Background Estimates from Primakov π^0 Production

RPC Technical Note #19

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Abstract

Simulations of the E160 detector system have not taken into account π^0 production from the Primakov effect. However, reasonable estimates of Primakov rates show that they will not be a significant background for E160.

1 Introduction

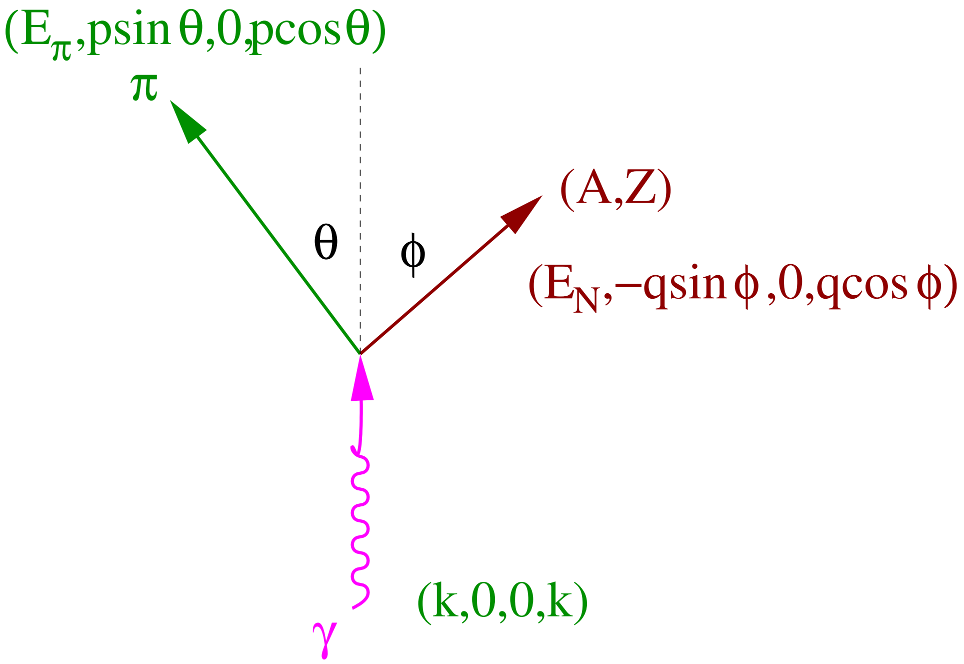

Figure 1: Kinematics and four-vectors for the Primakov effect.

Large numbers of neutral pions can be emitted at small angles in an experiment with real photons due to the Primakov[1] effect. In this case, the photon scatters from the Coulomb field of a nucleus and produces a π^0 in the final state. The real and virtual photons fuse to form the

π^0 in a process reversed from $\pi^0 \to \gamma\gamma$ decay. For E160 and other experiments with a real photon beam, we wish to know the backgrounds created by these Primakov photons.

The Primakov cross section is given by[2]

$$\frac{d\sigma_P}{d\Omega} = \Gamma_{\gamma\gamma}\frac{8\alpha Z^2}{m_\pi^3}\frac{\beta^3 k^4}{q^4}|F_{em}(q)|^2 \sin^2\theta \tag{1}$$

in which $\Gamma_{\gamma\gamma}$ is the decay width for $\pi^0 \to \gamma\gamma$, α is the fine structure constant, Z is the nuclear charge, m_π is the pion mass, β is the pion velocity, k is the incident photon energy, θ is the laboratory angle of pion emission, q is the three-momentum transfer to the nucleus, and F_{em} is the nuclear electromagnetic form factor corrected for absorption of pions. Since Primakov pions are produced peripherally, the absorption factor in the form factor can be neglected[2]. In general, pion production is a sum of Primakov, nuclear coherent and incoherent production, together with interference. However, for small angles, the Primakov effect dominates by 2 to 3 orders of magnitude (see Figs. 3-4 in Ref. [3]).

2 Formalism

In order to evaluate Eq. 1 one needs to work out the kinematics of the process. Fig. 1 shows the 4-vectors which yield three equations for energy and momentum conservation:

$$\begin{aligned} k + M_N &= E_N + E_\pi & (2)\\ k &= p\cos\theta + q\cos\phi & (3)\\ 0 &= p\sin\theta - q\sin\phi & (4) \end{aligned}$$

in which M_N is the mass of the nucleus, p is the momentum of the pion, $E_\pi = \sqrt{m_\pi^2 + p^2}$ is the energy of the pion, $E_N = \sqrt{M_N^2 + q^2}$ is the energy of the recoiling nucleus, and ϕ is the recoil angle of the nucleus. From Eqs. 3 and 4 we find that

$$q^2 = q^2\sin^2\phi + q^2\cos^2\phi = k^2 - 2kp\cos\theta + p^2 \tag{5}$$

and from Eq. 2 we find that

$$(k + M_N - E_\pi)^2 - M_N^2 = q^2 \tag{6}$$

Combining Eqs. 5 and 6 to eliminate q^2 yields

$$(k + M_N)E_\pi = \frac{1}{2}m_\pi^2 + kM_N + kp\cos\theta \tag{7}$$

Squaring this and collecting terms yields

$$\begin{aligned} ap^2 + bp + c &= 0 & (8)\\ a &= (k + M_N)^2 - k^2\cos^2\theta & (9)\\ b &= -k\cos\theta(m_\pi^2 + 2kM_N) & (10)\\ c &= (k + M_N)^2 m_\pi^2 - \frac{1}{4}m_\pi^4 - k^2 M_N^2 - kM_N m_\pi^2 & (11) \end{aligned}$$

which can be solved for p.

parameter	value
α	1/137
m_π	0.134 GeV
$\Gamma_{\gamma\gamma}$	7.73×10^{-9} GeV[3]
$F_{em}(q)$	$3[\sin qR + qR\cos qR]/(qR)^3$[4]
R	$r_0 A^{1/3}$ with $r_0 = 1.3$ fm[5]
$\hbar c$	0.197 GeV-fm

Table 1: Parameters for the Primakov formula.

3 Calculation

The perl script ~`griff/jpsi/primakov/primakov.pl` calculates the kinematics and cross sections for the Primakov effect. The input parameters are shown in Table 1

Engelbrecht[4] claims that the features of the Primakov cross section at fairly small angles are insensitive to the form factor F_{em} and that the uniform density model of the nucleus is a good enough approximation under the circumstances.

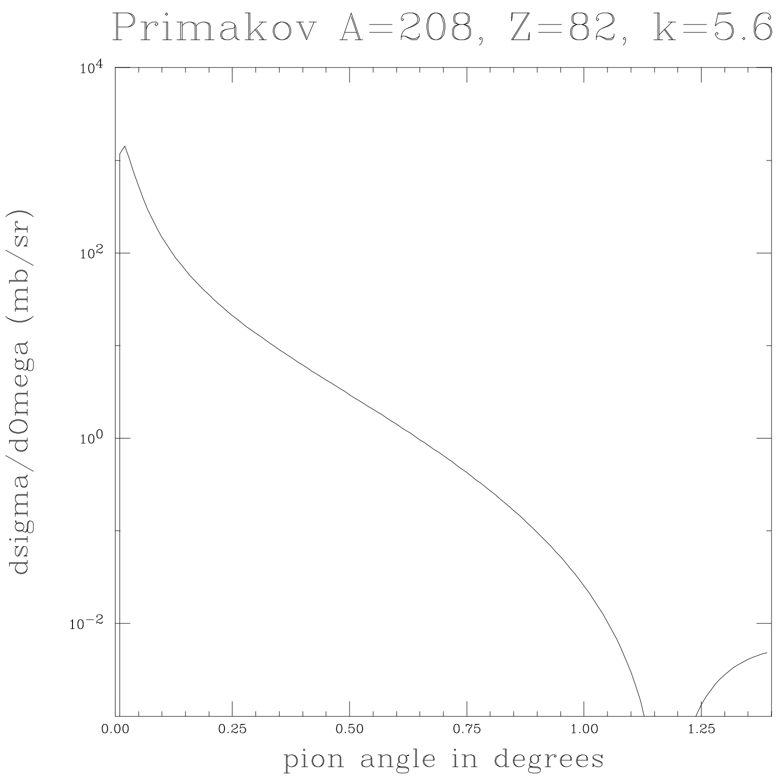

Figure 2: Primakov cross sections for $k = 5.6$ GeV photons.

4 Results

Fig. 2 shows the calculation of the Primakov cross section for the Jefferson Lab beam energy of 5.6 GeV. This is compared with Fig. 4 in Ref. [3] to test the current program for errors. The first minimum in the cross section can be pushed around by varying r_0, but in the region of large cross section `primakov.pl` agrees with the JLab calculations.

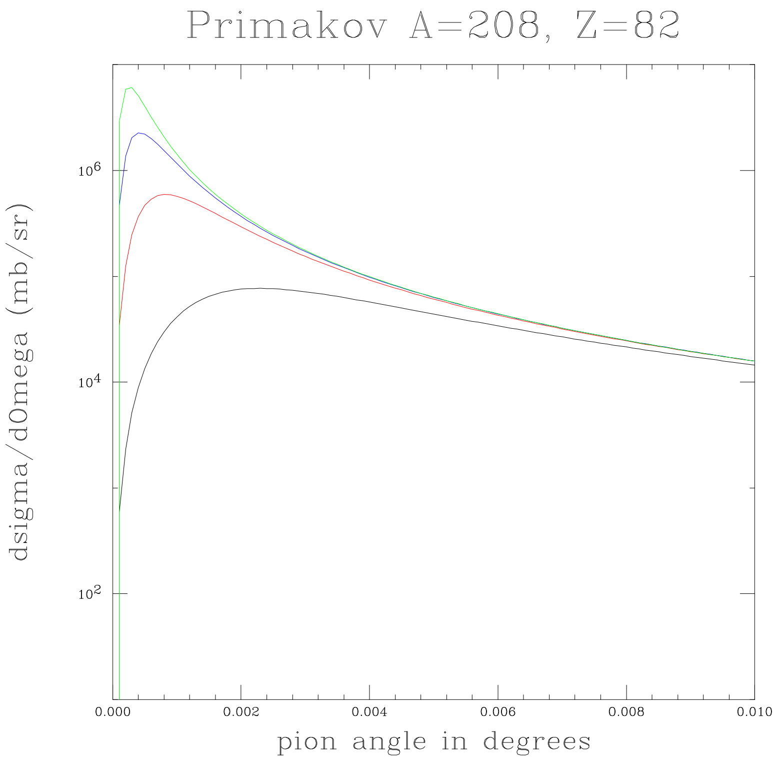

Figure 3: Primakov cross sections on lead for 15 (lowest curve), 25, 35 and 45 (highest curve) GeV photon energies.

Fig. 3 shows the Primakov cross sections for $^{208}_{82}$Pb at beam energies of 15, 25, 35 and 45 GeV. The peak position is approximately at $\theta = m_\pi^2/2k^2$[2].

Fig. 4 shows the dependence of the Primakov cross section on the target materials Be, Al, Cu, Sn, Au and Pb. The rapid rise of the cross section with heavy nuclei comes from the Z^2-dependence of the cross section. Therefore, our worst problems will come from a lead target.

We can numerically integrate the Primakov cross section to obtain total π_0 rates from the effect. At incident photon energies above 15 GeV, the peak angle is less than 0.002°, so we can essentially expect the π_0's to travel down the beam pipe. Table 2 lists the integrated Primakov cross sections for various materials and beam energies, together with the estimated rates for E160.

5 Conclusions

Primakov pions will not produce significant backgrounds for E160.

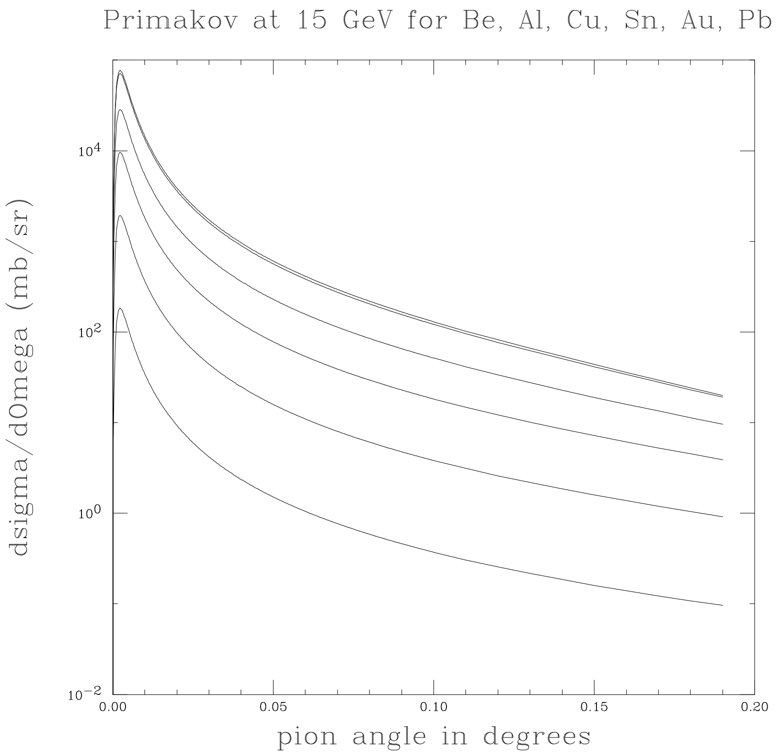

Figure 4: Dependence of the Primakov cross section for $k = 15$ GeV on the different E160 target materials. Curves from the bottom to the top correspond to Be, Al, Cu, Sn, Au and Pb.

References

[1] H. Primakov, Phys. Rev. **31** (1951) 899.

[2] G. Bellettini, *et al.*, Il Nuovo Cimento **66** (1970) 243.

[3] Jefferson Lab Experiment E-99-014, D. Dale, S. Danagoulian, A. Gasparian, R. Miskimen, *et al.*, "A Precision Measurement of the Neutral Pion Lifetime via the Primakoff Effect."

[4] C. Engelbrecht, Phys. Rev. **133** (1964) B988.

[5] R. Herman and R Hofstadter, *High-Energy Electron Scattering Tables*, (Stanford U. Press, Stanford, 1960) pp62–63.

photon energy k (GeV)	target material	Primakov cross section σ_P (μb)	π^0 rate per spill
15	Al	0.3	3
25	Al	0.4	4
35	Al	0.4	4
45	Al	0.4	4
15	Cu	1.6	16
25	Cu	1.8	18
35	Cu	1.9	19
45	Cu	2.0	20
15	Sn	4.5	45
25	Sn	5.1	51
35	Sn	5.5	55
45	Sn	5.8	58
15	Pb	11.6	116
25	Pb	13.1	131
35	Pb	14.1	141
45	Pb	14.9	149

Table 2: Primakov cross sections σ_P integrated over solid angle. Rates per spill for the experiment are calculated assuming 10^7 incident photons per spill and 1 b^{-1} targets.