ON THE TRIDENT CALCULATION FOR E-144

T.Koffas and A.C.Melissinos

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1 Commentary

We have taken another look at the calculation of the trident rate done by Christian and Kirk (note of 2/28/97) and which is refered to in the PRL article. Here we discuss some new results and comment on the validity of the calculation. The conclusion remains the same as before, namely that the trident rate is substantially lower than the rate for the 2-step process

$$\begin{aligned} e + \omega &\rightarrow e + \gamma \\ \gamma + n\omega &\rightarrow e^+e^- \end{aligned}$$

Nevertheless this is surprising in view of the fact that for the nonlinear Compton scattering (where everything can be calculated) the one-step (virtual electron) process has the same rate as the two-step process (involving the rescattering of a real electron).

There are, of course, differences between the two reactions. In the e^+e^- production case the final state is the same wether the indermediate photon is real or virtual. In the nonlinear case the final states are different, having an extra photon in the process where the electron is real. Another possible difference is that we are dealing with a photon propagator in one case vs. an electron propagator in the other.

An intuitive interpretation for the low trident rate is that for our parameters (i.e. very near threshold for e^+e^- production) the γ energy should be as high as possible. The Compton cross-section (2-step process) peaks at the end-point, whereas the virtual photon spectrum (tridents) has a $1/E_\gamma$ dependence thus depressing the photon-photon scattering rate.

This last observation also bears on the validity of the W-W approximation to our problem. In general the W-W method works well when the "emitting" particle is disturbed very little by the radiation of the virtual photons. We are very far from this condition, the electron loosing 2/3 of its energy when radiating the photon needed to produce the pair. Note also that we have not included interference terms.

Be this as it may, the agreement of the 2-step calculation with the data is an indication that the trident rate can not be very high. How do we know that the values of η assigned to the data points are correct? This follows from the good agreement of the E-cal scan data with the calculation.

2 Extension of the trident calculation

In the original calculation (note of 2/28/97) by Christian and Kirk the upper energy of the virtual photon has been significantly limited below that of the real (Compton backscattered) photons. This is evident from Table 1 of the note and its impact on the trident rate is shown in Fig. 6 of the note. This comes about because in evaluating the <u>kinematics</u> only diagrams as in Fig. 1 were considered. Here ω_V are the W-W virtual photons and ω_0 the real laser photons.

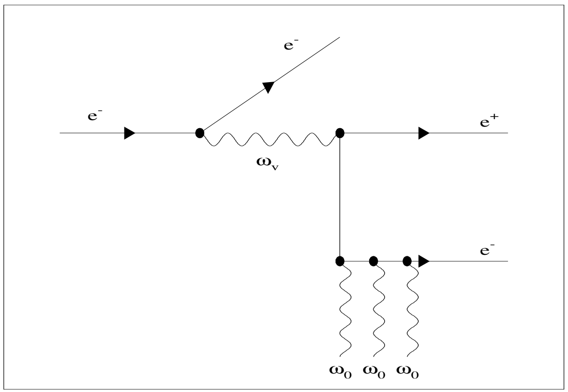

Figure 1:

The overall kinematic constraints are

$$(E_0 + n\omega_0)^2 \quad > \quad 3m^2 \tag{1}$$
$$(M_{\text{pair}} + m)^2 \quad < \quad (E_0 + n\omega_0)^2 \tag{2}$$

For head-on collisions Eqs. (1,2) give the bounds

$$n \quad > \quad \frac{2m^2}{E_0\omega_0} \tag{3}$$
$$M \quad < \quad M_{max} = m\left(\sqrt{1 + \frac{4nE_0\omega_0}{m^2}} - 1\right) \tag{4}$$

The ambiguity arises in translating these bounds into limits on ω_V and k_V. Since the intermediate photon is virtual, the constraint

$$\omega_V^2 - k_V^2 = 0 \tag{5}$$

need not apply. One can then use $\omega_V(\text{max}) = E_0$, the incoming electron energy (see the dashed curves in Fig. 6 of the CB/KTM note). The propagation of a virtual photon far off the mass shell contradicts the spirit of the W-W approximation and therefore we assume that (5) is satisfied.

We can now impose kinematic constraints on ω_V using the diagram of Fig. 1

$$(2m)^2 < (\omega_V + n\omega_0)^2 < M_{max}^2 \tag{6}$$

For head-on collisions, and using (5) this gives

$$\omega_V(min) = \frac{m^2}{n\omega_0} \tag{7}$$

$$\omega_V(max) = E_0 - \frac{m^2}{2n\omega_0}\left(\sqrt{1+\frac{4E_0 n\omega_0}{m^2}} - 1\right) \tag{8}$$

The constraints of Eqs. (7,8) are the ones used in the CB/KTM note and are applicable when all the laser photons are absorbed by the produced pair (this is obviously the case when only one external photon participates in the trident reaction as in Fig. 2 of the CB/KTM note).

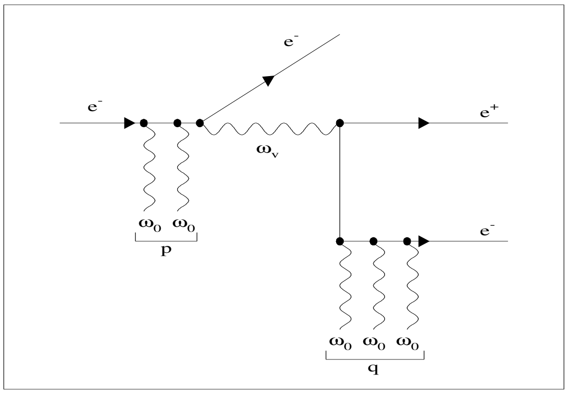

Figure 2:

However we are in the multiphoton regime and we know that we can satisfy the constraint (3) by absorbing p photons on the incoming electron and q photons on the produced pair as shown in Fig. 2, as long as

$$p + q = n \tag{9}$$

This leaves Eqs. (3,4) unchanged but Eq. (6) now reads

$$(2m)^2 < (\omega_V + q\omega_0)^2 < M_{max}^2 \tag{10}$$

For head-on collisions this translates into

$$\omega_V(min) = \frac{m^2}{q\omega_0} \tag{11}$$

$$\omega_V(max) = \frac{n}{q}\left[E_0 - \frac{m^2}{2n\omega_0}\left(\sqrt{1+\frac{4E_0 n\omega_0}{m^2}} - 1\right)\right] \tag{12}$$

If we introduce the "recoil parameter"

$$x = \frac{4E_0\omega_0}{m^2} = \frac{4\gamma\omega_0}{m} \tag{13}$$

we can write

$$\omega_V(min) = \frac{4E_0}{qx} \tag{14}$$

$$\omega_V(max) = \frac{n}{n-p}E_0\left(1+\frac{2}{nx}-\frac{1}{nx}\sqrt{1+nx}\right) \tag{15}$$

The above limits can be directly compared to the limits on the real photon which participates in the 2-step process

$$\omega_\gamma(min) = \omega_V(min) = \frac{4E_0}{qx} \tag{16}$$

$$\omega_\gamma(max) = E_0\frac{px}{1+px} \tag{17}$$

Typical results for these kinematic limits are given in Table 1, where we used

$$m^2 \rightarrow \overline{m}^2 = m^2(1+\eta^2) = 1.1m^2 \qquad (\eta \sim 0.3) \tag{18}$$

$$x = 4E_0\omega_0/\overline{m}^2 = 1.52 \tag{19}$$

$$n > 8/x = 5.2 \tag{20}$$

TABLE I

	$n=6$			
$p=0$	$q=6$	$\omega_V^{(min)}$=20.4	$\omega_V^{(max)}$=24.3	$\omega_\gamma^{(max)}$=0
$p=1$	$q=5$	=24.5	=29.2	=28.1
$p=2$	$q=4$	=30.6	=36.5	=35.1
$p=3$	$q=3$	=40.7	$> E_0$	=38.2
	$n=7$			
$p=0$	$q=7$	$\omega_V^{(min)}$=17.5	$\omega_V^{(max)}$=25.5	$\omega_\gamma^{(max)}$=0
$p=1$	$q=6$	=20.4	=29.8	=28.1
$p=2$	$q=5$	=24.5	=35.7	=35.1
$p=3$	$q=4$	=30.6	=44.6	=38.2
$p=4$	$q=3$	=40.7	$> E_0$	=40.0
etc.				

Table 1: Example of limits on the virtual photon energy when $n=6$ or $n=7$ laser photons are absorbed. Here p is the number of laser photons absorbed by the incoming electron and q the number of laser photons absorbed by the outgoing pair. Note that $p+q=n$.

Whether one should stop when $\omega_V > E_0$ is debatable, but since we adopted (5) this seems reasonable. A more important issue is how we take account of the coupling of the laser photons to the initial electron when $p \neq 0$. In the 2-step calculation this is explicitly accounted for by calculating the (multi) photon backscattered Compton cross section. Here we multiply the W-W cross section by an additional factor of $(\eta^2)^p$.

3 Results

The above modifications were included in the program and the trident rate was calculated for different assumptions. The e^+e^- spectra, number of laser photons absorbed and the corresponding rates are self-consistent. Furthermore the case $q = n$ ($p = 0$) reproduces the results of the previous calculation. We tabulate the results as the ratio of the new trident

TABLE II		
	$N_{e^+e^-}/N_{p=0}$	$N_{e^+e^-}/N_{2step}$
$p = 0$ assumption	1.0	7.5×10^{-4}
limits as in Table I	2.7	2×10^{-3}
limit $\omega_\nu^{(max)} = E_0$	48	3.5×10^{-2}
limit $\omega_\nu^{(max)} = E_0$ and set $\eta^{2p} = 1$	1.3×10^4	9.4

Table 2: e^+e^- rate from trident production compared to the 2-step process as a function of different assumptions on the energy of the virtual photon. Calculated for $\eta = 0.326$.

rate (a) over the $q = n$ trident rate and (b) over the 2-step process rate. We conclude that there is no evidence that the trident process can contribute significantly to the e^+ rate observed in E-144.

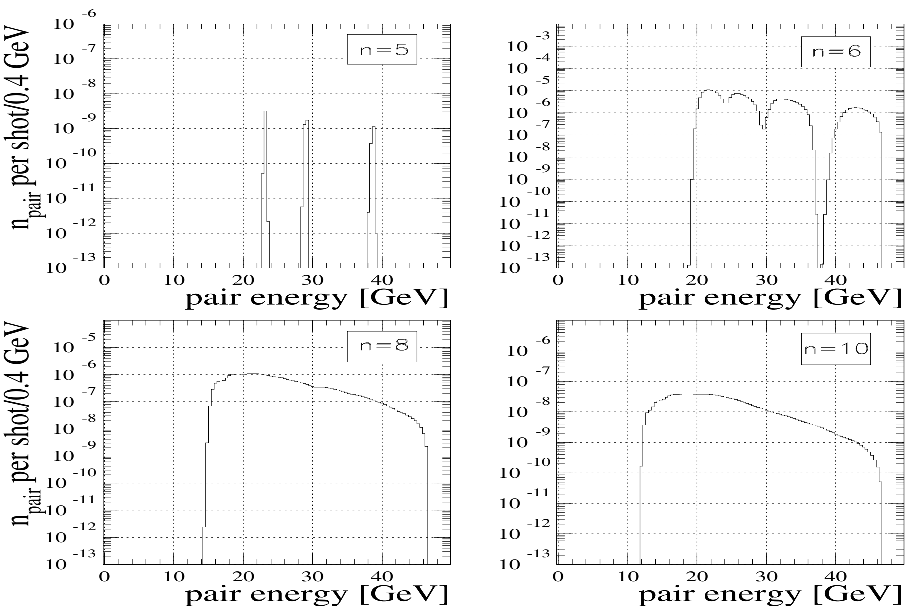

Figure 3: Calculated rates for the trident process, as presented in Fig. 2, for $\eta = 0.326$. The maximum energy of the virtual photon involved ω_v^{max}, is taken from Eq. 15.

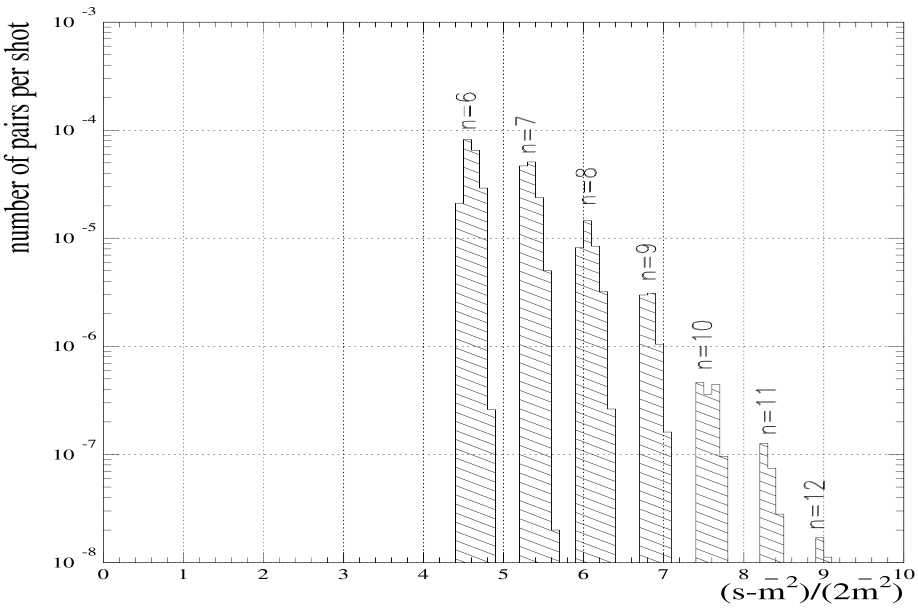

Figure 4: Contributions to the rate of the trident production as a function of the invariant quantity $(s - \bar{m}^2)/2\bar{m}^2$.

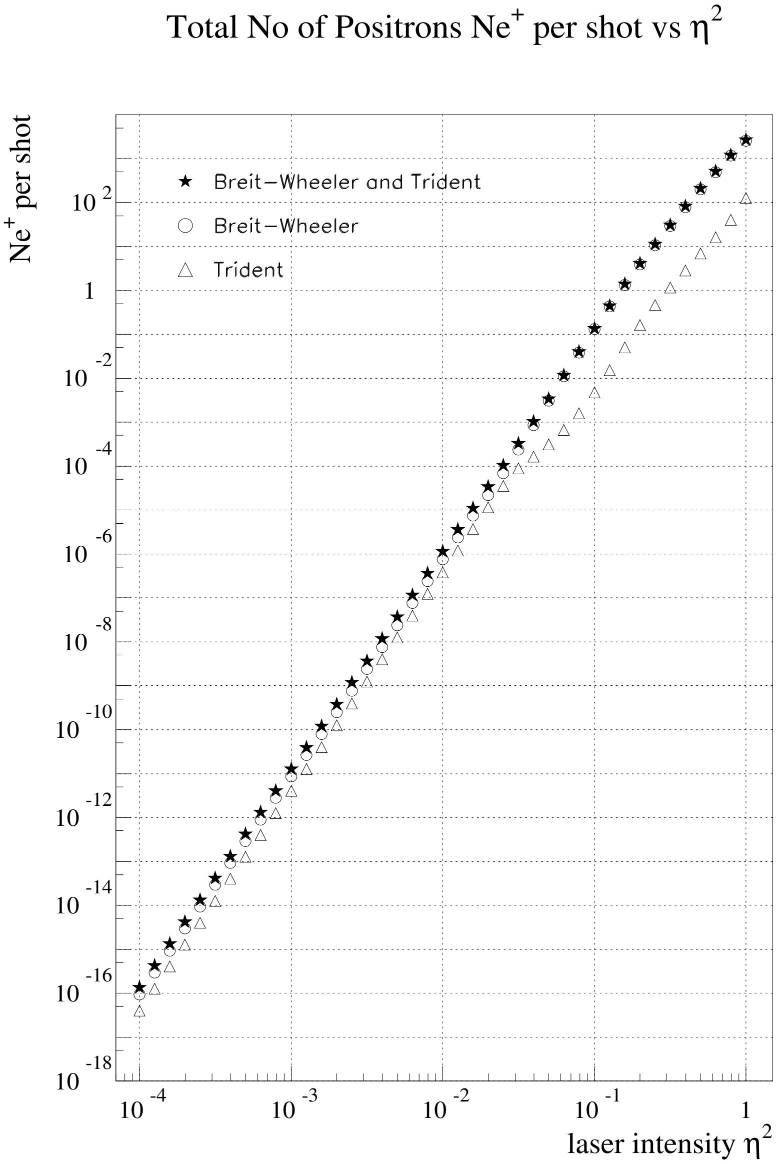

Figure 5: Calculated rates for trident production (assuming $\omega_v^{max} = E_0$ always) and two-step Breit-Wheeler pair creation as a function of η.