Can a Compton Polarimeter Measure Linear Photon Polarization?

RPC Technical Note #16

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1 Summary

SLAC experiments E161 and E159 will measure the circular polarization of incident photons by way of Compton polarimetry. The question has arisen whether the polarimeter designed for these two experiments is capable of determining the linear polarization of photons used in the E160 experiment. Because the linear polarization asymmetries are less than 10^{-4} for photon energies of 15, 25, and 35 GeV, a measurement of the linear polarization with a Compton polarimeter would be extremely difficult and time-consuming.

2 Formalism

Tolhoek[1] has worked out the formalism for Compton scattering for an electron at rest in all of the possible polarization configurations. The differential scattering cross section is given by

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2m_e^2}\Phi(\mathbf{k_0}, \mathbf{k}, \boldsymbol{\xi^0}, \boldsymbol{\xi}, \boldsymbol{\zeta^0}, \boldsymbol{\zeta}) \tag{1}$$

in which α/m_e is the classical electron radius, $\alpha = 1.44$ MeV-fm is the fine structure constant, $m_e = 0.511$ MeV is the electron mass, $\mathbf{k_0}$ ($\mathbf{k}$) is the incident (final) photon momentum, $\boldsymbol{\xi^0}$ ($\boldsymbol{\xi}$) is the incident (final) photon polarization vector, and $\boldsymbol{\zeta^0}$ ($\boldsymbol{\zeta}$) is the incident (final) electron polarization vector. The function Φ can be expressed as

$$\Phi = \sum_{i=0}^{4} \Phi_i \tag{2}$$

in which Φ_i itself is a sum of functions that depend on each combination of i polarization vectors.

The polarization variables are such that for electrons $\boldsymbol{\zeta}$ points in the direction of the electron spin and $||\boldsymbol{\zeta}|| = P_e$ is the polarization of the electron ensemble. In the case of photons, each component represents a different polarization state: ξ_1 measures the linear polarization in the scattering plane, ξ_2 measures the linear polarization normal to the scattering plane, and ξ_3 measures the circular polarization. For left-handed polarization $\xi_3 > 0$. The 1 and 2-directions are chosen such that the unit vectors parallel to the vector potentials are

$$\mathbf{e_1^0} = \mathbf{e_1} = (\mathbf{k_0} \times \mathbf{k})/|\mathbf{k_0} \times \mathbf{k}| \tag{3}$$

and

$$\mathbf{e_2^0} = (\hat{\mathbf{k}}_\mathbf{0} \times \mathbf{e_1^0}); \qquad \mathbf{e_2} = (\hat{\mathbf{k}} \times \mathbf{e_1}) \tag{4}$$

in which $\hat{\mathbf{k}}_\mathbf{0}$ and $\hat{\mathbf{k}}$ are the unit vectors in the direction of the incident and final photons.

For the cases of observing up to two of the polarization states, Tolhoek has worked out the functions in the cross section (for units $\hbar = 1, c = 1, m_e = 1$):

$$\begin{aligned}
\Phi_0 &= \frac{1}{8}[(1+\cos^2\theta) + (k_0 - k)(1-\cos\theta)] && (5)\\
\Phi_1(\boldsymbol{\xi}^\mathbf{0}) &= \frac{1}{8}\xi_1^0 \sin^2\theta && (6)\\
\Phi_1(\boldsymbol{\xi}) &= \frac{1}{8}\xi_1 \sin^2\theta && (7)\\
\Phi_1(\boldsymbol{\zeta}^\mathbf{0}) &= 0 && (8)\\
\Phi_1(\boldsymbol{\zeta}) &= 0 && (9)\\
\Phi_2(\boldsymbol{\xi}^\mathbf{0}, \boldsymbol{\xi}) &= \frac{1}{8}[(1+\cos^2\theta)\xi_1^0\xi_1 + 2\cos\theta(\xi_2^0\xi_2 + \xi_3^0\xi_3) + (k_0-k)\cos\theta(1-\cos\theta)\xi_3^0\xi_3] && (10)\\
\Phi_2(\boldsymbol{\xi}^\mathbf{0}, \boldsymbol{\zeta}^\mathbf{0}) &= -\frac{1}{8}\xi_3^0(1-\cos\theta)\boldsymbol{\zeta}^\mathbf{0}\cdot(\mathbf{k_0}\cos\theta + \mathbf{k}) && (11)\\
\Phi_2(\boldsymbol{\xi}, \boldsymbol{\zeta}^\mathbf{0}) &= -\frac{1}{8}\xi_3(1-\cos\theta)\boldsymbol{\zeta}^\mathbf{0}\cdot(\mathbf{k_0} + \mathbf{k}\cos\theta) && (12)\\
\Phi_2(\boldsymbol{\xi}^\mathbf{0}, \boldsymbol{\zeta}) &= -\frac{1}{8}\xi_3^0(1-\cos\theta)[\boldsymbol{\zeta}\cdot(\mathbf{k_0}\cos\theta + \mathbf{k}) - \frac{(1+\cos\theta)(k_0+k)}{(k_0-k+2)}\boldsymbol{\zeta}\cdot(\mathbf{k_0}-\mathbf{k})] && (13)\\
\Phi_2(\boldsymbol{\xi}, \boldsymbol{\zeta}) &= -\frac{1}{8}\xi_3(1-\cos\theta)[\boldsymbol{\zeta}\cdot(\mathbf{k_0}\cos\theta + \mathbf{k}) - \frac{(1+\cos\theta)(k_0+k)}{(k_0-k+2)}\boldsymbol{\zeta}\cdot(\mathbf{k_0}-\mathbf{k})] && (14)
\end{aligned}$$

and

$$\begin{aligned}
\Phi_2(\boldsymbol{\zeta}^\mathbf{0}, \boldsymbol{\zeta}) = & -\frac{1}{8}\big\{(\boldsymbol{\zeta}^\mathbf{0}\cdot\boldsymbol{\zeta})[(1+\cos^2\theta) + \frac{1}{2}(k_0-k)\sin^2\theta] \\
& - \frac{1}{2}(k_0-k)[\boldsymbol{\zeta}^\mathbf{0}\cdot(\hat{\mathbf{k}}_\mathbf{0}+\hat{\mathbf{k}})\boldsymbol{\zeta}\cdot(\hat{\mathbf{k}}_\mathbf{0}+\hat{\mathbf{k}}) + \boldsymbol{\zeta}^\mathbf{0}\cdot(\hat{\mathbf{k}}_\mathbf{0}\times\hat{\mathbf{k}})\boldsymbol{\zeta}\cdot(\hat{\mathbf{k}}_\mathbf{0}\times\hat{\mathbf{k}})] \\
& + \frac{1}{2}(k_0+k)(1+\cos\theta)(\boldsymbol{\zeta}^\mathbf{0}\times\boldsymbol{\zeta})\cdot(\hat{\mathbf{k}}_\mathbf{0}\times\hat{\mathbf{k}}) \\
& + \frac{(1+\cos\theta)(k_0-k)}{(k_0-k+2)}\boldsymbol{\zeta}\cdot(\mathbf{k_0}-\mathbf{k})\boldsymbol{\zeta}^\mathbf{0}(\hat{\mathbf{k}}_\mathbf{0}+\hat{\mathbf{k}}) \\
& - \frac{(1+2\cos\theta-\cos^2\theta)}{(k_0-k+2)}\boldsymbol{\zeta}\cdot(\mathbf{k_0}-\mathbf{k})\boldsymbol{\zeta}^\mathbf{0}(\mathbf{k_0}-\mathbf{k})\big\}
\end{aligned} \tag{15}$$

Each of these functions is odd; *e.g.* $\Phi(-\boldsymbol{\xi}) = -\Phi(\boldsymbol{\xi})$. Since $-\boldsymbol{\xi}$ is the opposite polarization of $\boldsymbol{\xi}$, the sum $\Phi(-\boldsymbol{\xi}) + \Phi(\boldsymbol{\xi}) = 0$ corresponds to the unpolarized case. Therefore, only the Φ functions with arguments of the *observed* polarization vectors show up in the cross section. In practice one should average over the unobserved final polarization states and sum over the unobserved initial polarization states.

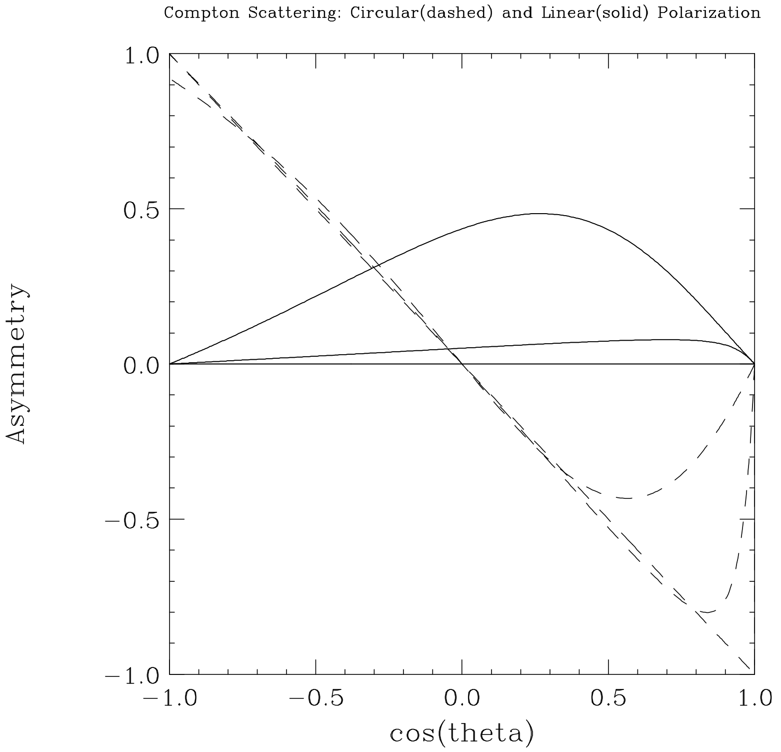

Figure 1: Compton asymmetries for linear (solid) and circular (dashed) polarization as a function of $\cos\theta$ for 1, 10 and 15000 MeV photons. In the limit of high energy, the curves become respectively a flat and a diagonal straight line.

2.1 Example

Let's consider the example $d\sigma(\boldsymbol{\xi^0}, \boldsymbol{\zeta^0})/d\Omega$ for which we know the initial electron and photon polarizations. Then

$$\begin{aligned}
\frac{d\sigma}{d\Omega} &= \frac{\alpha^2}{2m_e^2}\left(\frac{k}{k_0}\right)^2 \Big[\Phi(\boldsymbol{\xi^0}, \boldsymbol{\xi}, \boldsymbol{\zeta^0}, \boldsymbol{\zeta}) \\
&\quad + \Phi(\boldsymbol{\xi^0}, -\boldsymbol{\xi}, \boldsymbol{\zeta^0}, \boldsymbol{\zeta}) + \Phi(\boldsymbol{\xi^0}, \boldsymbol{\xi}, \boldsymbol{\zeta^0}, -\boldsymbol{\zeta}) + \Phi(\boldsymbol{\xi^0}, -\boldsymbol{\xi}, \boldsymbol{\zeta^0}, -\boldsymbol{\zeta})\Big] \\
&= \frac{2\alpha^2}{m_e^2}\left(\frac{k}{k_0}\right)^2 \Big[\Phi_0 + \Phi_1(\boldsymbol{\xi^0}) + \Phi_1(\boldsymbol{\zeta^0}) + \Phi_2(\boldsymbol{\xi^0}, \boldsymbol{\zeta^0})\Big] \qquad (16)
\end{aligned}$$

Plugging in the Φ's yields (modulo a factor of two that I don't understand)

$$\begin{aligned}
\frac{d\sigma(\boldsymbol{\xi^0}, \boldsymbol{\zeta^0})}{d\Omega} &= \frac{\alpha^2}{2m_e^2}\left(\frac{k}{k_0}\right)^2 \big[(1+\cos^2\theta) + (k_0 - k)(1-\cos\theta) \\
&\quad + \xi_1^0 \sin^2\theta - \xi_3^0(1-\cos\theta)\boldsymbol{\zeta^0}\cdot(\mathbf{k_0}\cos\theta + \mathbf{k})\big] \qquad (17)
\end{aligned}$$

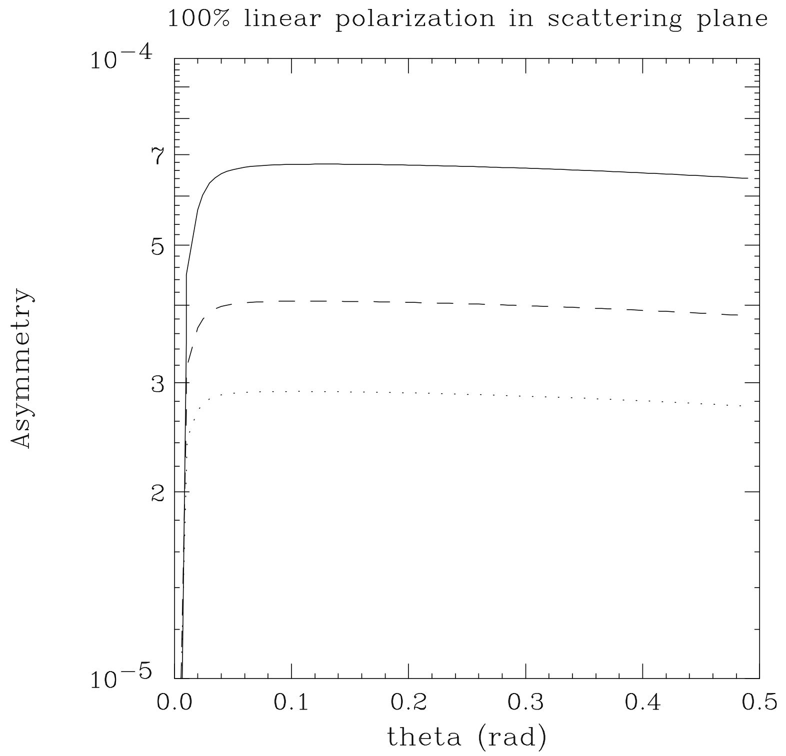

Figure 2: Linear polarization asymmetries for $k_0 = 15$ (solid), 25 (dashed) and 35 (dotted) GeV.

3 Results

One can easily show the following two identities:

$$\begin{aligned} \frac{k}{k_0} &= 1 + k_0(1 - \cos\theta)/m_e \qquad \text{and} \\ \frac{k_0}{k} &= 1 - k(1 - \cos\theta)/m_e \end{aligned} \tag{18}$$

The cross section for Compton scattering of linearly polarized photons with their polarization vector in the plane of scattering from Eq. 17 is then

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2m_e^2}\left(\frac{k}{k_0}\right)^2\left[\frac{k}{k_0} + \frac{k_0}{k} - \sin^2\theta + P_\gamma \sin^2\theta\right] \tag{19}$$

in which k_0 is the incident photon energy, k is the scattered photon energy, θ is the lab scattering angle, and P_γ is the photon polarization. There is no term in this expression that depends on linear polarization along an axis perpendicular to the scattering plane. By contrast, the equivalent cross section for circularly polarized photons and longitudinally polarized electrons is

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2m_e^2}\left(\frac{k}{k_0}\right)^2\left[\frac{k}{k_0} + \frac{k_0}{k} - \sin^2\theta + P_\gamma P_e \cos\theta(1 - \cos\theta)\frac{(k + k_0)}{m_e}\right] \tag{20}$$

in which P_e is the electron polarization.

We can form asymmetries as a ratio of the cross section difference for opposite polarizations divided by the cross section sum. In this case, the asymmetry for 100% linear polarization is

$$A_{\mathrm{lin}} = \frac{\sin^2\theta}{\frac{k}{k_0} + \frac{k_0}{k} - \sin^2\theta} \tag{21}$$

and the asymmetry for 100% circular polarization and 100% electron polarization is

$$A_{\mathrm{circ}} = \frac{\cos\theta(1-\cos\theta)\frac{(k+k_0)}{m_e}}{\frac{k}{k_0} + \frac{k_0}{k} - \sin^2\theta} \tag{22}$$

Fig. 1 shows the asymmetries A_{lin} (solid lines) and A_{circ} (dashed lines) for incident energies from 1 MeV to 15 GeV. Clearly, A_{lin} becomes small rapidly as the beam energy increases. Fig. 2 shows A_{lin} for beam energies appropriate to E160. The asymmetries in this case are well below 10^{-4} and decrease with beam energy. These small asymmetries make measuring linear polarization with a Compton polarimeter practically impossible.

References

[1] H.A. Tolhoek, Rev. Mod. Phys. **28**(1956)277.