PROPOSAL TO MEASURE THE 
A-DEPENDENCE OF $J/\psi$ AND $\psi'$ 
PHOTOPRODUCTION 
E160

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Representing the E160 Collaboration

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• What do we know about $J/\psi$ and $\psi'$ production and their interactions with hadrons? 
• What can SLAC contribute?  
• Details of the E160 experiment.
E160 Collaboration

- University of California Los Angeles
- Jefferson Lab
- University of Massachusetts
- DAPNIA-Saclay
- Institut für Kernphysik Mainz
- Los Alamos National Laboratory
- Mississippi State University
- Old Dominion University
- Ruhr-Universität Bochum
- Smith College
- Stanford Linear Accelerator Center
- University of Virginia
- College of William and Mary
- Yerevan Physics Institute
• Charmonium production: perturbative interactions, non-perturbative formation.

• Cross sections: perturbative $\rightarrow$ kinematic dependence; non-perturbative $\rightarrow$ uncalculable magnitude.

• No direct evidence for correct perturbative diagrams (e.g. photoproduction explained by both color singlet and octet models).

• Charmonium-nucleon interaction cross sections are not well known or understood.

• Yet charmonium production is used to probe gluon distributions and hot nuclear matter.
Estimates of $\sigma_{\text{tot}}^{\psi N}$ using vector meson dominance are small.

Estimates from the $A$-dependence of $J/\psi$ hadroproduction cluster around 6 mb.

Estimates from the $A$-dependence of $\psi'$ hadroproduction are between 7 and 30 mb.

Estimate from the sole extraction from $A$-dependent photoproduction measurements is about 4 mb.
Coherence and Formation Lengths

- Coherence length $\ell_c$
- Formation length $\ell_F$

nuclear radius
Length Scales

- Coherence Length: lifetime of a $c\bar{c}$ fluctuation
  \[ \ell_c \leq \frac{2E_{J/\psi}}{m_{J/\psi}^2} = 0.04 E_{J/\psi} \text{ fm} \]

- Formation Length: time of evolution from point-like perturbative interaction to full on-shell $J/\psi$.
  \[ \ell_F \approx \frac{2E_{J/\psi}}{m_{J/\psi}^2 - m_{J/\psi}^2} = 0.1 E_{J/\psi} \text{ fm} \]

- Nuclear Radius: \( R = 1.2A^{\frac{1}{3}} \text{ fm} = 4 - 7 \text{ fm} \)

- Internucleon Spacing: \( \langle r \rangle = \left( \frac{4\pi R^3}{A} \right)^{\frac{1}{3}} = 1.9 \text{ fm} \)
Advantages of Photoproduction

- Photons do not have multiple interactions with the nucleons in a nucleus.
- Elastic $\psi$ production is relatively simple.
- Photons do not appreciably excite the 1P charmonium states $\chi_c$, which radiatively decay into $J/\psi$’s.
- Both $\ell_c$ and $\ell_F$ change with $E_\gamma$. 
Charm Photoproduction

- Representative data
Paradox of Vector Meson Dominance

- \( \sigma_{\text{tot}}^{\psi N} \propto \frac{d\sigma}{dt}(\gamma N \rightarrow \psi N)|_{t=0} \) by the optical theorem.

- This results in \( \sigma_{\text{tot}}^{\psi N} \leq 1 \text{ mb} \) which is much less than the 3-10 mb values obtained from \( A \)-dependence studies.

- A virtual meson of mass \( m \) must have a transverse size \( \sim \frac{1}{m} \) by uncertainty-principle arguments.

- Interaction cross sections scale as the square of the transverse size. Therefore,

\[
\frac{\sigma_{\text{tot}}^{\psi N}}{\sigma_{\text{tot}}^{\psi'}} = \left( \frac{m_{J/\psi}}{m_{\psi'}} \right)^2 \approx 0.7
\]

- By the same scaling argument,

\[
\sigma_{\text{tot}}^{\psi N} = \left( \frac{r_{J/\psi}}{r_\pi} \right)^2 \sigma_{\text{tot}}^{\pi N} = 2 - 5 \text{ mb}
\]

- \( r_\pi \approx 0.65 \text{ fm}; \sigma_{\text{tot}}^{\pi N} \approx 25 \text{ mb} \).

- \( r_{J/\psi} \approx 0.2 - 0.3 \text{ fm} \).

- \( r_{\psi'} \approx 2r_{J/\psi} \).

- \( \sigma_{\text{tot}}^{\psi N} / \sigma_{\text{tot}}^{\psi N} \approx 4 \).
Why Might VMD Fail?

- The fluctuation of a photon might not always be a vector meson.
- Frankfurt and Strikman: Color transparency insures that the pre-meson is of small size at the time of interaction with the nucleon.
- Hufner and Kopeliovich: The $c\bar{c}$ fluctuation has a wavefunction $|c\bar{c}\rangle = \alpha_1|J/\psi\rangle + \alpha_2|\psi'\rangle + ...$
- Ducati and Mariotto: Elastic $J/\psi$ photoproduction could originate from color octet states rather than pomeron exchange.

How can one study the problem?

- E160 uses nuclei of various sizes to probe the $J/\psi$ at a range of times during its formation.
A-Dependence of $J/\psi$ Production

- Comparisons to Drell-Yan data indicate that shadowing does not explain the suppression of $J/\psi$'s in nuclei.
- Data are fit to a universal form $A^\alpha$, which is roughly the same for $J/\psi$ and $\psi'$.
- If the error bars on the FNAL data are realistic, the $A$-dependence of $J/\psi$ production is not a power law.
- $\sigma^{pA\rightarrow\psi}/A\sigma^{pN\rightarrow\psi} = \exp(-L\rho_0\sigma^\psi_{\text{tot}})$ with $L \propto A^{1/3}$ fits approximately to a power law in $A$ as long as $\sigma^\psi_{\text{tot}}$ is constant.
- FNAL E866/NuSea Collaboration, Phys. Rev. Lett. 84(00)3256.

- $\psi'$ has stronger $A$-dependence than $J/\psi$ at $x_F \approx 0$, suggesting that $\sigma_{\psi'N}^{A} > \sigma_{\psi N}^{A}$.

- $J/\psi$ and $\psi'$ have similar, but enhanced $A$-dependencies at large $x_F$, suggesting that $\sigma_{\psi'N}^{A} = \sigma_{\psi N}^{A}$.

- $\alpha$ is small for large $x_F$ and also for small $p_T^2$. 
Glauber Approximation for $\sigma_{\text{tot}}^{\psi N}$

- Data as shown in B. Müller, nucl-th/9806023
- 
  \[
  \frac{\sigma^{pA \rightarrow \psi}}{A\sigma^{pN \rightarrow \psi}} = e^{-L\rho_0\sigma_{\text{tot}}^{\psi N}}.
  \]

- $L$: length of the absorption trajectory.
- $\rho_0$: the nuclear density.
- $\sigma_{\text{tot}}^{\psi N}$ is assumed to be constant.
- Procedure works for nucleus-nucleus collisions.
Glauber Approximation (con’t)

- Fits to $pA$ data from Hüfner, Kopeliovich and Polleri, hep-ph/0010282
- Open circles: Glauber-model fit.
- Solid circles: with corrections for gluon anti-shadowing.
- Lines: theoretical expectations for color singlet (lower) and octet (upper) absorption mechanisms.
- $\sigma_{\text{tot}}^\psiN$ is clearly not well-known.
*J/ψ* Suppression for Heavy Ions

- From B. Müller, nucl-th/980623:
- It's still unclear whether *J/ψ* is a reliable probe for quark-gluon plasmas.
- Suppression in light targets is the result of the $A^{0.92}$ dependence of the $pA \rightarrow J/ψX$ cross section.
- The color octet model provides a credible mechanism, but there is no direct evidence for it.
- Available evidence points toward a novel suppression in Pb+Pb collisions.
- Color singlet absorption on hadronic comovers does not explain the Pb+Pb data.
Experiment E160

- SLAC can make a significant contribution to the understanding of charmonium production and propagation using a coherent bremsstrahlung photon beam with maximum energy 50 GeV.
- The proposed experiment E160 eliminates the overwhelming hadronic backgrounds with an absorber that only muons can penetrate.
- $J/\psi$’s and $\psi'$’s are reconstructed from $\mu^+\mu^-$ pairs.
- Muon angles and momenta are determined by two dipole magnets and 3 planes of hodoscopes.
E160 Spectrometer

- 29D32 Dipole Magnet
  - 25.4 cm gap; ±36 cm width; ≈ 1 m length.
  - $\int B \cdot dl = 22 \text{ kG-m}$.
- 70D43 LASS Dipole Magnet
  - 102 cm gap; ±45 cm width; 109 cm length.
  - $\int B \cdot dl = 25 \text{ kG-m}$.
  - 4000 A from 2 existing 1.6 MW power supplies.
  - mirror plates removed.
- 2.2 m alumina (Al$_2$O$_3$) absorber with tungsten core.
- 3 planes of scintillator hodoscopes.
E160 Hodoscopes

- 3 planes at 4.2, 5.7 and 7.0 m from target; 1 phototube per plastic scintillator finger.
- 2 X sub-planes with 2/3 overlap of 1.5 cm thick fingers; segmented into top and bottom halves.
- 1 Y sub-plane with 2 cm thick fingers; segmented into left and right halves.

<table>
<thead>
<tr>
<th>Plane</th>
<th>$x$ (cm)</th>
<th>$y$ (cm)</th>
<th>fingers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>±60</td>
<td>±60</td>
<td>300</td>
</tr>
<tr>
<td>2</td>
<td>±55</td>
<td>±80</td>
<td>300</td>
</tr>
<tr>
<td>3</td>
<td>±50</td>
<td>±100</td>
<td>300</td>
</tr>
</tbody>
</table>
E160 Hodoscopes (con’t)

- leading-edge discriminators
- multi-hit TDC’s:
  - 0.5 ns granularity
  - leading-edge triggers only
  - $< 7$ ns input pulse width
  - $\approx 11$ ns dead time
- $\sigma_x = 0.15$ cm; $\sigma_y = 0.6$ cm.
- 100 hits per plane per 500 ns spill.
- muon tracks every 10-30 ns.
- Muons with $p_T^2 < 0.5$ GeV$^2$ rarely trigger all 3 planes.
 Targets

<table>
<thead>
<tr>
<th>element</th>
<th>Z</th>
<th>$A$</th>
<th>$\rho$ (g/cm$^3$)</th>
<th>g/cm$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Be</td>
<td>4</td>
<td>9.012</td>
<td>1.848</td>
<td>3.26</td>
</tr>
<tr>
<td>Al</td>
<td>13</td>
<td>26.980</td>
<td>2.700</td>
<td>3.36</td>
</tr>
<tr>
<td>Cu</td>
<td>29</td>
<td>63.546</td>
<td>8.960</td>
<td>2.57</td>
</tr>
<tr>
<td>Pb</td>
<td>82</td>
<td>207.200</td>
<td>11.350</td>
<td>1.27</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>element</th>
<th>cm rad. lengths</th>
<th>int. lengths</th>
<th>b$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Be</td>
<td>1.764</td>
<td>0.050</td>
<td>0.043</td>
</tr>
<tr>
<td>Al</td>
<td>1.245</td>
<td>0.140</td>
<td>0.032</td>
</tr>
<tr>
<td>Cu</td>
<td>0.287</td>
<td>0.200</td>
<td>0.019</td>
</tr>
<tr>
<td>Pb</td>
<td>0.112</td>
<td>0.200</td>
<td>0.007</td>
</tr>
</tbody>
</table>

- Targets mounted on a movable target ladder
- Thicknesses chosen to obtain up to 2 b$^{-1}$ without exceeding 0.2 radiation length.
**Beam Intensity and Rates**

- Electron beam energies: 26.7, 43.3 and 48.3 GeV.
- Upper: photon intensities (flux times energy).
- Lower: quasi-elastic $J/\psi$ events per day.
- Dashed lines: incoherent radiation.
Acceptance and Resolution Calculations

- Monte Carlo simulation for muons only
- Accounts for:
  - Multiple scattering in absorber and detectors
  - Energy loss in absorber
  - Beam size
  - Target length
  - Quasi-elastic sample of $J/\psi$ and $\psi'$ events
  - $e^{3t}$ distribution with no $A$-dependence
  - $1 + \cos \theta^*$ angular distribution of $\mu^+\mu^-$
  - Bethe-Heitler backgrounds
**Invariant Mass Spectra**

- Invariant mass of $\mu^+\mu^-$ pairs.
- Dashed line: quasi-elastic, elastic and inelastic Bethe-Heitler backgrounds.
- Solid line: total spectrum with $J/\psi$ and $\psi'$ peaks.
- Cuts on each muon: $\theta > 40$ mr; $p > 2.5$ GeV.

<table>
<thead>
<tr>
<th>Peak Photon Energy (GeV)</th>
<th>15.0</th>
<th>25.0</th>
<th>35.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J/\psi$ Mass Resolution (GeV)</td>
<td>0.120</td>
<td>0.127</td>
<td>0.142</td>
</tr>
<tr>
<td>$J/\psi$ Energy Resolution (GeV)</td>
<td>0.54</td>
<td>1.04</td>
<td>1.80</td>
</tr>
<tr>
<td>$J/\psi \ p_T^2$ Resolution (GeV$^2$)</td>
<td>0.09</td>
<td>0.11</td>
<td>0.14</td>
</tr>
<tr>
<td>$J/\psi$ Acceptance</td>
<td>0.07</td>
<td>0.17</td>
<td>0.24</td>
</tr>
<tr>
<td>$\psi'$ Acceptance</td>
<td>0.01</td>
<td>0.16</td>
<td>0.23</td>
</tr>
</tbody>
</table>
Physics Model

\[ \frac{d\sigma}{dt} = \sigma_0 \sum_i A^{\alpha_i} r_i a_i e^{a_i t} \]

\[ \sigma(\gamma N \rightarrow J/\psi N) = (10.3 \pm 0.7 \text{ nb}) \left[ \frac{\sqrt{s}}{10 \text{ GeV}} \right]^{0.80 \pm 0.04} \]

<table>
<thead>
<tr>
<th>( i )</th>
<th>process</th>
<th>( \alpha_i )</th>
<th>( r_i )</th>
<th>( a_i ) (GeV(^{-2}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>coherent</td>
<td>1.4</td>
<td>0.14</td>
<td>33</td>
</tr>
<tr>
<td>2</td>
<td>quasi-elastic</td>
<td>0.94</td>
<td>1</td>
<td>2.0</td>
</tr>
<tr>
<td>3</td>
<td>inelastic</td>
<td>1</td>
<td>0.30</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Curves for Be (dotted), Al (dashed), Cu (dot-dashed) and Pb (solid) and \( E_\gamma = 20 \text{ GeV} \).
Physics Model (con’t)

- Simulated $t$ distribution for $J/\psi$ production
  - counts per GeV$^2$ vs. $t$
  - $t$-resolution of 0.1 GeV$^2$
  - 8000 quasi-elastic events
  - 1000 coherent events (x)
  - 2000 inelastic events (squares)
  - total events shown as diamonds
Elastic $J/\psi$ Kinematics

Elastic $J/Psi$ Kinematics

![Graph showing Elastic $J/\psi$ Kinematics](image)

<table>
<thead>
<tr>
<th>$k$ (GeV)</th>
<th>$s$ (GeV$^2$)</th>
<th>$k^*$ (GeV)</th>
<th>$k'^*$ (GeV)</th>
<th>$E_\psi^*$ (GeV)</th>
<th>$t_{\text{min}}$ (GeV$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>29.0</td>
<td>2.61</td>
<td>1.63</td>
<td>3.50</td>
<td>-0.170</td>
</tr>
<tr>
<td>25</td>
<td>47.8</td>
<td>3.39</td>
<td>2.67</td>
<td>4.09</td>
<td>-0.036</td>
</tr>
<tr>
<td>35</td>
<td>66.5</td>
<td>4.02</td>
<td>3.42</td>
<td>4.61</td>
<td>-0.024</td>
</tr>
</tbody>
</table>

* indicates CM frame

$k$: incident photon lab momentum

$k'$: $\psi$ momentum

$p_T^2$: $\psi$ transverse momentum squared

$s, t$: Mandelstam variables

$E_\psi$: $\psi$ energy
## Conservative Statistical Error Estimates

<table>
<thead>
<tr>
<th>variable</th>
<th>coherent (stat. error)</th>
<th>QE (stat. error)</th>
<th>inelastic (stat. error)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$\pm 1$ GeV$^{-2}$</td>
<td>$\pm 0.04$ GeV$^{-2}$</td>
<td>$\pm 0.2$ GeV$^{-2}$</td>
</tr>
<tr>
<td>$\sigma_{\gamma A \to \psi}$</td>
<td>11%</td>
<td>3%</td>
<td>12%</td>
</tr>
<tr>
<td>$\alpha_{J/\psi}$</td>
<td>$\pm 0.044$</td>
<td>$\pm 0.004$</td>
<td>$\pm 0.017$</td>
</tr>
<tr>
<td>$\alpha_{\psi'}$</td>
<td>$\pm 0.044$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>total $\sigma_{\psi N}$</th>
<th>total $\sigma_{\psi' N}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pm 0.3$ mb</td>
<td>$\pm 3$ mb</td>
</tr>
</tbody>
</table>

- Expected statistical accuracy is based on the simple 3-component model and 10,000 $J/\psi$ events for each target and energy.
- $a$: exponential slope parameter of $t$-distribution
- $\sigma_{\gamma A \to \psi}$: $\psi$ photoproduction cross section
- $\alpha_{J/\psi}$: $A^\alpha$ fit to $\sigma_{\gamma A \to \psi}$
- $\sigma_{\psi N}$: extraction of $\psi N$ total cross section from $A$-dependence of $\sigma_{\gamma A \to \psi}$
Statistical Error Estimates (con’t)

- $\Delta a$: extracted from fits to Monte Carlo data
- $\Delta \sigma_{\gamma A \rightarrow \psi}$ (quasi-elastic and inelastic): obtain $\sigma_{\gamma A \rightarrow \psi}$ from measured counts in $0.3 < -t < 2$ GeV$^2$; use systematics of $a$ to correct for events with $-t < 0.3$ and $-t > 2$.
- $\Delta \sigma_{\gamma A \rightarrow \psi}$ (coherent): QE counts subtracted from $0 < -t < 0.3$ GeV$^2$.
- $\Delta \alpha$: errors on a linear fit to $\sigma_{\gamma A \rightarrow \psi}$ Monte Carlo data for 4 targets.
- $\Delta \sigma^N_{\psi,\text{tot}}$: propagation of errors for the form
  \[
  \frac{\sigma_{\gamma Pb \rightarrow \psi}/207}{\sigma_{\gamma Be \rightarrow \psi}/9} = e^{-(L_{Pb}-L_{Be})\rho_0 \sigma^N_{\psi,\text{tot}}}
  \]
  (with 4 targets errors will be proportionately smaller if $\sigma^N_{\psi,\text{tot}}$ does not change with $\ell_F$).
Expected Quasi-Elastic Results

Measured Slope Parameters for quasi–elastic t–distributions

- Slope of the quasi-elastic t-distribution extracted from photoproduction experiments (square points):
  - 11 GeV, Cornell75, Gittelman, et al.
  - 55 GeV, FNAL76, Nash, et al.
  - 90 GeV, CERN NA14, Barate, et al.
  - 850-32400 GeV, ZEUS97, Breitweg, et al.
  - 360-43300 GeV, H1-00, Aldoff, et al.
- Estimated errors for E160 at 15, 25, and 35 GeV photon beam energies (dots).
\( \alpha \) from Photoproduction

Photoproduction

- Photoproduction data from SLAC at 17 GeV and FNAL at 120 GeV
- Estimated errors for E160
Error Estimates on $\sigma_{\psi N}^{\psi N}$

Total Cross Sections

[Graph showing data points and error bars for different production mechanisms and estimations of cross sections as a function of $E_{\psi}$ (GeV).]
Request

- 2 weeks of checkout at low rates
- 2 months of production running at 120 Hz
- Resources to obtain scintillator material for hodoscopes
- Use of the LASS and 29D36 dipole magnets
- Resources to obtain several hundred phototubes for hodoscopes
- Upgrade to the data acquisition system to handle 16kB per spill
- Commissioning of a coherent bremsstrahlung beam line for ESA
**Request (con’t)**

<table>
<thead>
<tr>
<th></th>
<th>26.7</th>
<th>43.3</th>
<th>48.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron Energy (GeV)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electron Current (10^{10}/spill)</td>
<td>2.0</td>
<td>1.2</td>
<td>1.0</td>
</tr>
<tr>
<td>Peak Photon Energy (GeV)</td>
<td>15.0</td>
<td>25.0</td>
<td>35.0</td>
</tr>
<tr>
<td>Linear Polarization</td>
<td>0.69</td>
<td>0.69</td>
<td>0.48</td>
</tr>
<tr>
<td>$J/\psi$ per day total</td>
<td>1494.9755.10025.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$J/\psi$ per day in Peak</td>
<td>423.2988.3713.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$J/\psi$ goal per target</td>
<td>2000.8000.8000.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>days (at 100% efficiency)</td>
<td>9.5</td>
<td>10.7</td>
<td>8.6</td>
</tr>
</tbody>
</table>

- Summary of beam parameters, $J/\psi$ rates, and running time:
  - 0.0007 radiation length diamond
  - 3 mm diameter collimator
  - Average 1 b$^{-1}$ target thickness
  - Spectrometer as described
  - Linear polarization is given at the main coherent peak.
Conclusions

- E160 will study $J/\psi$ and $\psi'$ production and propagation through nuclei with error bars small enough to make a difference for interpretations of $J/\psi$ suppression in relativistic heavy-ion collisions. This suppression remains one of the most promising indicators of quark-gluon plasma formation.

- E160 will measure fundamental charmonium cross sections $d\sigma/dt$ and $\sigma_{\text{tot}}^{\psi'}$ with at least a factor of three improvement in accuracy over an energy range where one would be surprised not to find large variations produced by $\ell_F$.

- E160 will provide data using a simple probe, the results of which are easier to interpret than hadron-induced reactions.

- E160 will make the first ever measurement of the $A$-dependence of $\psi'$ photoproduction and test the claim that $\sigma_{\text{tot}}^{\psi'}$ is as large as 25 mb.

- E160 will constrain models for the creation and interaction of $J/\psi$ and $\psi'$ particles, and shed light on the VMD mystery.
Elastic $J/\psi$ Kinematics (con't)

\[ E_\psi = \sqrt{m_\psi^2 + k^2} \]
\[ E^*_\psi = \sqrt{m_\psi^2 + k'^2} \]
\[ s = M^2 + 2Mk \]
\[ u = -2ME_\psi + M^2 + m_\psi^2 \]
\[ k^* = \frac{(s-M^2)}{2\sqrt{s}} \]
\[ k'^2 = \frac{(s-M^2-m_\psi^2)^2-4M^2m_\psi^2}{4s} \]
\[ t = -2M(k - E_\psi) \]
\[ t = m_\psi^2 - 2k(E_\psi - k' \cos \theta) \]
\[ t = m_\psi^2 - 2k^*(E^*_\psi - k'^* \cos \theta^*) \]
\[ t_{\text{min}} = m_\psi^2 - 2k^*(E^*_\psi - k'^*) \]
\[ -(t - t_{\text{min}}) = 4k^*k'^* \sin^2 \frac{\theta^2}{2} \]
\[ p_T^2 = (k'^* \sin \theta^*)^2 \]
\[ p_T^2 = 4k'^2 \sin^2 \frac{\theta^*}{2} \cos^2 \frac{\theta^*}{2} \]
\[ k^*_{\text{muon}} = \sqrt{m_\psi^2/4 - m_{\text{muon}}^2} \]

$M$ is the nucleon mass; $m_\psi$ is the $\psi$ mass; $k^*_{\text{muon}}$ is the momentum of a decay muon in the $\psi$ CM frame.
Charmonium States

- $\eta_c(1S)$ $I^G(J^{PC}) = 0^+(0^{-+})$ $m = 2979.8$ MeV
  - $\Gamma = 13.2$ MeV

- $J/\psi(1S)$ $I^G(J^{PC}) = 0^-(1^{--})$ $m = 3096.88$ MeV
  - $\Gamma = 87$ keV
  - $BR(\rightarrow \mu^+\mu^-) = (6.01 \pm 0.19)\%$

- $\chi_{c0}(1P)$ $I^G(J^{PC}) = 0^+(0^{++})$ $m = 3417.3$ MeV
  - $\Gamma = 14$ MeV
  - $BR(\rightarrow \gamma J/\psi) = (6.6 \pm 1.8) \times 10^{-3}$

- $\chi_{c1}(1P)$ $I^G(J^{PC}) = 0^+(1^{++})$ $m = 3510.53$ MeV
  - $\Gamma = 0.88$ MeV
  - $BR(\rightarrow \gamma J/\psi) = (27.3 \pm 1.6)$

- $\chi_{c2}(1P)$ $I^G(J^{PC}) = 0^+(2^{++})$ $m = 3556.17$ MeV
  - $\Gamma = 2.00$ MeV
  - $BR(\rightarrow \gamma J/\psi) = (13.5 \pm 1.1)$

- $\psi'(2S)$ $I^G(J^{PC}) = 0^-(1^{--})$ $m = 3686.00$ MeV
  - $\Gamma = 277$ keV
  - $BR(\rightarrow \mu^+\mu^-) = (7.7 \pm 1.7) \times 10^{-3}\%$
  - $BR(\rightarrow \gamma J/\psi) = (54.2 \pm 3.0)\%$