rad s}^{-1}, \quad R \sim 10 \, \text{km} \)
  - Voltage \( \sim 3 \times 10^{16} \, \text{V}; \) \( I \sim 3 \times 10^{14} \, \text{A}; \) \( P \sim 10^{31} \, \text{W} \)
- Magnetar
  - \( B \sim 10^{14} \, \text{G}; \) \( P \sim 10^{37} \, \text{W} \)
- Massive Black Hole in AGN
  - \( B \sim 10^4 \, \text{G}; \) \( P \sim 10^{39} \, \text{W} \)
- GRB
  - \( B \sim 10^{16} \, \text{G}; \) \( P \sim 10^{42} \, \text{W} \)

EM energy density \( \gg \) particle energy density

Energy is extracted electromagnetically: Poynting flux
Pulsar basics: spindown

Pulsar = magnetized rotating conducting sphere (w/plasma)

\[ \dot{\varepsilon}_{\text{rot}} = I \Omega \dot{\Omega} = \frac{10^{31} I_{45} \dot{P}_{15}}{P^3} \text{ erg/s} \]

Vacuum spindown (Pacini 68, Ostriker&Gunn 69) order of mag.:

1) Rotation of B field:
\[ E \sim \frac{\Omega r}{c} B \]

2) Light cylinder (wave zone):
\[ R_L = c / \Omega \]

3) Sweepback:
\[ B_\phi (R_L) \approx B_p (R_L) \]

4) Radial Poynting flux:
\[ \frac{1}{4\pi} (\bar{E} \times \bar{B})_r = \frac{1}{4\pi} E_p B_\phi \]

5) Energy loss:
\[ \dot{\varepsilon}_{\text{rot}} \sim 4\pi R_L^2 \frac{c}{4\pi} [(\bar{E} \times \bar{B})_r]_{r=R_L} \approx -\Omega R_L^3 B^2 (R_L) \]

6) Dipole:
\[ B \propto \mu / r^3 \]
\[ \dot{\varepsilon}_{\text{rot}} \approx -\frac{\mu^2 \Omega^4}{c^3} f(\chi) \]

If energy loss is Poynting dominated a dipole with plasma will spindown similarly (but with different \( f(\chi) \))
Features of GJ ('69) picture:

- Corotating magnetosphere
  \[ \vec{E} = -\frac{\vec{v}}{c} \times \vec{B} = -\frac{\vec{\Omega}}{c} \times \vec{R} \times \vec{B} \]
  \[ \frac{1}{4\pi} \nabla \cdot \vec{E} = \rho_{GJ} = -\frac{\vec{\Omega} \cdot \vec{B}}{2\pi c} \]

- Charge-separated flow

- Field distorted by particle currents

- Energy loss -- Poynting \( \vec{B}_\phi \times \vec{E}_\theta \) also = current x voltage. No need for obliquity for spindown!

- Open circuit

- Null surface -- Holloway’s (73) paradox. Can’t fill closed zone w/right charge. Pair creation unlikely there. Gap/curr. closure? Is GJ picture viable?
Force-free electrodynamics
(Scharleman ‘73, Michel ‘73):

• Magnetosphere filled with plasma
  so \( \vec{E} \cdot \vec{B} = 0 \)

• Inertia is low, only EM forces are important.

Determine field structure of axisymmetric rotator:

Steady state Maxwell equations:

\[
\rho = \frac{1}{4\pi} \nabla \cdot \vec{E} \quad \vec{j} = \frac{c}{4\pi} \nabla \times \vec{B}
\]

Ideal MHD:

\[
\vec{E} = -\frac{\vec{v}}{c} \times \vec{B} = \frac{\Omega R}{c} \vec{B}_{pol} \times \hat{\phi}
\]

Flux function and poloidal current:

\[
\vec{B} = \nabla \Psi \times \nabla \phi + I(\Psi) \nabla \phi
\]

Force-free:

\[
\rho \vec{E} + \vec{j} \times \vec{B} = 0
\]
**Global Structure: Pulsar equation**

**Force-free electrodynamics**
(Sharleman ‘73, Michel ‘73):

- Magnetosphere filled with plasma so \( \vec{E} \cdot \vec{B} = 0 \)

- Inertia is low, only EM forces are important.

Determine field structure of axisymmetric rotator:

\[
\rho = \frac{1}{4\pi} \nabla \cdot \vec{E} \quad \vec{j} = \frac{c}{4\pi} \nabla \times \vec{B}
\]

Ideal MHD:

\[
\vec{E} = -\frac{\vec{v}}{c} \times \vec{B} = \frac{\Omega R}{c} \vec{B}_{pol} \times \hat{\phi}
\]

Flux function and poloidal current:

\[
\vec{B} = \nabla \Psi \times \nabla \phi + I(\Psi) \nabla \phi
\]

Force-free:

\[
\rho\vec{E} + \vec{j} \times \vec{B} = 0
\]

**Steady state Maxwell equations:**
Cross-field balance: relativistic Grad-Shafranov equation

Second order elliptic nonlinear PDE

Singular surface at $x=1$ (regularity condition)

Poloidal field structure is determined by current enclosed by flux surfaces. $I(\Psi)$ is an unknown function of an unknown function!!!

To solve it requires a guess for the current.
Properties of CKF solution:

Smooth at the light cylinder

E<B everywhere (force free ok)

\[ \Psi_{open} = 1.36 \Psi_{pc} \]

Distributed return current over 20% of the polar cap. Bulk of return current (>90%) is in the current sheet.

No significant centrifugal acceleration inside LC: sigma problem is in the wind.

Asymptotically monopolar field -- implies acceleration of the wind
Extrapolation to oblique rotator pulsar magnetosphere: time-dependent solution
Plasma supply: pair production

Polar cap is a space-charge limited accelerator. Accelerated primary particles radiate CR, and pair produce in the strong field. Pair cascade shorts out E*B. \( \gamma_{\text{primary}} \sim 10^7 \), \( \gamma_{\text{secondary}} \sim 10^{2-3} \)

Acceleration is driven by departures from GJ charge density

\[
\nabla \cdot \vec{E} = 4\pi (\rho - \rho_{GJ})
\]

Arons & Scharleman 79

Pair formation through CR (in young pulsars) or nonresonant inverse Compton scattering on thermal photons (most pulsars). Multiplicities due to ICS are smaller \( (10^2) \) than from CR. Hibschmann & Arons 01
Plasma supply: outer gaps

Outer gaps form at the intersection of \( \Omega \cdot \vec{B} = 0 \) surface and the closed zone (Cheng, Ho, Ruderman'86). Vacuum forms if the current is charge-separated (Holloway 73). Pair production, acceleration and gamma-ray emission result. Outer gaps disappear for large pair outflow.

Advantage -- geometry of high-energy emission. However, basic electrodynamics of how these gaps operate is still not well constrained.
**Plasma supply: slot gaps**

- **Pair-free zone near last open field-line**
  - Slower acceleration
  - Pair formation front at higher altitude
  - Slot gap forms between conducting walls
- **$E_{\parallel}$ acceleration is not screened**

![Diagram of plasma supply and slot gaps](image-url)
**Plasma supply: slot gaps**

- **Pair-free zone near last open field-line**

- Slower acceleration

- Pair formation front at higher altitude

- Slot gap forms between conducting walls

- $E_{\parallel}$ acceleration is not screened
**Fundamental unsolved problem:**

What happens to a strongly magnetized rotating conducting sphere with no work function?

Does it form a magnetosphere and/or a wind? If so, what are its properties?

**Strategy:** investigate qualitative behavior using numerical simulations.

**Particle-in-cell method:**

- Collect currents at the cell centers
- Find fields on the mesh (Maxwell’s eqs)
- Interpolate fields to particles positions
- Move particles under Lorentz force

Can handle vacuum gaps, counterstreaming, space-charge flows
How does plasma know about spin of the star?

Induced quadrupole + monopole

\[ E_r = \frac{2}{3} \phi_0 \frac{a}{r^2} + \phi_0 \frac{a^3}{r^4} (1 - 3\cos^2 \theta) \]

\[ E_\theta = -\phi_0 \frac{a^3}{r^4} \sin 2\theta \quad \phi_0 = \Omega B a^2 / c \]

Extraction of opposite signs of charge on poles and equator.

Vacuum field contains central charge and surface charge.

Behavior of charges outside the conductor is governed by the \( E \cdot B = 0 \) surfaces.

Trapping regions
• Non-neutral configuration: dome+disk solution
• Plasma-filled $E \cdot B = 0$ surface, shearing flow.
• Vacuum gaps. Similar to Michel et al ‘85,’01

Simulation comes to equilibrium where no more charge is emitted. No net wind!

Is GJ picture really wrong?
- Stationary solution -- emission stops. Stable to pair production in gaps
- Ion overdensity at 1.5 R -- differential rotation. Essential to have E*B=0.
- Field lines that are not filled with plasma to the star -- rotate differentially
- Dome in corotation at GJ density. Fieldlines shorted to the star.

Is the aligned charge-separated rotator dead?
Can plasma fill the magnetosphere?
Not if it is unable to spread across the field lines!
Diocotron instability:

Particle dynamics is ExB drift.

Wavebreaking in the shearing flow similar to Kelvin-Helmholtz instability.

Azimuthal charge perturbation leads to radial ExB drift.

Typical unstable mode is a multiple of rotation frequency

\[ \text{(diocotron frequency} = \frac{\omega_p^2}{2\omega_c} \) \]

Grows in radius due to injection of new plasma from the surface
Implications for GJ model

If the closed zone cannot be supplied with GJ charge density from the star, the plasma near the star loses corotation and becomes unstable to diocotron instability which transports the charge to return magnetosphere to corotation. Holloway’s paradox resolved! (important even for pair production)

Nonaxisymmetry is important even for aligned rotators!

Dead aligned pulsars may still spin down with wind!
Global structure of the magnetosphere is beginning to be understood through computer simulations.

Reconciling global structure with local charge supply is a new theoretical frontier. The gaps have to react to global stresses in the magnetosphere to provide the current required. This opens the opportunity to study time-variable phenomena such as drifting subpulses, microstructures, etc.

Goldreich-Julian corotating magnetosphere is a dynamical consequence of the induced electric fields and plasma reaction in the dipole geometry.

Charge adjustment in the closed zone is carried out via diocotron instability. Transport across magnetic field lines is possible even if the plasma is strongly magnetized.

Modeling in full 3D is essential even for aligned rotators.

Interaction of the return current with the gaps is crucial to what GLAST will see. We will have a direct diagnostic of the return current.