Gideon Alexander
Erez Reinherz-Aronis
Tel-Aviv University
February 21, 2006

Addendum to our E166 asymmetry analysis:

A more bounded $\Delta(Asymmetry)$ option

In our analysis of the E166 September 2005 run, as described in our note from December 19, 2005 (here refereed to as Note1), we extracted the positron asymmetries and estimated their polarisation while adopting a rather conservative approach to the error calculations. As defined in Note1, the positron asymmetry is given by:

$$Asy \;=\; \frac{<(Signal - Bg)_{-60}> \;-\; <(Signal - Bg)_{+60}>}{<(Signal - Bg)_{-60}> \;+\; <(Signal - Bg)_{+60}>} \;=\; \frac{L \;-\; R}{L \;+\; R} \tag{1}$$

where for convenience we denoted

$$L \;= <(Signal - Bg)_{-60}> \quad \text{and} \quad R \;= <(Signal - Bg)_{+60}> \;\; .$$

The error associated with the Asy is then:

$$\Delta Asy \;=\; \frac{2}{(L+R)^2}\sqrt{(L\Delta R)^2 + (R\Delta L)^2}\;. \tag{2}$$

Now, assuming a Gaussian error, the $\Delta L/L$ and $\Delta R/R$ are strictly speaking equal to

$$\frac{\Delta L}{L} = \frac{\sigma_L}{L}\frac{1}{\sqrt{N_L}} \quad \text{and} \quad \frac{\Delta R}{R} = \frac{\sigma_R}{R}\frac{1}{\sqrt{N_R}}$$

where σ_L and σ_R are the variance square roots of the corresponding L and R distributions. Throughout our analysis described in Note1 we have tried to avoid the use of σ_L and σ_R since the experimental distributions were fitted rather poorly by the Gaussian shape. Therefore we did set $\sigma_L/L \;=\; \sigma_R/R \;=\; 1$ to obtain the asymmetry error expression

$$\Delta Asy \;=\; \frac{2LR}{(L+R)^2}\sqrt{1/N_L + 1/N_R}\;. \tag{3}$$

which yields rather conservative values since in our case one has $\frac{\sigma_L}{L} \;\leq\; 1$ and $\frac{\sigma_R}{R} \;\leq\; 1$. An option for a more bounded error can be obtained if one adopts an approximate value for σ_L and σ_R through the relation

$$\sigma^2 \approx s^2 \;=\; \frac{1}{N-1}\sum_i^N (x_i - \bar{x})^2$$

where x_i are the individual measurements and $\bar{x}$ is their average (see e.g. Barlow's Statistics book, Chapters 5.2.2 and 7.3). In this case the error on the asymmetry is given by

$$\Delta Asy \;\approx\; \frac{2LR}{(L+R)^2}\sqrt{\left[\frac{s_L}{L}\right]^2\frac{1}{N_L} + \left[\frac{s_R}{R}\right]^2\frac{1}{N_R}} \tag{4}$$

Following this procedure we recalculated the errors on the asymmetry values for all the cases given in the Tables of our Note1 and found that the bounded errors are lower typically by a factor two to three in comparison to those estimated in the conservative approach. In Table 1 of the present note we compare the conservative asymmetry errors (in the rectangular brackets) with the more bounded values (bold faced) using the identical conditions of Table 8 in our Note1. Using the asymmetry results given in Table 1 and the analysing power presented in the E166 proposal, we plot in Fig. 1 the calculated polarisation of the positrons as a function of their energy where they are compared to the expectations shown in the proposal.

e^+ Energy [MeV]	Spect. current [A]	Asymmetry	Δ(Asymmetry)	
3.65	100	0.0050	[0.0015]	**0.0012**
4.38	120	0.0088	[0.0016]	**0.0007**
5.11	140	0.0091	[0.0012]	**0.0004**
5.84	160	0.0100	[0.0017]	**0.0007**
6.57	180	0.0089	[0.0017]	**0.0009**
e^- Energy [MeV]	Spect. current [A]	Asymmetry	Δ(Asymmetry)	
5.84	160	0.0145	[0.0018]	**0.0004**

Table 1: The positron asymmetry as a function of the energy using a signal − background with a gain 1, 15 bits applying a 1.5σ cut around the maximum counts. The data is not divided by the tor6130 value. Two values are given for the error on the asymmetry. The conservative errors are given in the rectangular brackets whereas the bold faced errors are those calculated by setting $s^2 = \sigma^2$ (see text).

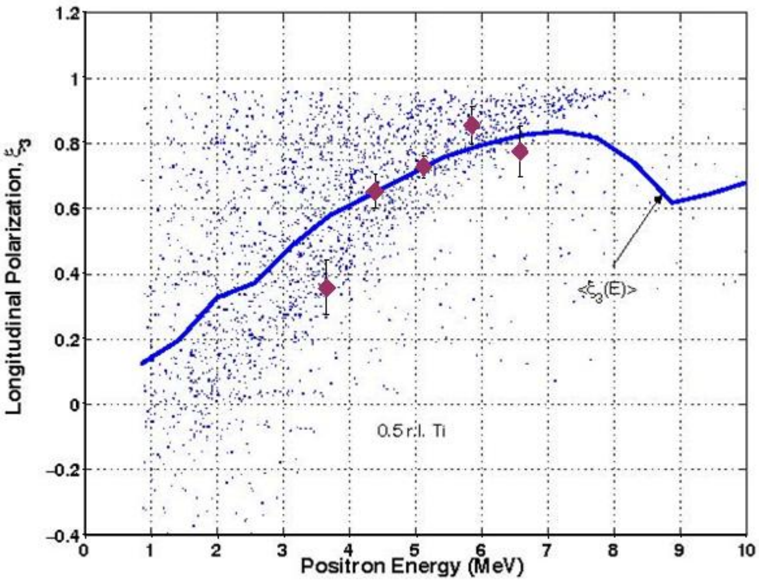

Figure 1: The measured positron polarisation values (using gain 1, high sensitivity data) with their bounded statistical errors as a function of their energy superimposed on the expectation (solid line) given in the E166 experiment proposal.