E144: Polarization effects in positron production and multi-photon Compton scattering in interaction of intense laser with relativistic electrons.

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March 8, 1997

1 Introduction and definitions

We will consider the process of electron-positron pair creation in interaction of relativistic electrons with laser beam. Pairs can be created in laser focus by two processes:

1. Two step process when high energy real photons are created from backscattering of laser photons from the electron beam (Compton scattering) and high energy photons interact with laser photons with creation of e^+e^- pair.

2. One step trident production process; it has virtual intermediate high energy photon.

In this note we will consider polarization effects for the 1-st process. We can expect that trident production will have similar polarization behavior because the essential difference between two processes is in intermediate photon state: photon is real for two step process and virtual for trident production; and presumably not in polarization.

Compton scattering of an electron in a field of intense electromagnetic wave is the interaction of the electron with polarized photons. Backscattered photons (and scattered electrons) are polarized depending on polarization of the laser.

The consequent e^+e^- pair production by polarized photons interacting with the laser photons is the interaction of two polarized particles and polarization effects will define entirely the cross-section of pair production.

In this note we will consider polarization effects in nonlinear Compton scattering of an electron for linear and circular polarized laser (Sections 2 and 3); pair production for linear and circular polarized laser (Sections 4 and 5). Discussion of polarization effects for E144 two-step pair production process will be discussed in Section 6. In E144 positrons are produced below kinematic threshold for one laser photons (we need more than 4 laser photons to produce a pair). Polarization effects above kinematic threshold for one laser photon are discussed in section 7 on example of 250 GeV electron beam and 5 eV laser.

What will **not** be considered in this note is the degree of polarization of scattered electrons; polarization of positrons in e^+e^- pair production and effects of polarization of incoming electron beam. We will do it elsewhere.

Definitions

Laser intensity is characterized by dimensionless parameter η as the "mean square" of the vector-potential of the electromagnetic field ($\hbar = c = 1$):

$$\eta = \frac{e\sqrt{-\overline{A^2}}}{m}$$

where m-electron mass, e-unit charge, A-vector potential of the electromagnetic wave.

For instance circular polarized laser with 4-vector potential $A = (0,\, a\cos(kx),\, a\sin(kx),\, 0)$ (a-amplitude) will have $\eta = ea/m$; linear polarized laser with 4-vector potential $A = (0,\, a\cos(kx),\, 0,\, 0)$ of the same amplitude a will have $\eta = 1/\sqrt{2}ea/m$.

Density of photons ρ_l with energy ω is related to the laser intensity η^2 by the formula:

$$\rho_l = \frac{\omega\, m^2}{e^2}\eta^2 = \frac{\alpha^2}{4\pi}\frac{\omega}{m}\frac{1}{r_0^3}\eta^2$$

where we use constants:

$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137}; \qquad r_0 = \frac{e^2}{4\pi\, m} = 2.82\, 10^{-9}\, \mu m\, .$$

For E144 green and IR laser ($\omega = 2.34$ and $\omega = 1.17$ eV):

$$\begin{aligned}
\rho_l &= 0.866\, 10^{15}\, \eta^2 \left[\frac{1}{\mu m^3}\right], \quad \eta^2 = 1.15\, 10^{-15}\, \rho_l\, ; \text{ (For green laser)}\, , \\
\rho_l &= 0.433\, 10^{15}\, \eta^2 \left[\frac{1}{\mu m^3}\right], \quad \eta^2 = 2.30\, 10^{-15}\, \rho_l\, . \text{ (For IR laser)}\, .
\end{aligned}$$

Field intensities $\eta \approx 1$ are reachable with existing laser technology. For instance diffraction limited spot $A = 2\sigma_x\sigma_y = 6.37$ μm^2 of 1.5 ps long ($\sigma_z = 191$ μm) 1 Joule green laser pulse will have (N_l is the number of laser photons in a pulse of total laser power P: $N_l = \frac{P}{\omega}$):

$$\begin{aligned}
\rho_{l,max} &= \frac{N_l}{(2\pi)^{3/2}\sigma_x\sigma_y\sigma_z} = 0.87\, 10^{15} \left[\frac{1}{\mu m^3}\right], \\
\eta^2_{max} &= 1.00
\end{aligned}$$

1.1 Kinematics of Compton scattering of an electron.

Consider reaction:

$$e^- + s\,\omega \rightarrow e^- + \gamma$$

where $s\omega$ stands for "s" laser photons, γ for backscattered high energy photon.

Incoming laser photon and electron have 4-momentums k_1 and p_1; backscattered high energy photon and electron have 4-momentums k_2 and p_2.

4-momentums of electrons are modified when they go through the region of a strong electromagnetic field; their kinematics is defined by quasi-momentums q_1 and q_2:

$$\begin{aligned} q_1 &= p_1 + \frac{\eta^2 m^2}{2(k_1 p_1)} k_1\,, \qquad q_1^2 = {m_*}^2 = m^2(1+\eta^2)\,; \\ q_2 &= p_2 + \frac{\eta^2 m^2}{2(k_1 p_2)} k_1\,, \qquad q_2^2 = {m_*}^2 = m^2(1+\eta^2)\,. \end{aligned} \tag{1}$$

Invariant dimensionless parameters u, u_s, x and center of mass energy square $\tilde{s}$ are defined as:

$$\begin{aligned} u &= \frac{k_1 k_2}{k_1 p_2}\,, \qquad u_s = \frac{s\,x}{1+\eta^2}\,, \qquad x = \frac{2 k_1 p_1}{m^2}\,, \\ \tilde{s} &= (k_1 + p_1)^2 = (1+x)\,m^2\,. \end{aligned}$$

Center Of Mass (COM) frame is defined by condition $s\vec{k_1} + \vec{q_1} = 0$ where s is the number of laser photons. In the COM frame electron scatters by the angle $\cos\Theta = [-1, 1]$ which corresponds to the range of parameter $u = [0, u_s]$:

$$u = \frac{u_s(1-\cos\Theta)}{2 + u_s(1+\cos\Theta)} = [0 \ldots u_s]$$

Note that COM frame is different for different orders s and different field intensities η.

1.2 Kinematics of e^+e^- pair production.

Consider reaction:

$$\gamma + s\,\omega \to e^+ + e^-$$

where again $s\omega$ stands for "s" laser photons interacting with high energy photon γ.

Incoming laser photon and high energy photon have 4-momentums k_1 and k_2; electron and positron have 4-momentums p_1 and p_2; their momentums are modified in the laser field according to (1).

Invariant dimensionless parameters u, u_s, x and center of mass energy square $\tilde{s}$ are defined as:

$$\begin{aligned} u &= \frac{(k_1 k_2)^2}{4(k_1 p_1)(k_1 p_2)}\,, \qquad u_s = \frac{s\,x}{1+\eta^2}\,, \qquad x = \frac{k_1 k_2}{2\,m^2}\,, \\ \tilde{s} &= (k_1 + k_2)^2 = 4x\,m^2\,. \end{aligned}$$

COM frame is defined by condition $s\vec{k_1} + \vec{k_2} = 0$ where s is the number of laser photons. In the COM frame electron (or positron) can scatter by an angle $\cos\Theta = [-1, 1]$ which corresponds to the parameter u in the range $u = [1, u_s]$:

$$u = \frac{u_s}{u_s - (u_s - 1)\cos^2\Theta} = [1...u_s]$$

COM frame is different for different orders s and different field intensities η.

1.3 More definitions.

For both Compton scattering and e^+e^- pair production we will write down a few useful expressions which will be used later. As it will be shown later cross-sections for Compton scattering and pair production are expressed with the use of functions $A_i(s, \alpha, \beta)$ for the linear polarization and Bessel functions $J_s(\alpha)$ for the circular polarization. These functions depend on parameters α and β.

For linear polarized laser:

$$\begin{aligned} \alpha_{Lin} &= -2\sqrt{2}s\frac{\sqrt{u(u_s - u)}}{u_s}\frac{\eta}{\sqrt{1+\eta^2}}\cos\phi\,, \\ \beta &= \frac{u\,s}{2u_s}\frac{\eta^2}{1+\eta^2}\,. \end{aligned} \tag{2}$$

For circular polarized laser:

$$\alpha_{Circ} = -2s\frac{\sqrt{u(u_s - u)}}{u_s}\frac{\eta}{\sqrt{1+\eta^2}} \tag{3}$$

Note that:

$$\alpha_{Lin} = \sqrt{2}\ \alpha_{Circ}\ \cos\phi$$

where factor $\sqrt{2}$ is exclusively due to different definitions of η as a function of peak field intensity a for two polarizations.

The above expressions for α and β have exactly the same form for Compton scattering and pair production; though variables u, u_s have different definitions for two processes.

2 Polarization of high energy photons produced in interaction of intense linear polarized laser with unpolarized electron beam.

We will consider unpolarized electron beam, as it was the case for E-144 experiment when polarization of electrons was not always known.

Consider linear polarized laser light propagating along the Z-axis. Polarization of the photon $\vec{e_a}$ is in $X-Y$ plane with an angle ϕ to the X-axis; polarization is characterized by 4-dimensional vector $e_a = (0\,,\cos\phi\,,\sin\phi\,,0)$. Consider head-on collision of the photon(s) with an electron propagating in the $-Z$ direction; after collision electron scatters in the plane $X-Z$ by the angle Θ in the COM frame.

Note that $17°$ crossing angle between electron and laser beams in the LAB frame for E144 configuration is taken into account automatically by invariant definition of kinematic variables u,u_s and $\tilde{s}$: In the COM frame collisions occur head-on.

Consider two states of linear polarization of backscattered photon: e_1 with polarization vector in the scattering plane, and e_2 with polarization vector perpendicular to the scattering plane:

$$\begin{aligned} e_1 &= (0, \cos\Theta, 0, -\sin\Theta)\,, \\ e_2 &= (0, 0, 1, 0)\,. \end{aligned}$$

We will ignore the polarization of outgoing electron.

We will call σ_1 the cross-section of the process with photon backscattered into polarization state e_1, and σ_2 with photon backscattered into polarization state e_2.

Using Form and Maple we derive for two polarizations:

$$\begin{aligned} d\sigma_1 &= 2r_0^2 \frac{m^2}{(\tilde{s}-m^2)}\frac{1}{\eta^2}\frac{du\,d\phi}{(1+u)^2} \\ &\quad \left[-2A_0^2 + 2\eta^2(A_1^2\cos(2\phi) - A_0A_2) + \eta^2\left(2+\frac{u^2}{(1+u)}\right)(A_1^2 - A_0A_2)\right]\,, \\ d\sigma_2 &= 2r_0^2 \frac{m^2}{(\tilde{s}-m^2)}\frac{1}{\eta^2}\frac{du\,d\phi}{(1+u)^2} \\ &\quad \left[-2\eta^2(A_1^2\cos(2\phi) - A_0A_2) + \eta^2\left(2+\frac{u^2}{(1+u)}\right)(A_1^2 - A_0A_2)\right]\,. \end{aligned}$$

and for total unpolarized cross-section ($\sigma_{Unp} = \sigma_1 + \sigma_2$):

$$d\sigma_{Unp} = 2r_0^2 \frac{m^2}{(\tilde{s}-m^2)}\frac{1}{\eta^2}\frac{du\,d\phi}{(1+u)^2}\left[-2A_0^2 + 2\eta^2\left(2+\frac{u^2}{(1+u)}\right)(A_1^2 - A_0A_2)\right]\,. \qquad (4)$$

Unpolarized cross-section $d\sigma_{Unp}$ still depends on ϕ (through functions A_0, A_1 and A_2) and hence we should keep $d\phi$ in the above expression.

One can see that the dimension of the cross-section is defined by r_0^2. Due to the factor $\frac{m^2}{(\tilde{s}-m^2)}$ cross-section of the Compton scattering asymptotically goes as $1/\tilde{s}$ at high energies. At E144 intermediate energies:

$$
\begin{aligned}
\sqrt{\tilde{s}} &= 2.63\, m\,, \quad \left(\frac{m^2}{\tilde{s}-m^2}\right)^{-1} = x = 1.63 \qquad \text{for green laser}\,,\\
\sqrt{\tilde{s}} &= 1.82\, m\,, \quad \left(\frac{m^2}{\tilde{s}-m^2}\right)^{-1} = x = 0.82 \qquad \text{for IR laser}\,,
\end{aligned}
$$

We can rewrite the above formulae in more symmetric way:

$$
\begin{aligned}
d\sigma_1 &= \frac{1}{2} d\sigma_{Unp} + d\sigma_{pol}\,,\\
d\sigma_2 &= \frac{1}{2} d\sigma_{Unp} - d\sigma_{pol}\,,\\
d\sigma_{pol} &= 2r_0^2 \frac{m^2}{(\tilde{s}-m^2)} \frac{1}{\eta^2} \frac{du\, d\phi}{(1+u)^2}\\
&\quad \left[-A_0^2 + 2\eta^2 (A_1^2 (1+2\cos(2\phi)) - A_0 A_2)\right]\,. \qquad (5)
\end{aligned}
$$

Cross-sections $d\sigma_1$ and $d\sigma_2$ are not useful for experimental measurements. From experimental point of view it is important to calculate differential cross-sections for backscattered photons which have the linear polarization parallel $\sigma_{||}$ or perpendicular $\sigma_{\perp}$ to the polarization of the laser.

For photon backscattered at some angle Θ with the direction of the original laser photons parallel or perpendicular polarization condition is not well-defined in general case. What does it mean that two linear polarizations of two photons are parallel if photons have some angle between them? We should go to some frame where two photons will have parallel momentums $\vec{k_1} \parallel \vec{k_2}$, then in such frame we can define parallel or perpendicular orientation of photon polarizations. The process for which the polarization of backscattered photons will be important is the consequent pair creation in the same laser focus. In general case transformation from the COM frame of Compton scattering to the COM frame of a new process (pair creation) should be made. This transformation depends on the number of photons involved in both processes and field intensity and in general case is not trivial.

We will use the following approximation which is valid for the most of the energy spectrum of backscattered photons except the lowest energies. Consider backscattered photons with angles Θ close to $180°$ in the COM frame. We know from kinematics that those photons have maximum energy in the LAB frame and will have a major contribution to the pair creation. For those photons we will neglect correction due to the angle of $\Theta - 180°$ and calculate cross-sections $\sigma_{||}$ and $\sigma_{\perp}$ for "almost" parallel and perpendicular orientation of polarizations. For that we note that both polarizations can be presented as combinations of polarizations e_1 and e_2 for backscattered photons:

$$
\begin{aligned}
e_{||} &= e_1 \cos\phi - e_2 \sin\phi\,,\\
e_{\perp} &= e_1 \sin\phi + e_2 \cos\phi\,.
\end{aligned}
$$

where ϕ is the angle between laser polarization and scattering plane $X-Z$. Then we derive:

$$\begin{aligned}
d\sigma_{||} &= \frac{1}{2} d\sigma_{Unp} + d\sigma_{Pol}\,, \\
d\sigma_{\perp} &= \frac{1}{2} d\sigma_{Unp} - d\sigma_{Pol}\,, \\
d\sigma_{Pol} &= 2r_0^2 \frac{m^2}{(\tilde{s}-m^2)} \frac{1}{\eta^2} \frac{du\, d\phi}{(1+u)^2} \\
&\left[-A_0^2\left(1+\frac{(u_s-u)}{u}(1+\eta^2)(1-\cos(2\phi))\right)+2\eta^2(A_1^2-A_0A_2)\right]\,. \qquad (6)
\end{aligned}$$

Note that for $\phi = 0$ these cross-sections are equivalent to 5.

We will use parameter σ defined as:

$$\sigma = 2\,\eta^2\left[\frac{s}{4\beta}-\frac{1}{2}-\left(\frac{\alpha}{8\beta}\right)^2\right] \qquad (7)$$

Using expression (2) for parameters α and β we can express σ in terms of kinematic variables u, u_s and $\cos(2\phi)$:

$$\sigma = 1+\frac{(u_s-u)(\eta^2+1)(1-\cos(2\phi))}{2u}$$

We can see that polarization term in (6) can be presented using variable σ:

$$d\sigma_{Pol} = 2r_0^2 \frac{m^2}{(\tilde{s}-m^2)} \frac{1}{\eta^2} \frac{du\, d\phi}{(1+u)^2}\left[-A_0^2(2\sigma-1)+2\eta^2(A_1^2-A_0A_2)\right]\,. \qquad (8)$$

We can also rewrite $\sigma_{||}$ and $\sigma_{\perp}$ in (6):

$$\begin{aligned}
d\sigma_{||} &= 2r_0^2 \frac{m^2}{(\tilde{s}-m^2)} \frac{1}{\eta^2} \frac{du\, d\phi}{(1+u)^2} \\
&\left[-2A_0^2\sigma+4\eta^2\left(1+\frac{u^2}{4(1+u)}\right)(A_1^2-A_0A_2)\right]\,, \\
d\sigma_{\perp} &= 2r_0^2 \frac{m^2}{(\tilde{s}-m^2)} \frac{1}{\eta^2} \frac{du\, d\phi}{(1+u)^2} \\
&\left[-2A_0^2(1-\sigma)+\eta^2\frac{u^2}{1+u}(A_1^2-A_0A_2)\right]\,. \qquad (9)
\end{aligned}$$

This form of cross-sections "coincide" with one by Narozhny et al derived for e^+e^- pair creation by two head-on colliding photons with parallel and perpendicular polarizations. "Coincide" means that cross-sections for Compton scattering should be rewritten from one for positron production using the rules that discussed in Section 4.

Note that both cross-sections 6 and 9 are valid for photons with close to 180° backscattering angle and hence their limits at 0° or equivalently $u \rightarrow 0$ are not valid.

Polarization P(u) of the backscattered γ-beam is defined as:

$$P(u) = \frac{d\sigma_{||} - d\sigma_{\perp}}{d\sigma_{||} + d\sigma_{\perp}}$$

Fraction of photons with polarization $\sigma_{||}$ will be:

$$P_{||}(u) = \frac{d\sigma_{||}}{d\sigma_{||} + d\sigma_{\perp}} \quad = \frac{1+P}{2}$$

One can also define the average polarization of the backscattered photons $< P >$ (for each order s):

$$< P >= \frac{1}{\sigma_{Unp}} \int P(u) \frac{d\sigma_{Unp}}{du} du = \frac{\sigma_{||} - \sigma_{\perp}}{\sigma_{||} + \sigma_{\perp}}$$

The difference $(d\sigma_{||} - d\sigma_{\perp})$ is the difference between polarization-conserved process and polarization-change processes. Hence positive polarization of backscattered photons in our case means that more photons backscatter with the same polarization as original laser photons.

Some useful expressions can be derived for polarized and unpolarized cross-sections at the limit $u \rightarrow u_s$ for arbitrary field intensity η. This limit is especially interesting because at $u = u_s$ backscattered photons have maximum energy and hence have dominant contribution to e^+e^- pair creation.

By evaluating expressions for $d\sigma_{||}$ and $d\sigma_{\perp}$ at the limit $u \rightarrow u_s$ we get:

$$\begin{aligned}
\lim_{u \rightarrow u_s} P(u) &= \frac{2(u_s+1)}{(u_s+1)^2+1} && \text{For odd } s = 1, 3, 5, ...; \\
\lim_{u \rightarrow u_s} P(u) &= 0 && \text{For even } s = 2, 4, 6, ...; \\
\lim_{u \rightarrow u_s} \frac{d\sigma_{Unp}}{du\, d\phi} &= 2r_0^2 \frac{m^2}{(\tilde{s}-m^2)} \eta^{2s-2} K_s \frac{1}{2} \left(\frac{1}{1+u_s} + \frac{1}{(1+u_s)^3} \right) && \text{For odd } s = 1, 3, 5, ...; \\
\lim_{u \rightarrow u_s} \frac{d\sigma_{Unp}}{du\, d\phi} &= 2r_0^2 \frac{m^2}{(\tilde{s}-m^2)} \eta^{2s-2} \frac{1}{4} \frac{{u_s}^2}{(1+u_s)^3} && \text{For even } s = 2, 4, 6, ...; \\
K_s &= 1, \left(\frac{3}{4}\right)^2, \left(\frac{25}{32}\right)^2, ... \text{for } s = 1, 3, 5, ...
\end{aligned}$$

I failed to find exact analytical evaluation for constants K_s for arbitrary s; one can see though that K_s is decreasing from 1 to 1/2 such that cross-sections for the odd and even s asymptotically approach each other at $s \rightarrow \infty$.

The above results are obtained with the help of Maple, they are valid *for arbitrary field intensity* η. Note that for large η the maximum energy of backscattered photons:

$$\omega_{max} = \omega(u_s) = E_{tot} \frac{s\, x}{1 + s\, x + \eta^2}$$

decreases with the field intensity. Still the above expressions for polarization and cross-sections are valid at the u_s.

Dependence of polarization $P(u)$ for different orders s versus the kinematic variable u and photon energy ω/E_{tot} is shown on Fig. 1. Two plots are equivalent if correspondence between u and ω is taken into account:

$$\omega = E_{tot}\,\frac{u}{1+u}$$

Total cross-section for each order s of the multi-photon Compton scattering has the leading term proportional to $1/\eta^{2s-2}$. In addition the total cross-section depends on the field intensity because the range of u decreases with the field intensity for each order s.

For instance for $s = 1$ the total cross-section and average polarization up to the terms proportional to η^2:

$$\begin{aligned}
\lim_{x\to 0}\sigma^{(1)}_{Unp} &= 4\pi r_0^2\frac{m^2}{(\tilde{s}-m^2)}\,\frac{2}{3}x\,(1-1.35\eta^2)\,;\\
\lim_{x\to 0} < P >^{(1)} &= \frac{1}{2}\,(1-0.33\eta^2)\,;\\
\lim_{x\to\infty}\sigma^{(1)}_{Unp} &= 4\pi r_0^2\frac{m^2}{(\tilde{s}-m^2)}\,\left(\frac{1}{4}\,(1-5\eta^2)+\frac{1}{2}\ln(x)\right)\,;\\
\lim_{x\to\infty} < P >^{(1)} &= 0\,.
\end{aligned}$$

For $s = 2$ the total cross-section and average polarization up to the terms proportional to η^4:

$$\begin{aligned}
\lim_{x\to 0}\sigma^{(2)}_{Unp} &= 4\pi r_0^2\frac{m^2}{(\tilde{s}-m^2)}\,\eta^2\,\frac{7}{10}x\,(1-2.82\eta^2)\,;\\
\lim_{x\to 0} < P >^{(2)} &= \frac{2}{3}\,(1-0.34\eta^2)\,;\\
\lim_{x\to\infty}\sigma^{(2)}_{Unp} &= 4\pi r_0^2\frac{m^2}{(\tilde{s}-m^2)}\,\eta^2\,\left(\frac{3}{4}\,(1-1.83\eta^2)\right)\,;\\
\lim_{x\to\infty} < P >^{(2)} &= 0\,.
\end{aligned}$$

Average polarization $< P >$ for linear polarized laser as a function of kinematic variable u_s is shown on Fig. 2. We plotted $< P >$ versus variable u_s and not x because in this form it doesn't depend on field intensity η. Average polarization is higher for small u_s and goes to 0 at $u_s \to \infty$ (see above). Average polarization is also higher for higher orders s at the same u_s (But for the same x average polarization is lower for higher s). In E144 not the average polarization $< P >$ but the differential polarization at the limit u_s, $P(u_s)$ is important because only the highest energy photons contribute to the pair production. If positrons were produced above threshold (as in case of 250 GeV electron beam) then the average polarization $< P >$ is important for *total* positron production rate but $P(u_s)$ will define the rate of positrons produced with *maximum energy*.

Examples of unpolarized and polarized differential cross-sections as a function of the fractional photon energy for $s = 1, 2, 3, 4$ and $\eta = 0.1$ are shown on Fig.3.

Finally we will derive the leading terms of the total cross-sections (4),(6),(9) for $s = 1$ expanded in series of field intensity η. Unpolarized cross-section should be in the form of Klein-Nishina formula that provides cross-check for calculations.

Expanding expression $\frac{1}{\eta^2}\left[-2A_0^2 + 2\eta^2\left(2 + \frac{u^2}{(1+u)}\right)(A_1^2 - A_0 A_2)\right]$ in series of η and integrating over ϕ, then over $u = [0...x]$ we get:

$$\begin{aligned}
\frac{d\sigma_{Unp}}{du} &= 2\pi r_0^2 \frac{m^2}{\tilde{s} - m^2}\left(\frac{1}{(u+1)^3} + \frac{1}{u+1} - 4\frac{u}{(u+1)^2\,x} + 4\frac{u^2}{(u+1)^2\,x^2}\right), \\
\sigma_{Unp} &= 2\pi r_0^2 \frac{m^2}{\tilde{s} - m^2}\left(\frac{x^3 + 18x^2 + 32x + 16}{2\,x\,(x+1)^2} + \frac{x^2 - 4x - 8}{x^2}\ln(x+1)\right). \qquad (10)
\end{aligned}$$

Which is well known Klein-Nishina formula for Compton scattering.

The same expansion for polarized cross-section $\sigma_{||}$:

$$\begin{aligned}
\frac{d\sigma_{||}}{du} &= 2\pi r_0^2 \frac{m^2}{\tilde{s} - m^2}\left(\frac{1}{2(u+1)^3} + \frac{1}{2(u+1)} - 2\frac{u}{(u+1)^2\,x} + 3\frac{u^2}{(u+1)^2\,x^2}\right), \\
\sigma_{||} &= 2\pi r_0^2 \frac{m^2}{\tilde{s} - m^2}\left(\frac{x^3 + 22x^2 + 44x + 24}{4\,x\,(x+1)^2} + \frac{x^2 - 4x - 12}{2\,x^2}\ln(x+1)\right).
\end{aligned}$$

And for $\sigma_\perp$:

$$\begin{aligned}
\frac{d\sigma_\perp}{du} &= 2\pi r_0^2 \frac{m^2}{\tilde{s} - m^2}\left(\frac{1}{2(u+1)^3} + \frac{1}{2(u+1)} - 2\frac{u}{(u+1)^2\,x} + \frac{u^2}{(u+1)^2\,x^2}\right), \\
\sigma_\perp &= 2\pi r_0^2 \frac{m^2}{\tilde{s} - m^2}\left(\frac{x^3 + 14x^2 + 20x + 8}{4\,x\,(x+1)^2} + \frac{x^2 - 4x - 4}{2\,x^2}\ln(x+1)\right).
\end{aligned}$$

One can easily cross-check that at $u = u_s$:

$$\begin{aligned}
\lim_{u\to u_s}\frac{d\sigma_{||}}{du} &= 4\pi r_0^2 \frac{m^2}{(\tilde{s} - m^2)}\,\frac{1}{2}\left[\frac{1}{2}\left(\frac{1}{1+u_s} + \frac{1}{(1+u_s)^3}\right) + \frac{1}{(1+u_s)^2}\right]; \\
\lim_{u\to u_s}\frac{d\sigma_\perp}{du} &= 4\pi r_0^2 \frac{m^2}{(\tilde{s} - m^2)}\,\frac{1}{2}\left[\frac{1}{2}\left(\frac{1}{1+u_s} + \frac{1}{(1+u_s)^3}\right) - \frac{1}{(1+u_s)^2}\right]; \\
\lim_{u\to u_s} P(u) &= \frac{2(u_s+1)}{(u_s+1)^2 + 1}.
\end{aligned}$$

in agreement with the result that was derived previously.

Some examples of polarization $P(u)$ and polarized cross-sections for linear polarized laser for $s = 1$ and $s = 2$ laser photons are presented on Fig.4 and Fig.5 for E144 green laser and 46.6 GeV electron beam.

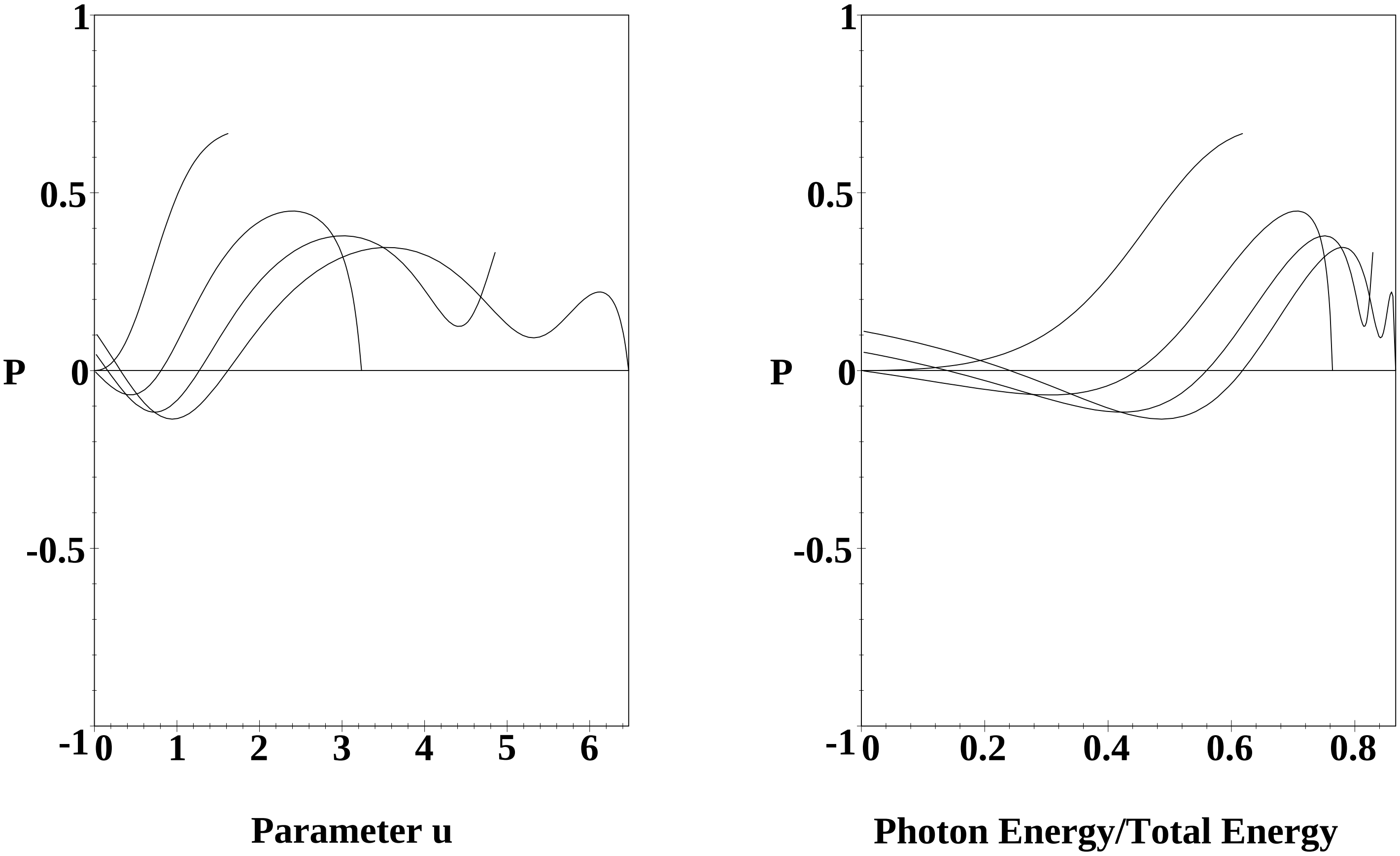

Figure 1: s=1,2,3,4 polarization versus u and fractional photon energy for E144 Green linear polarized laser and 46.6 GeV electron beam. Higher orders s have higher $u_{max} = u_s$ and higher photon energy.

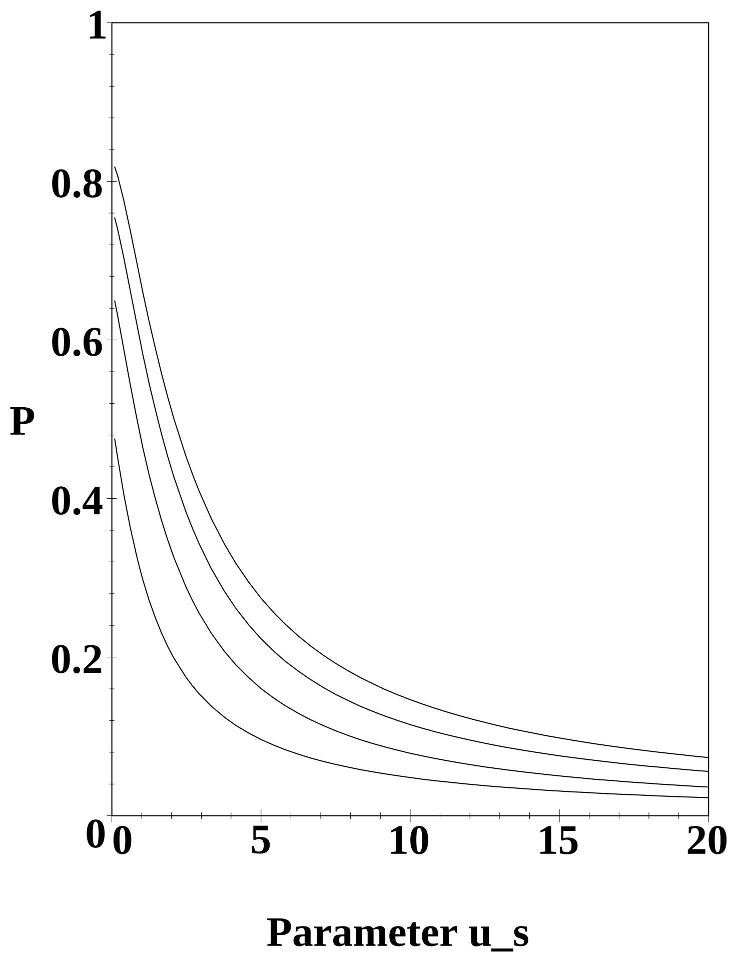

Figure 2: s=1,2,3,4 average polarization $< P >$ versus x for linear polarized laser. Higher orders s have higher polarization. (This plot is for arbitrary laser frequency, electron beam energy and field intensity)

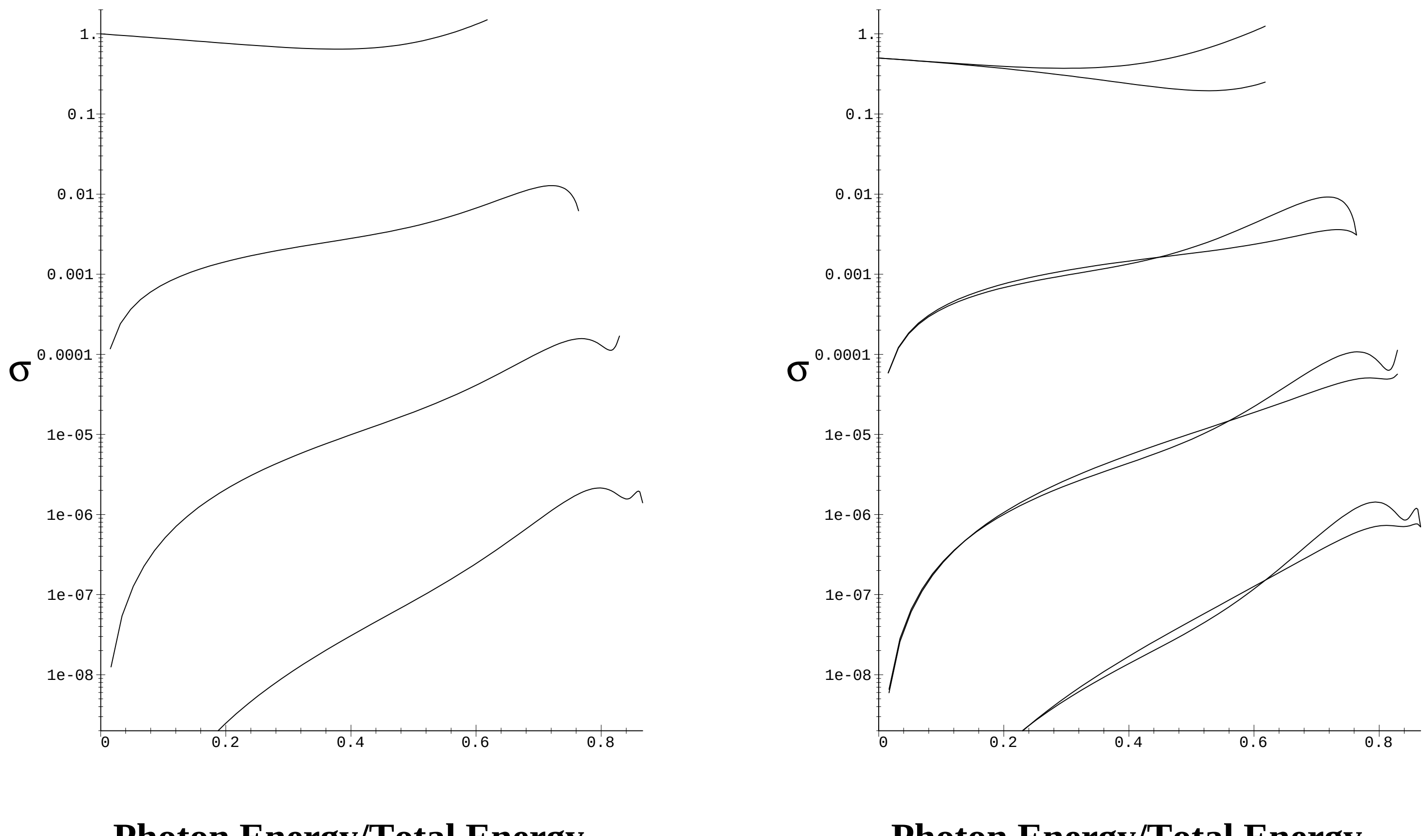

Figure 3: s=1,2,3,4 unpolarized (left plot) and polarized (right plot) cross-sections versus the fractional photon energy ω/E_{tot} for linear polarized Green E144 laser, 46.6 GeV electron beam and $\eta = 0.1$. Higher orders s have higher ω_{max} and lower cross-sections (scaled as $1/\eta^{(2s-2)}$). Vertical scale is in $2r_0^2 m^2/(\tilde{s} - m^2)$ units.

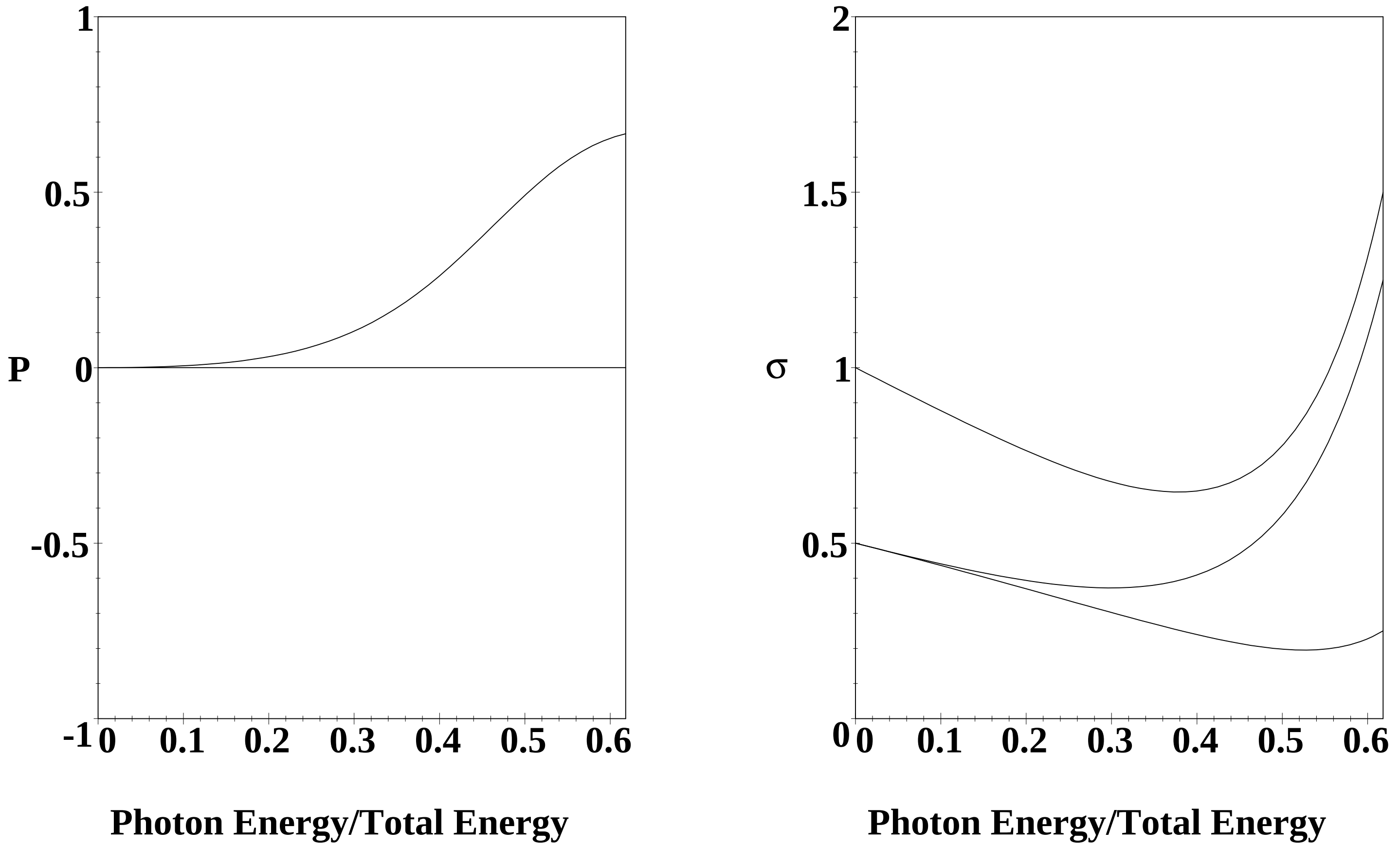

Figure 4: s=1 Compton scattering, linear polarized laser Polarization of backscattered photons (left plot) and differential cross-sections $\frac{d\sigma_{Unp}}{d\omega\, d\phi}$, $\frac{d\sigma_{\|}}{d\omega\, d\phi}$ and $\frac{d\sigma_{\perp}}{d\omega\, d\phi}$ (right plot) versus the photon energy for E144 configuration with green laser and field intensity $\eta = 0.1$. Cross-sections are in $2r_0^2 m^2/(\tilde{s} - m^2)$ units.

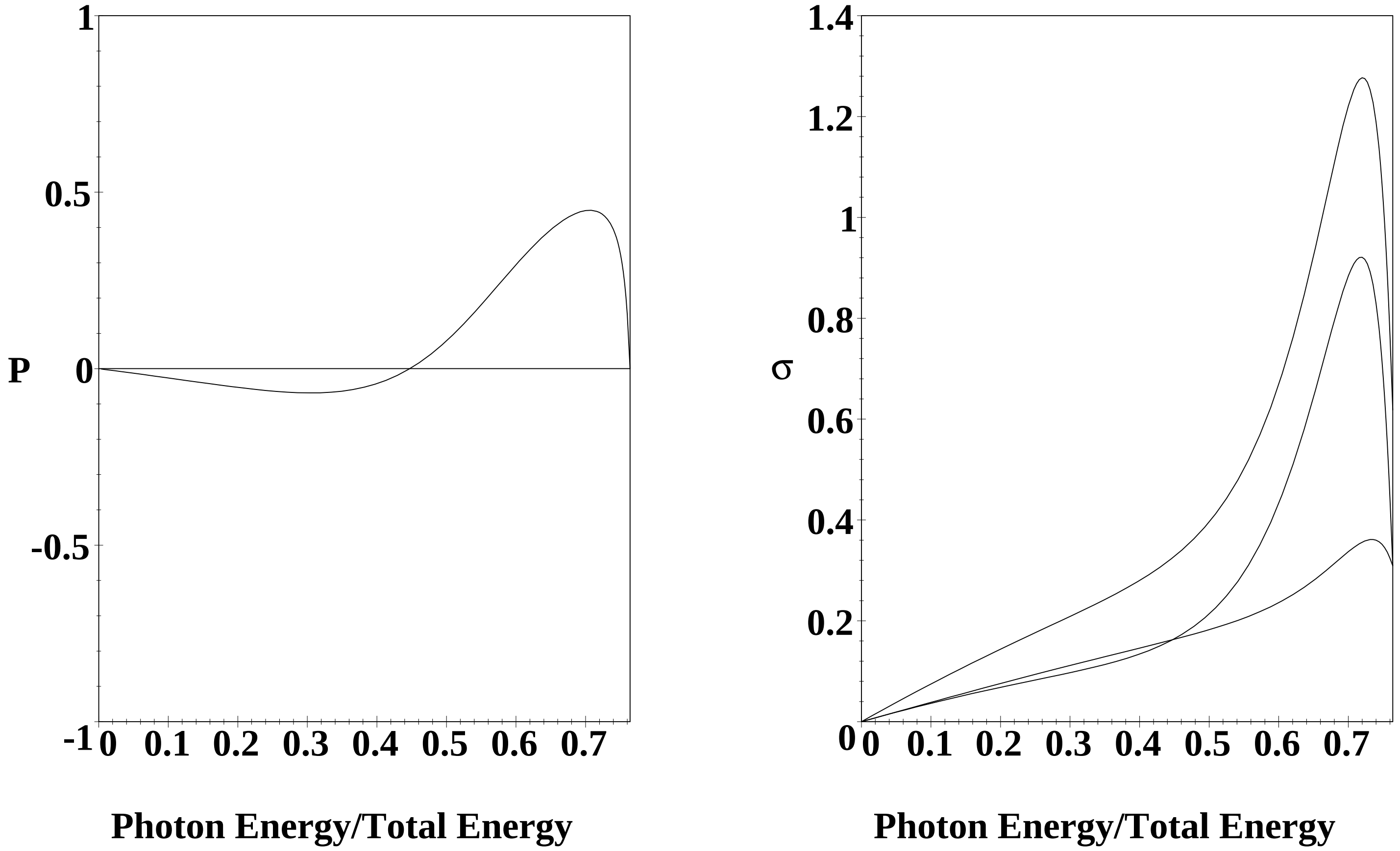

Figure 5: s=2 Compton scattering, linear polarized laser. Polarization of backscattered photons (left plot) and differential cross-sections $\frac{d\sigma_{Unp}}{d\omega\, d\phi}$, $\frac{d\sigma_{\|}}{d\omega\, d\phi}$ and $\frac{d\sigma_{\perp}}{d\omega\, d\phi}$ (right plot) versus the photon energy for E144 configuration with green laser and field intensity $\eta = 0.1$. Cross-sections are in $2r_0^2 m^2/(\tilde{s} - m^2)$ units. Cross-sections for s=2 are additionally multiplied by factor $1/\eta^2$.

3 Polarization of high energy photons produced in interaction of intense circular polarized laser with unpolarized electron beam.

Consider laser light that is circularly polarized with positive helicity.

Helicity of the photon is the projection of its spin to the direction of the momentum $\vec{k}$. Because photons are massless particles they can be in 2 helicity states: +1 (positive helicity) and -1 (negative helicity). Polarization of the circular polarized photon with positive helicity is characterized by polarization 4-vector $e_a = 1/\sqrt{2}(0\,,1\,,+i\,,0)$ and polarizations with negative helicity by 4-vector $e_a = 1/\sqrt{2}(0\,,1\,,-i\,,0)$

We will ignore (sum over) the polarization of scattered electron.

Consider two states of circular polarization of backscattered photons: e_p for photons with positive helicity, and e_n for photons with negative helicity:

$$\begin{aligned} e_p &= \frac{1}{\sqrt{2}}(0\,,\cos\Theta\,,+i\,,-\sin\Theta)\,, \\ e_n &= \frac{1}{\sqrt{2}}(0\,,\cos\Theta\,,-i\,,-\sin\Theta)\,. \end{aligned}$$

where Θ is the scattering angle in the COM frame.

Then using Form and Maple we derive:

$$\begin{aligned} d\sigma_p &= \frac{1}{2}d\sigma_{Unp} + d\sigma_{Pol}\,, \\ d\sigma_n &= \frac{1}{2}d\sigma_{Unp} - d\sigma_{Pol}\,, \\ d\sigma_{Pol} &= 4\pi r_0^2 \frac{m^2}{(\tilde{s}-m^2)}\frac{1}{\eta^2}\frac{du}{(1+u)^2} \\ & \left[\frac{\eta\sqrt{1+\eta^2}}{2}\left(1+\frac{u^2}{2(1+u)}\right)(J_{s-1}-J_{s+1})J_s\frac{2u-u_s}{\sqrt{u(u_s-u)}}\right]\,. \end{aligned} \tag{11}$$

where $d\sigma_p$ is the differential cross-section for photons with positive helicity (helicity-conserved process) and $d\sigma_n$ is the differential cross-section for photon with negative helicity (helicity-flip process). $d\sigma_{Unp}$ is the cross-section for unpolarized process:

$$d\sigma_{Unp} = 4\pi r_0^2 \frac{m^2}{(\tilde{s}-m^2)}\frac{1}{\eta^2}\frac{du}{(1+u)^2}\left[-2\,{J_s}^2+\eta^2\left(1+\frac{u^2}{2\,(u+1)}\right)(\,{J_{s-1}}^2+{J_{s+1}}^2-2\,{J_s}^2\,)\right]$$

Bessel functions J_s are functions of argument α. Using the following relation between Bessel functions J_{s-1}, J_{s+1},J_s and equation (3):

$$\alpha\,J_{s-1}+\alpha\,J_{s+1} = 2s\,J_s\,,\quad \alpha = -2s\frac{\sqrt{u(u_s-u)}}{u_s}\frac{\eta}{\sqrt{1+\eta^2}}$$

we can rewrite polarized cross-sections (11) in the form:

$$
\begin{aligned}
d\sigma_p &= 4\pi r_0^2 \frac{m^2}{(\tilde{s}-m^2)} \frac{1}{\eta^2} \frac{du}{(1+u)^2} \\
&\left[-{J_s}^2 + \frac{\eta^2}{2} \left(1+\frac{u^2}{2\,(u+1)}\right) \left(2(1-\frac{u}{u_s})\,{J_{s-1}}^2 + 2\frac{u}{u_s}\,{J_{s+1}}^2 - 2\,{J_s}^2\right)\right], \\
d\sigma_n &= 4\pi r_0^2 \frac{m^2}{(\tilde{s}-m^2)} \frac{1}{\eta^2} \frac{du}{(1+u)^2} \\
&\left[-{J_s}^2 + \frac{\eta^2}{2} \left(1+\frac{u^2}{2\,(u+1)}\right) \left(2\frac{u}{u_s}\,{J_{s-1}}^2 + 2(1-\frac{u}{u_s})\,{J_{s+1}}^2 - 2\,{J_s}^2\right)\right] \qquad (12)
\end{aligned}
$$

Polarization $P(u)$ of the backscattered γ-beam is defined as:

$$
P(u) = \frac{d\sigma_n - d\sigma_p}{d\sigma_n + d\sigma_p}
$$

The fraction of photons with polarization σ_n:

$$
P_n(u) = \frac{d\sigma_n}{d\sigma_n + d\sigma_p} \quad = \frac{1+P}{2}
$$

The average polarization of the backscattered photons $< P >$ (for each order s):

$$
< P >= \frac{1}{\sigma_{Unp}} \int P(u) \frac{d\sigma_{Unp}}{du} du = \frac{\sigma_n - \sigma_p}{\sigma_p + \sigma_n}
$$

Note that the difference $(d\sigma_n - d\sigma_p)$ is the difference between helicity-flip and helicity-conserved processes. Hence positive polarization in our definition means that more photons of the maximum energy backscatter by flipping their helicity (as it will be shown later). This has a simple classical analogy: if polarization of the original laser photon is characterized by the electric field vector $\vec{E}$ rotating clockwise along the Z-axis of propagation of the photon then the photon backscattered by $180°$ will have the same polarization vector $\vec{E}$ rotating in the same direction but the photon will propagate in the opposite direction and hence its helicity will flip. In some sense helicity-flip process for circular polarized laser is similar to the polarization-conserved process (parallel polarization between two photons) for linear polarized laser.

Again by analogy with linear polarized laser some useful expressions can be derived for polarized and unpolarized cross-sections at the limit $u \to u_s$. Evaluating expressions for $d\sigma_p$ and $d\sigma_n$ at the limit $u \to u_s$ we get:

$$
\begin{aligned}
\lim_{u\to u_s} P(u) &= 1 \quad \text{For any } s\,; \\
\lim_{u\to u_s} \frac{d\sigma_{Unp}}{du} &= 4\pi r_0^2 \frac{m^2}{(\tilde{s}-m^2)} \frac{1}{2} \left[\frac{1}{1+u_s} + \frac{1}{(1+u_s)^3}\right] \quad \text{For } s=1\,; \\
\lim_{u\to u_s} \frac{d\sigma_{Unp}}{du} &= 0 \quad \text{For } s>1\,.
\end{aligned}
$$

Note again that the above results are valid *for arbitrary field intensity* η. By comparing them with similar results for linear polarized laser one can conclude that circular laser has higher polarization for photons with maximum energy (1); it has the same (as it should be) unpolarized cross-section for s=1 Compton scattering at $u \to u_s$ (2) and its unpolarized cross-section goes to 0 at $u \to u_s$ for higher orders $s > 1$ (3). This fact can be explained by the total angular momentum conservation. Initial state has the total spin of the electron + s-laser photons ($s \pm 1/2$); final state for exactly backscattered photons has the total spin ($1 \pm 1/2$) and no relative angular momentum; the only s when they can be equal in the spin-flip process is $s = 1$:

$$\begin{aligned} 1 \pm \frac{1}{2} &= 1 \pm \frac{1}{2} \\ s \pm \frac{1}{2} &\neq 1 \pm \frac{1}{2} \quad \text{For any } s > 1 \end{aligned}$$

Maximum energy of backscattered photons:

$$\omega_{max} = \omega(u_s) = E_{tot} \frac{s\,x}{1 + s\,x + \eta^2}$$

decreases with the field intensity.

Dependence of polarization $P(u)$ for different orders s versus the kinematic variable u and photon energy ω/E_{tot} is shown on Fig. 6.

Total cross-section for each order s of the multi-photon Compton scattering has the leading term proportional to $1/\eta^{2s-2}$. In addition the total cross-section depends on the field intensity because the range of u decreases with the field intensity for each order s.

For $s = 1$ the total cross-section and average polarization up to the terms proportional to η^2:

$$\begin{aligned} \lim_{x \to 0} \sigma^{(1)}_{Unp} &= 4\pi r_0^2 \frac{m^2}{(\tilde{s} - m^2)} \frac{2}{3} x\,(1 - 1.4\eta^2)\,; \\ \lim_{x \to 0} < P >^{(1)} &= 0\,; \\ \lim_{x \to \infty} \sigma^{(1)}_{Unp} &= 4\pi r_0^2 \frac{m^2}{(\tilde{s} - m^2)} \left(\frac{1}{4}(1 - 6\eta^2) + \frac{1}{2}\ln(x) \right)\,; \\ \lim_{x \to \infty} < P >^{(1)} &= -1\,. \end{aligned}$$

Cross-sections are very much similar to the linear polarization case except that η-dependence is a little different: $(1 - 1.4\eta^2)$ instead of $(1 - 1.35\eta^2)$ and $(1 - 6\eta^2)$ instead of $(1 - 5\eta^2)$ Average photon polarization for $s = 1$ circular laser changes from 0 to -1 for x from 0 to ∞.

For $s = 2$ the total cross-section and average polarization up to the terms proportional to η^4:

$$\lim_{x \to 0} \sigma^{(2)}_{Unp} = 4\pi r_0^2 \frac{m^2}{(\tilde{s} - m^2)} \eta^2 \frac{4}{5} x\,(1 - \eta^2)\,;$$

$$\begin{aligned}
\lim_{x\to 0} < P >^{(2)} &= 0\,; \\
\lim_{x\to\infty} \sigma^{(2)}_{Unp} &= 4\pi r_0^2 \frac{m^2}{(\tilde{s}-m^2)}\,\eta^2\,; \\
\lim_{x\to\infty} < P >^{(2)} &= -\frac{1}{3}\,.
\end{aligned}$$

Average polarization $< P >$ for circular polarized laser as a function of kinematic variable u_s is shown on Fig. 7. We plotted $< P >$ versus variable u_s and not x because in this form it doesn't depend on field intensity η. Average polarization is higher for large u_s : it goes from 0 to $-\frac{1}{2s-1}$ for u_s from 0 to ∞ (see above). In E144 not the average polarization $< P >$ but the differential polarization at the limit u_s, $P(u_s)$ is important because only the highest energy photons contribute to the pair production. If positrons were produced above threshold (as in case of 250 GeV electron beam) then the average polarization $< P >$ is important for *total* positron production rate but $P(u_s)$ will define the rate of positrons produced with *maximum energy*.

Examples of unpolarized and polarized differential cross-sections as a function of the fractional photon energy $s = 1, 2, 3, 4$ and $\eta = 0.1$ are shown on Fig.8 for 46.6 GeV electron beam and Green laser (E144).

Finally we will derive the leading terms of the total cross-sections (11) for $s = 1$ expanded in terms of field intensity η. Unpolarized cross-section is again was checked to be the Klein-Nishina formula identical to expansion for linear polarized laser (10).

For σ_n we get:

$$\begin{aligned}
\frac{d\sigma_n}{du} &= 2\pi r_0^2 \frac{m^2}{\tilde{s}-m^2}\left(\frac{u^3}{(u+1)^3\,x} + 2\frac{u^2}{(u+1)^2\,x^2}\right)\,, \\
\sigma_n &= 2\pi r_0^2 \frac{m^2}{\tilde{s}-m^2}\left(\frac{2x^3+13x^2+18x+8}{2\,x\,(x+1)^2} - \frac{3x+4}{x^2}\ln(x+1)\right)\,.
\end{aligned}$$

And for σ_p:

$$\begin{aligned}
\frac{d\sigma_p}{du} &= 2\pi r_0^2 \frac{m^2}{\tilde{s}-m^2}\left(\frac{1}{(u+1)^3} + \frac{1}{(u+1)} - \frac{u(u+2)^2}{(u+1)^3\,x} + 2\frac{u^2}{(u+1)^2\,x^2}\right)\,, \\
\sigma_p &= 2\pi r_0^2 \frac{m^2}{\tilde{s}-m^2}\left(-\frac{x^2-6x-8}{2\,x\,(x+1)} + \frac{x^2-x-4}{x^2}\ln(x+1)\right)\,.
\end{aligned}$$

One can easily cross-check that at $u = u_s$:

$$\begin{aligned}
\lim_{u\to u_s} \frac{d\sigma_n}{du} &= 4\pi r_0^2 \frac{m^2}{(\tilde{s}-m^2)}\,\frac{1}{2}\left(\frac{1}{1+u_s} + \frac{1}{(1+u_s)^3}\right)\,; \\
\lim_{u\to u_s} \frac{d\sigma_p}{du} &= 0\,, \\
\lim_{u\to u_s} P(u) &= 1\,.
\end{aligned}$$

in agreement with the result that was derived previously.

Some examples of polarization $P(u)$ and polarized cross-sections for circular polarized laser for N=1 and N=2 laser photons can be found on Fig. 9 and 10 for E144 Green laser and 46.6 GeV electron beam.

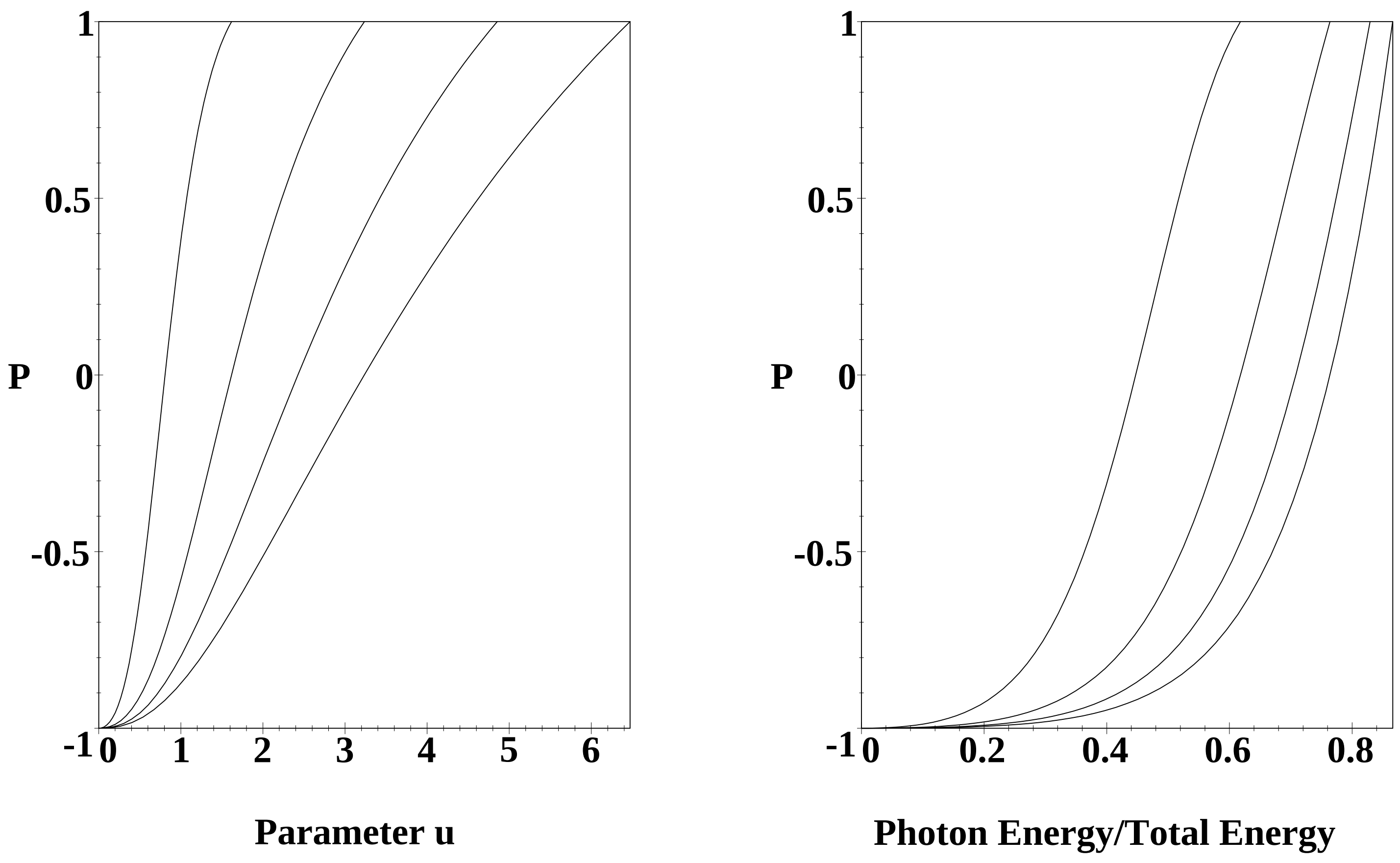

Figure 6: s=1,2,3,4 polarization versus u and fractional photon energy for E144 Green circular polarized laser and 46.6 GeV electron beam. Higher orders s have higher $u_{max} = u_s$ and higher photon energy.

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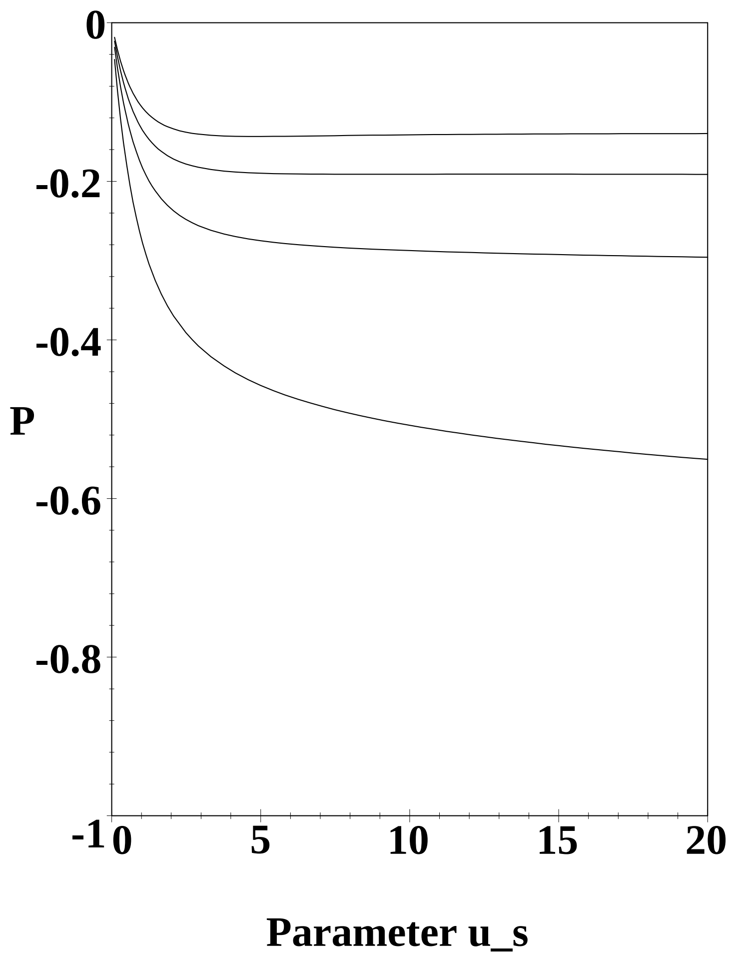

Figure 7: s=1,2,3,4 average polarization $< P >$ versus u_s for circular polarized laser. Higher orders s have higher (negative) polarization. (This plot is for arbitrary laser frequency, electron beam energy and field intensity)

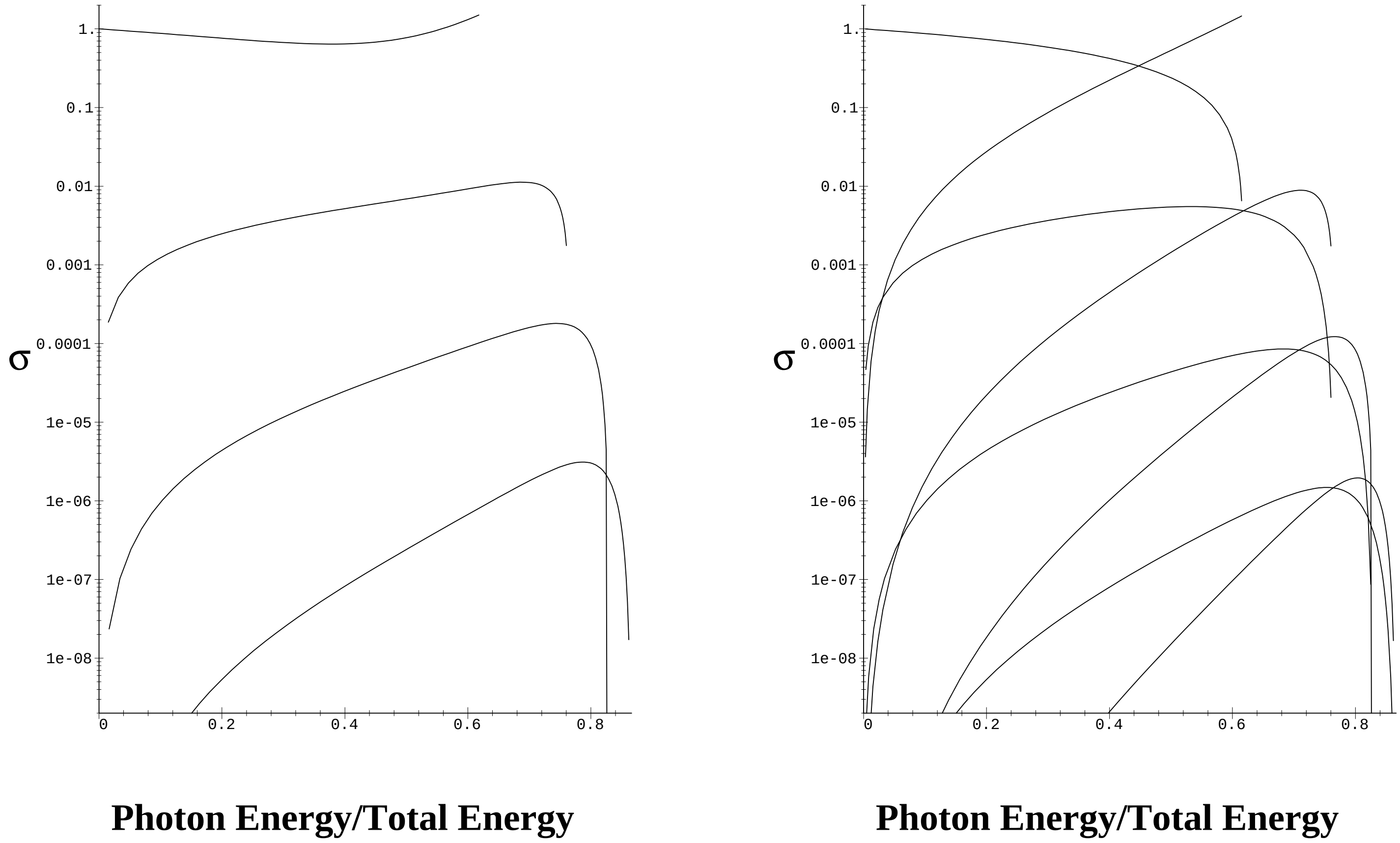

Figure 8: s=1,2,3,4 unpolarized (left plot) and polarized (right plot) cross-sections versus the fractional photon energy ω/E_{tot} for circular polarized Green E144 laser, 46.6 GeV electron beam and $\eta = 0.1$. Higher orders s have higher ω_{max} and lower cross-sections (scaled as $1/\eta^{(2s-2)}$). Vertical scale is in $2r_0^2m^2/(\tilde{s}-m^2)$ units.

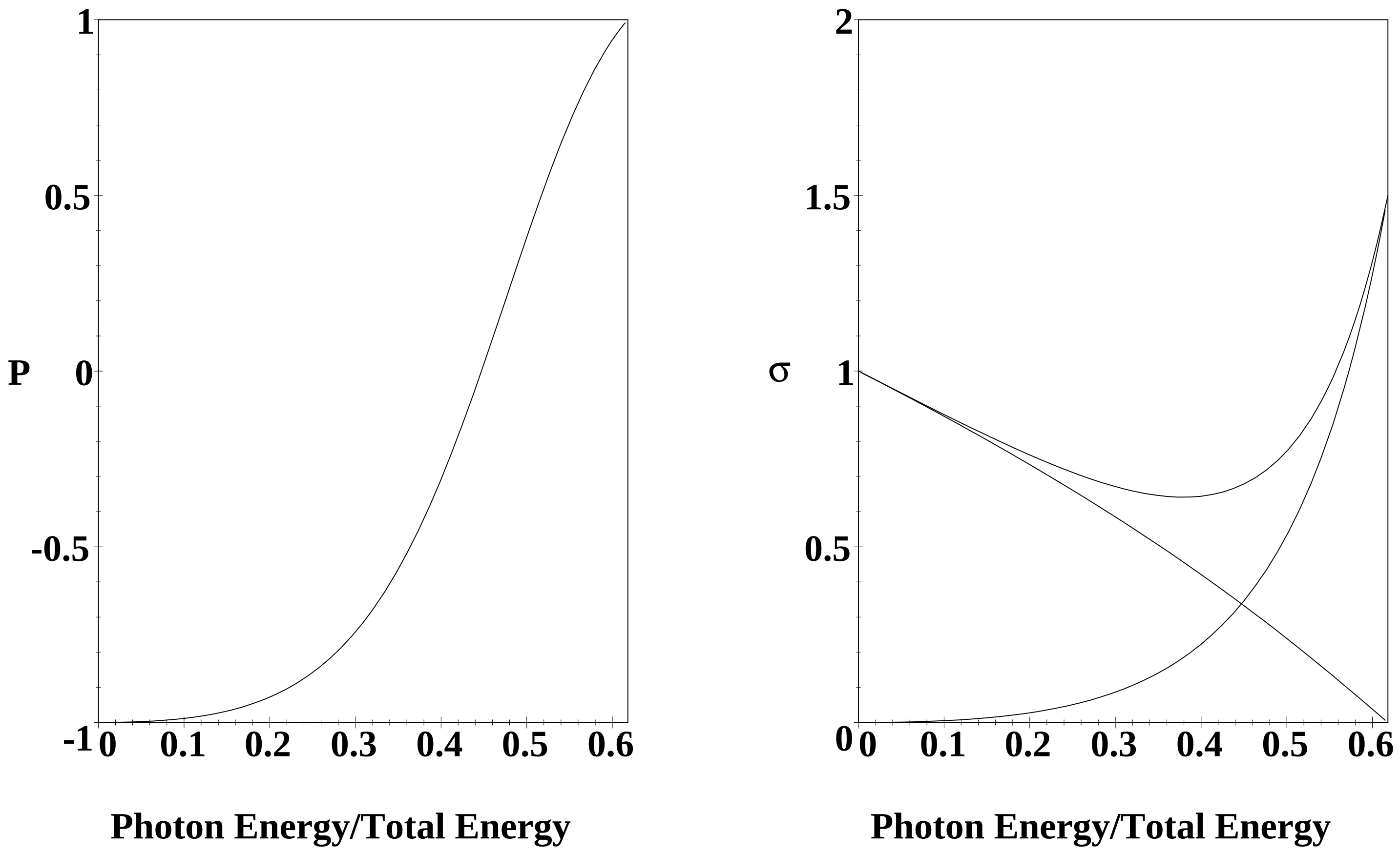

Figure 9: s=1 Compton scattering, circular polarized laser. Polarization of backscattered photons (left plot) and differential cross-sections $\frac{d\sigma_{Unp}}{d\omega\, d\phi}$, $\frac{d\sigma_{\|}}{d\omega\, d\phi}$ and $\frac{d\sigma_{\perp}}{d\omega\, d\phi}$ (right plot) versus the photon energy for E144 configuration with green laser and field intensity $\eta = 0.1$. Cross-sections are in $2r_0^2 m^2/(\tilde{s} - m^2)$ units.

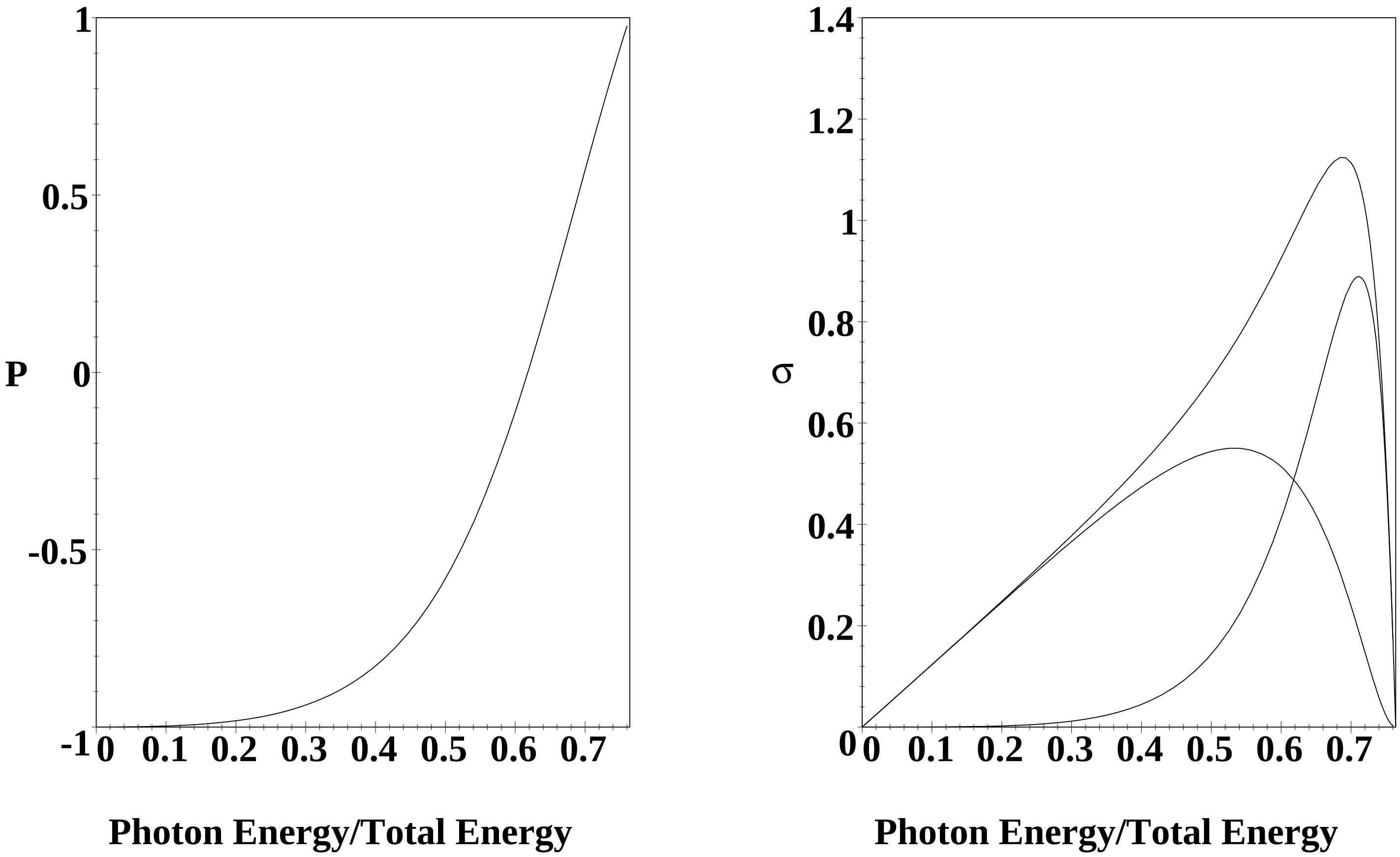

Figure 10: s=2 Compton scattering, circular polarized laser. Polarization of backscattered photons (left plot) and differential cross-sections $\frac{d\sigma_{Unp}}{du\,d\phi}$, $\frac{d\sigma_{\|}}{du\,d\phi}$ and $\frac{d\sigma_{\perp}}{du\,d\phi}$ (right plot) versus the photon energy for E144 configuration with green laser and field intensity $\eta = 0.1$. Cross-sections are in $2r_0^2 m^2/(\tilde{s} - m^2)$ units. Cross-sections for s=2 are additionally multiplied by factor $1/\eta^2$.

4 e^+e^- pair production in interaction of intense linear polarized laser with linear polarized photons.

We will consider process $sk_1 + k_2 \to q_1 + q_2$ where sk_1 stands for s laser photons, k_2 - high energy photon, q_1 and q_2 - electron and positron. Differential cross-sections can be immediately derived from those for the Compton scattering of the electron (6),(8), (9) $sk_1 + q_1 \to k_2 + q_2$ by substitution $k_2 \to -k_2$, $q_1 \to -q_1$.

To obtain cross-sections for pair production from those for Compton scattering one need to take into account the difference in definition of u, different final state phase space as well as different spin-statistics and normalization of wave functions in the initial state. We summarized this rules below:

$$
\begin{array}{rll}
\dfrac{u^2}{(u+1)} & \to -4u & \text{(definition of u)}\,, \\
\dfrac{m^2}{(\tilde{s}-m^2)} & \to \dfrac{m^2}{\tilde{s}} & \text{(massless photon in the initial state)}\,, \\
\dfrac{du}{(1+u)^2} & \to -\dfrac{du}{4\,u\sqrt{u(u-1)}} & \text{(final state phase space)}\,, \\
1 & \to 2\ (\times\ 2) & \text{(spin-statistics, polarized case)}\,, \\
1 & \to 1\ (\times\ 1) & \text{(spin-statistics, unpolarized case)}\,, \\
1 & \to 2\ (\times\ 2) & \text{(normalization in the initial state)}\,.
\end{array} \tag{13}
$$

More detailed explanation of these rules can be found in my thesis. For example spin-statistics factors arise from the rule that the averaging over polarizations of initial states and summation over polarizations of final states of particle involved in the reaction should be made.

Unpolarized cross-section for e^+e^- pairs produced in photon-photon interaction is:

$$
d\sigma_{Unp} = 2\,r_0^2 \frac{m^2}{\tilde{s}} \frac{1}{\eta^2} \frac{du\,d\phi}{u\sqrt{u(u-1)}} \left[A_0^2 + 2\eta^2(2u-1)(A_1^2 - A_0A_2)\right]
$$

Cross-sections for e^+e^- pairs produced by photons with parallel and perpendicular to the laser polarization will be:

$$
\begin{aligned}
d\sigma_{||} &= d\sigma_{Unp} + d\sigma_{Pol}\,, \\
d\sigma_{\perp} &= d\sigma_{Unp} - d\sigma_{Pol}\,, \\
d\sigma_{Pol} &= 2r_0^2 \frac{m^2}{\tilde{s}} \frac{1}{\eta^2} \frac{du\,d\phi}{u\sqrt{u(u-1)}} \\
& \left[+A_0^2\left(1 + \frac{(u_s-u)}{u}(1+\eta^2)(1-\cos(2\phi))\right) - 2\eta^2(A_1^2 - A_0A_2)\right]\,.
\end{aligned} \tag{14}
$$

Note that different spin-statistics of the initial state results in new relation between total and polarized cross-sections:

$$d\sigma_{Unp} = \frac{d\sigma_{||} + d\sigma_{\perp}}{2}$$

Using parameter σ (7) introduced by Narozhny et al we obtain by reversing equations (9) for Compton scattering:

$$\begin{aligned}
d\sigma_{||} &= 2r_0^2 \frac{m^2}{\tilde{s}} \frac{1}{\eta^2} \frac{du\, d\phi}{u\sqrt{u(u-1)}} \\
&\quad \left[+2A_0^2\sigma + 4\eta^2(u-1)(A_1^2 - A_0A_2)\right] , \\
d\sigma_{\perp} &= 2r_0^2 \frac{m^2}{\tilde{s}} \frac{1}{\eta^2} \frac{du\, d\phi}{u\sqrt{u(u-1)}} \\
&\quad \left[+2A_0^2(1-\sigma) + 4\eta^2 u(A_1^2 - A_0A_2)\right] , \\
\sigma &= 2\,\eta^2 \left[\frac{s}{4\beta} - \frac{1}{2} - \left(\frac{\alpha}{8\beta}\right)^2\right] \qquad (15)
\end{aligned}$$

Cross-sections (14) and (15) are equivalent.

Note that because the initial state photons collide at 180° in their COM frame polarized cross-sections are free from possible ambiguity in definition of parallel and perpendicular orientation of polarizations (ambiguity existing in Compton scattering is discussed in section 2).

We define differential asymmetry A in e^+e^- pairs production due to different polarizations of the photon:

$$A(u) = \frac{d\sigma_{||} - d\sigma_{\perp}}{d\sigma_{||} + d\sigma_{\perp}}$$

Total asymmetry A_{tot} for the pair production is:

$$A_{tot} = \frac{1}{\sigma_{Unp}} \int A(u) \frac{d\sigma_{Unp}}{du}\, du = \frac{\sigma_{||} - \sigma_{\perp}}{\sigma_{||} + \sigma_{\perp}}$$

First we will study analytical behavior of differential asymmetry $A(u)$.

Expressions derived below at the limit $u \to u_s$ are valid for arbitrary field intensity η. In the LAB frame $u = u_s$ corresponds to the minimum and maximum energy of the positron E_{e^+}. It will be shown below that for $u_s \gg 1$ most of the positrons are created near $u = u_s$ and contrary for $u_s \sim 1$ most of the positrons are created near $u = 1$.

By evaluating expressions for $d\sigma_{||}$ and $d\sigma_{\perp}$ at the limit $u \to u_s$ (the same way as we did for Compton scattering) we get:

$$\lim_{u \to u_s} A(u) = -\frac{1}{2u_s - 1} \quad \text{For odd } s = 1, 3, 5, ... ;$$

$$
\begin{aligned}
\lim_{u\to u_s} A(u) &= 0 \qquad\qquad \text{For even } s=2,4,6,\ldots;\\
\lim_{u\to u_s}\frac{d\sigma_{Unp}}{du\,d\phi} &= 2r_0^2\frac{m^2}{\tilde{s}}\eta^{2s-2}\,K_s\,\frac{1}{2}\,\frac{(2u_s-1)}{u_s\sqrt{u_s(u_s-1)}} \quad \text{For odd } s=1,3,5,\ldots;\\
\lim_{u\to u_s}\frac{d\sigma_{Unp}}{du\,d\phi} &= 2r_0^2\frac{m^2}{\tilde{s}}\eta^{2s-2}\,\frac{1}{2}\frac{u_s}{u_s\sqrt{u_s(u_s-1)}} \quad \text{For even } s=2,4,6,\ldots;\\
K_s &= 1,\left(\frac{3}{4}\right)^2,\left(\frac{25}{32}\right)^2,\ldots \quad \text{for } s=1,3,5,\ldots.
\end{aligned}
$$

(K_s are the same as for Compton scattering)

We should notice that the asymmetry $A(u)$ at $u\to u_s$ is **negative** - pair production by photons with the same polarization as the laser is **unfavorable**. Differential unpolarized cross-sections for different orders s have very similar form as those for the Compton scattering. Due to the fact that $K_s\to 1/2$ for large s differential unpolarized cross-sections for odd and even orders s asymptotically approach each other at $s\to\infty$. This is another similarity with the Compton scattering.

The above results are valid *for arbitrary field intensity* η.

For large η the "gap" of kinematically allowed positron (electron) energies is reduced by the field. The energy of the positron (electron) in the LAB frame that corresponds to invariant u is

$$
E_{e^+} = \frac{E_{Tot}}{2}\left(1\pm\sqrt{1-\frac{1}{u}}\right) \tag{16}
$$

One value of u corresponds to two energies of the positron symmetrical with respect to $E_{Tot}/2$ in the LAB frame. (These two energies correspond to two angles of the positron Θ and $(\pi-\Theta)$ in the COM frame). Because the differential cross-section should not be changed by replacing positron with electron it should be symmetric with respect to $E_{Tot}/2$ in the LAB frame. The range of positron energies that corresponds to $u=[1..u_s]$ is:

$$
\begin{aligned}
E_{e^+} &= \left[E_{Tot}/2 \quad .. \quad E_{Tot}/2\left(1\pm\sqrt{1-\frac{1}{u_s}}\right)\right],\\
[E_{min}..E_{max}] &= \left[E_{Tot}/2\left(1-\sqrt{1-\frac{1}{u_s}}\right) \quad .. \quad E_{Tot}/2\left(1+\sqrt{1-\frac{1}{u_s}}\right)\right] =\\
&= \quad E_{Tot}/2\left(1\pm\sqrt{1-\frac{1+\eta^2}{s\,x}}\right).
\end{aligned}
$$

From the last equation one can see that the energy range available for pair creation became smaller for higher field intensities η.

If positrons are created well above the threshold ($u_s\gg 1$) then the positron (electron) energy range is almost $(0..E_{Tot})$. And contrary for positrons created near the threshold ($u_s\simeq 1$) the energy range is small (compared to E_{tot}) and centered around $E_{Tot}/2$ as it is the case for E144 experiment.

At the limit $u \to 1$, evaluation of A(u) doesn't have simple analytical form in general case. Below the result for $s = 1$ (which is valid *for arbitrary field intensity* η) is shown:

$$\lim_{u\to 1} A(u) = -\frac{1}{u_s^2 + 2u_s - 2} \quad \text{For } s = 1\,.$$

In the LAB frame $u = 1$ corresponds to the energy of the positron equals the half of the total energy: $E_{e^+} = \frac{1}{2}E_{tot}$. Note that the asymmetry $A(u)$ is again negative.

Dependence of differential asymmetry $A(u)$ in positron production for different orders s versus the kinematic variable u and positron fractional energy E_{e^+}/E_{tot} is shown on Fig. 11. Two plots are equivalent if relation (16) between u and E_{e^+} is taken into account.

Total asymmetry A_{tot} in pair production for linear polarized laser as a function of kinematic variable $u_s > 1$ is shown on Fig. 12 for $\eta = 0.1$. Total asymmetry A_{tot} as a function of u_s is an oscillating function - higher the order s, more oscillations. In general second derivative $\partial^2 A_{tot}/\partial u_s^2$ has s regions with constant (positive or negative) sign. Asymptotic behavior of A_{tot} is the following:

$$\begin{aligned}
\lim_{u_s\to 1} A_{tot} &= -1 \qquad \text{For odd } s = 1, 3, 5, \ldots\,;\\
\lim_{u_s\to 1} A_{tot} &= 0 \qquad \text{For even } s = 2, 4, 6, \ldots\,;\\
\lim_{u_s\to \infty} A_{tot} &= 0 \qquad \text{For any } s\,.
\end{aligned}$$

From the above relations it follows that positron production is suppressed at the threshold ($A_{tot} = -1$) for odd powers s including important case $s = 1$ for positron production above the threshold.

Average polarization is higher for small x and goes to 0 at $x \to \infty$ (see above) by analogy with Compton scattering for linear polarized laser.

Examples of unpolarized and polarized differential cross-sections as a function of the fractional positron energy $s = 4, 5, 6$ and $\eta = 0.1$ are shown on Fig.13.

We will derive leading terms in η for the total cross-sections (15) for $s = 1$.

Expanding expression $\frac{1}{\eta^2}\left[A_0^2 + 2\eta^2(2u-1)(A_1^2 - A_0A_2)\right]$ in series of η and integrating over ϕ and then over $u = [0...x]$ we get:

$$\begin{aligned}
\frac{d\sigma_{Unp}}{du} &= 4\pi r_0^2 \frac{m^2}{\tilde{s}} \left(\frac{(2u-1)x^2 + 2u(x-u)}{2x^2 u\sqrt{u(u-1)}} \right)\,,\\
\sigma_{Unp} &= 4\pi r_0^2 \frac{m^2}{\tilde{s}} \left(\frac{3-w^4}{2} \ln\frac{1+w}{1-w} - w(2-w^2) \right)\,. \qquad (17)
\end{aligned}$$

where we used variable w:

$$w = \sqrt{1 - \frac{1}{x}} = \sqrt{1 - \frac{(2m)^2}{\tilde{s}}}$$

Variable w is another convenient representation of the energy available for creation of e^+e^- pair; the minimum,maximum energy of the positron (electron) is related to w by:

$$E_{e^+,max,min} = E_{Tot}\frac{1 \pm w}{2}$$

Equation (17) is well-known Breit-Wheeler equation.

Polarized cross-sections for $s = 1$ in the approximation of weak field are:

$$\begin{aligned}
\frac{d\sigma_{\|}}{du} &= 4\pi r_0^2 \frac{m^2}{\tilde{s}} \left(\frac{(2u-1)x^2 + 2u(x - \frac{3}{2}u)}{2x^2 u\sqrt{u(u-1)}} \right), \\
\sigma_{\|} &= 4\pi r_0^2 \frac{m^2}{\tilde{s}} \left(\frac{(5-3w^2)(1+w^2)}{4} \ln\frac{1+w}{1-w} - \frac{w(5-3w^2)}{2} \right). \qquad (18)
\end{aligned}$$

$$\begin{aligned}
\frac{d\sigma_{\perp}}{du} &= 4\pi r_0^2 \frac{m^2}{\tilde{s}} \left(\frac{(2u-1)x^2 + 2u(x - \frac{1}{2}u)}{2x^2 u\sqrt{u(u-1)}} \right), \\
\sigma_{\perp} &= 4\pi r_0^2 \frac{m^2}{\tilde{s}} \left(\frac{7-2w^2-w^4}{4} \ln\frac{1+w}{1-w} - \frac{w(3-w^2)}{2} \right). \qquad (19)
\end{aligned}$$

Note that the difference between parallel and perpendicular orientations of polarizations is in the term $(x - \frac{3}{2}u)$ versus $(x - \frac{1}{2}u)$ in differential cross-section. Parallel polarization cross-section is always less than perpendicular polarization: asymmetry is maximum at the $u = u_s$ ($u_s = x$ in the weak field approximation $s = 1$) and minimum at $u = 1$. Dependence of the total asymmetry versus x is shown on Fig. 14 for $s = 1$.

Finally we notice the asymptotic behavior of the Breit-Wheeler cross-sections for $w \to 0$ (positron production near the threshold):

$$\begin{aligned}
\sigma_{Unp}(w \to 0) &= 4\pi r_0^2 \frac{m^2}{\tilde{s}} \left(w + 2w^3 + O(w^4) \right), \\
\sigma_{\|}(w \to 0) &= 4\pi r_0^2 \frac{m^2}{\tilde{s}} \left(\frac{10}{3}w^3 + O(w^4) \right), \\
\sigma_{\perp}(w \to 0) &= 4\pi r_0^2 \frac{m^2}{\tilde{s}} \left(2w + \frac{2}{3}w^3 + O(w^4) \right).
\end{aligned}$$

Asymmetry in positron production near the threshold has the following asymptotic form:

$$\begin{aligned}
A_{tot}(w \to 0) &= -1 + \frac{10}{3}w^2 + O(w^3), \qquad \lim_{w\to 0} A = -1, \\
A_{tot}(x \to \infty) &= -\frac{1}{2x(\ln(4x)-1)} + O\left(\frac{1}{x^2}\right), \qquad \lim_{x\to\infty} A = -0.
\end{aligned}$$

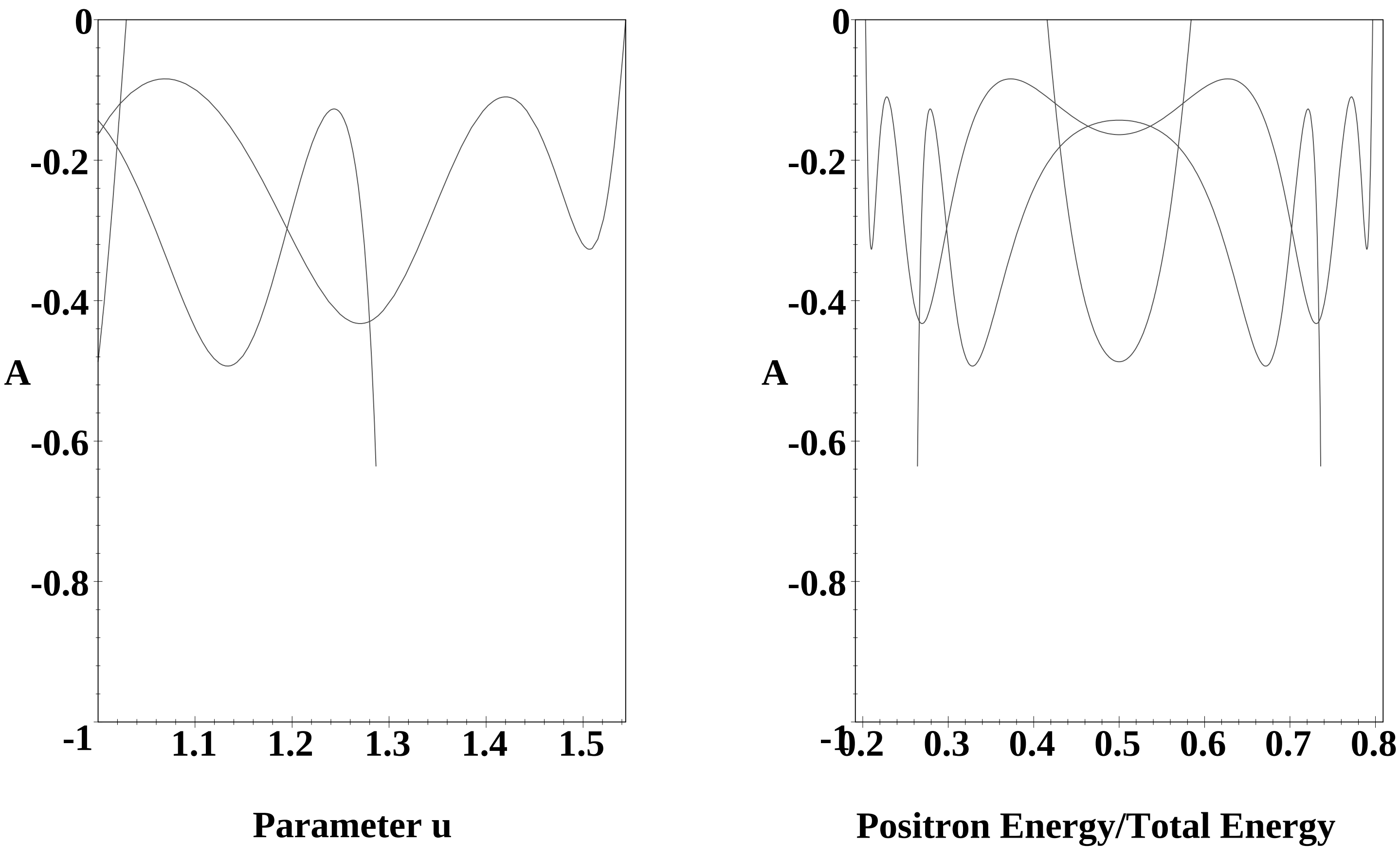

Figure 11: s=4,5,6 asymmetry versus u and fractional positron energy for E144 Green linear polarized laser and $E_{tot} = 29$ GeV photon. Higher orders s have higher $u_{max} = u_s$ and higher maximum positron energy.

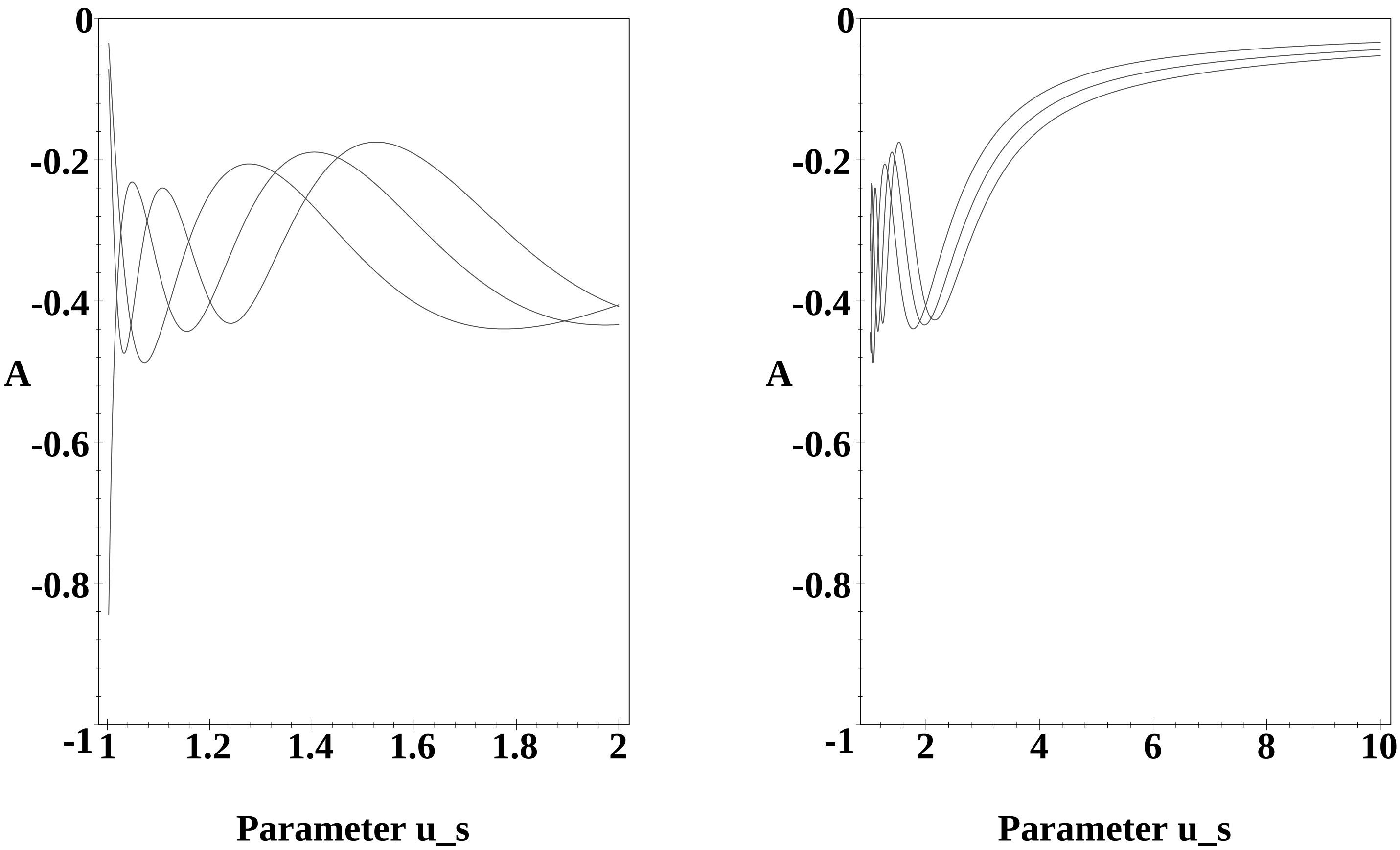

Figure 12: s=4,5,6 total asymmetry A_{tot} in pair production versus u_s for linear polarized laser. Two plots differs by horizontal scale only. (These plots are for arbitrary laser frequency, photon energy and field intensity; s=4,5,6 are shown because of the relevance to E144.)

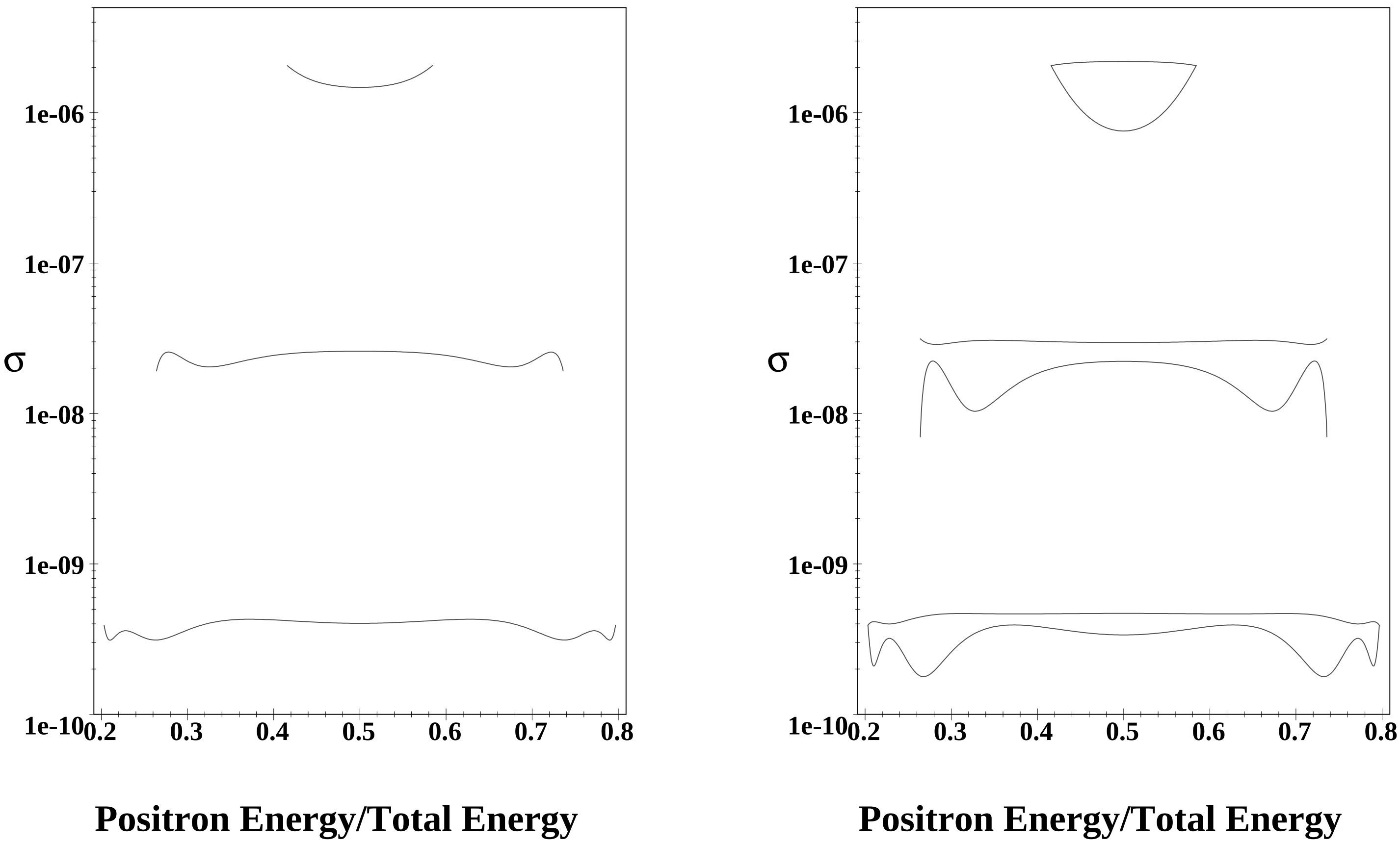

Figure 13: s=4,5,6 unpolarized (left plot) and polarized (right plot) differential cross-sections versus the fractional positron energy E_{e^+}/E_{tot} for linear polarized E144 Green laser, 29 GeV photons and $\eta = 0.1$. Higher orders s have higher $E_{e^+,max}$ and lower cross-sections (scaled as $1/\eta^{(2s-2)}$). Cross-sections are in $2r_0^2m^2/\tilde{s}$ units.

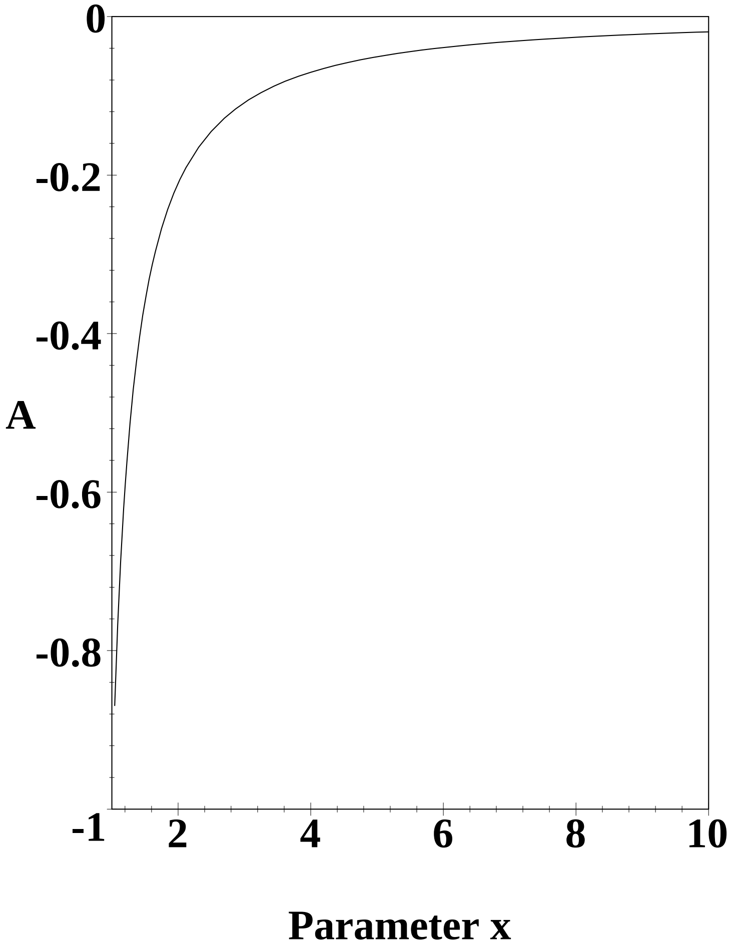

Figure 14: Total asymmetry in s=1 positron production in the focus of linear polarized laser versus the parameter x for arbitrary laser frequency.

5 e^+e^- pair production in interaction of intense circular polarized laser with circular polarized photons.

We will repeat calculations of the previous section for positron produced by circular polarized laser interacting with circular polarized photons.

Unpolarized cross-section for e^+e^- pair in photon-photon interaction is:

$$d\sigma_{Unp} = 4\pi r_0^2 \frac{m^2}{\tilde{s}} \frac{1}{\eta^2} \frac{du}{u\sqrt{u(u-1)}} \left[{J_s}^2 + \frac{\eta^2}{2}(2u-1)({J_{s-1}}^2 + {J_{s+1}}^2 - 2\,{J_s}^2) \right]$$

Cross-sections for e^+e^- produced by photon with the positive and negative helicities (laser assumed to have positive helicity):

$$\begin{aligned}
d\sigma_p &= d\sigma_{Unp} + d\sigma_{Pol} \,, \\
d\sigma_n &= d\sigma_{Unp} - d\sigma_{Pol} \,, \\
d\sigma_{Pol} &= 4\pi r_0^2 \frac{m^2}{\tilde{s}} \frac{1}{\eta^2} \frac{du}{u\sqrt{u(u-1)}} \\
& \left[\frac{\eta\sqrt{1+\eta^2}}{2}(2u-1)(J_{s-1} - J_{s+1}) J_s \frac{2u - u_s}{\sqrt{u(u_s - u)}} \right] \,.
\end{aligned} \tag{20}$$

Or using relation between Bessel functions $\alpha\, J_{s-1} + \alpha\, J_{s+1} = 2s\, J_s$ and equation (3):

$$\begin{aligned}
d\sigma_p &= 4\pi r_0^2 \frac{m^2}{\tilde{s}} \frac{1}{\eta^2} \frac{du}{u\sqrt{u(u-1)}} \\
& \left[-{J_s}^2 + \frac{\eta^2}{2}(2u-1) \left(2(1 - \frac{u}{u_s})\, {J_{s-1}}^2 + 2\frac{u}{u_s}\, {J_{s+1}}^2 - 2\, {J_s}^2 \right) \right] \,, \\
d\sigma_n &= 4\pi r_0^2 \frac{m^2}{\tilde{s}} \frac{1}{\eta^2} \frac{du}{u\sqrt{u(u-1)}} \\
& \left[-{J_s}^2 + \frac{\eta^2}{2}(2u-1) \left(2\frac{u}{u_s}\, {J_{s-1}}^2 + 2(1 - \frac{u}{u_s})\, {J_{s+1}}^2 - 2\, {J_s}^2 \right) \right]
\end{aligned} \tag{21}$$

$$d\sigma_{Unp} = \frac{d\sigma_p + d\sigma_n}{2}$$

Differential asymmetry A in e^+e^- pairs production is defined as:

$$A(u) = \frac{d\sigma_n - d\sigma_p}{d\sigma_n + d\sigma_p}$$

Total asymmetry A_{tot} for pair production is:

$$A_{tot} = \frac{1}{\sigma_{Unp}} \int A(u) \frac{d\sigma_{Unp}}{du}\, du = \frac{\sigma_n - \sigma_p}{\sigma_n + \sigma_p}$$

First we show analytical behavior of differential asymmetry $A(u)$.

By evaluating expressions for $d\sigma_p$ and $d\sigma_n$ at the limit $u \to u_s$ (maximum/minimum positron energy) we get:

$$\begin{aligned}
\lim_{u\to u_s} A(u) &= -1 \qquad \text{For any } s\,; \\
\lim_{u\to u_s} \frac{d\sigma_{Unp}}{du} &= 4\pi r_0^2 \frac{m^2}{\tilde{s}} \frac{1}{2} \frac{(2u_s - 1)}{u_s\sqrt{u_s(u_s-1)}} \qquad \text{for } s = 1\,; \\
\lim_{u\to u_s} \frac{d\sigma_{Unp}}{du} &= 0 \qquad \text{for } s > 1\,.
\end{aligned}$$

The above results are valid for arbitrary field intensity η.

Asymmetry $A(u)$ is negative at $u = u_s$. Production of positrons with maximum (minimum) energy is **strongly suppressed** by photons with the same spin direction ("n"-state) as one of the original laser photons. This result can be explained by the total angular momentum conservation: if high energy photon in the "n"-state interacts with laser in the "p" state with creation of the pair which goes exactly in the forward-backward direction (hence there is no relative angular momentum) then the total angular momentum of photons ≥ 2 can not be equated by the electron-positron pair which has (maximum) total angular momentum 1 for forward-backward production.

The above observation is an important argument against positron production by the laser in the same laser focus as one where high energy photons were created. Because photons of the maximum energy are almost 100% "n" photons (assuming "p" laser) they cannot produce positrons near $u \approx u_s$ of the spectrum (maximum energy) by interaction with the same "p"-polarized laser. This effect can be reversed if one can use the second laser focus opposite polarization ("n") for positron production.

At the limit $u \to 1$:

$$\lim_{u\to 1} A(u) = -\frac{u_s(2-u_s)}{u_s^2 + 2u_s - 2} \quad \text{for any } s$$

.

In the LAB frame $u = 1$ corresponds to the energy of the positron equals half of the total energy: $E_{e^+} = \frac{1}{2}E_{tot}$. Asymmetry $A(1)$ is negative below $u_s = 2$ and positive above $u_s = 2$ (see the above formula). If pair creation occurs near the threshold $u_s \approx 1$ then $A(1) = -1$.

Dependence of differential asymmetry $A(u)$ in positron production for different orders s versus the kinematic variable u and positron fractional energy E_{e^+}/E_{tot} is shown on Fig. 15. Two plots are equivalent if relation (16) between u and E_{e^+} is taken into account.

Total asymmetry A_{tot} in pair production for circular polarized laser as a function of kinematic variable u_s ($u_s \geq 1$) is shown on Fig. 16 for $\eta = 0.1$. Total asymmetry A_{tot} as a function of u_s is a slowly decreasing function. Higher order s, faster the decrease. Asymptotic behavior of A_{tot} is the following:

$$\begin{aligned} \lim_{u_s \to 1} A_{tot} &= -1 \qquad \text{For any } s\,; \\ \lim_{u_s \to \infty} A_{tot} &= +\frac{1}{2\,s-1}\,. \end{aligned}$$

From the above relations it follows that positron production is suppressed at the threshold ($A_{tot} = -1$) for all powers s including an important case $s = 1$ for positron production above the threshold.

Examples of unpolarized and polarized differential cross-sections as a function of the fractional positron energy for $s = 4, 5, 6, 7$ and $\eta = 0.1$ are shown on Fig.17.

We will derive leading terms in η for the total cross-sections (20) for $s = 1$.

Expanding expression $\frac{1}{\eta^2}\left[{J_s}^2 + \frac{\eta^2}{2}(2u-1)({J_{s-1}}^2 + {J_{s+1}}^2 - 2\,{J_s}^2)\right]$ in series of η and integrating over $u = [0...x]$ we get:

$$\begin{aligned} \frac{d\sigma_{Unp}}{du} &= 4\pi r_0^2 \frac{m^2}{\tilde{s}} \left(\frac{(2u-1)x^2 + 2u(x-u)}{2x^2 u\sqrt{u(u-1)}} \right)\,, \\ \sigma_{Unp} &= 4\pi r_0^2 \frac{m^2}{\tilde{s}} \left(\frac{3-w^4}{2} \ln\frac{1+w}{1-w} - w(2-w^2) \right)\,. \end{aligned} \tag{22}$$

It is of course the same Breit Wheeler formula as one obtained for linear polarized laser. Variable w is the same:

$$w = \sqrt{1-\frac{1}{x}} = \sqrt{1-\frac{(2m)^2}{\tilde{s}}}$$

Polarized cross-sections for $s = 1$ in the approximation of weak field:

$$\begin{aligned} \frac{d\sigma_p}{du} &= 4\pi r_0^2 \frac{m^2}{\tilde{s}} \left(\frac{u^2(2x-1)}{x^2 u\sqrt{u(u-1)}} \right)\,, \\ \sigma_p &= 4\pi r_0^2 \frac{m^2}{\tilde{s}} \left(\frac{(1-w^2)(1+w^2)}{2} \ln\frac{1+w}{1-w} + w(1+w^2) \right)\,. \end{aligned} \tag{23}$$

$$\begin{aligned} \frac{d\sigma_n}{du} &= 4\pi r_0^2 \frac{m^2}{\tilde{s}} \left(\frac{(x-u)(2ux-x-u)}{x^2 u\sqrt{u(u-1)}} \right)\,, \\ \sigma_n &= 4\pi r_0^2 \frac{m^2}{\tilde{s}} \left(\frac{5-w^4}{2} \ln\frac{1+w}{1-w} - w(5-w^2) \right)\,. \end{aligned} \tag{24}$$

The difference between "p" and "n" polarizations has the form:

$$\frac{d\sigma_p}{du} - \frac{d\sigma_n}{du} = 4\pi r_0^2 \frac{m^2}{\tilde{s}} \left(\frac{x(2u-x)(2u-1)}{x^2 u\sqrt{u(u-1)}} \right) .$$

The sign of the difference is defined by $(2u - x)$; because u varies from 1 to x "p" differential cross-section is always lower than "n" in the region $u = [1..x/2)$ and always higher that "n" in the region $u = (x/2..x]$ (they are equal for $u = x/2$).

The total asymmetry in the pair production will depend on the relative contributions from regions $u = [1..x/2)$ and $u = (x/2..x]$. It is negative for parameter x near the threshold where the pair creation is dominated by the region $u = 1$ and it is positive for x far from threshold where the pair creation is dominated by $u = x$. Dependence of the total asymmetry versus x for $s = 1$ is plotted on Fig. 18. On can see that $A_{tot} = 0$ corresponds to the parameter $x \approx 3.804$. Asymptotic behavior of the total asymmetry:

$$\begin{aligned}
A_{tot}(w \to 0) &= -1 + \frac{8}{3}w^2 + O(w^3), \qquad \lim_{w\to 0} A = -1\,, \\
A_{tot}(x \to \infty) &= +\frac{\ln(4x) - 3}{\ln(4x) - 1} + O\left(\frac{1}{x}\right), \qquad \lim_{x\to\infty} A = +1\,.
\end{aligned}$$

The asymptotic behavior of the Breit-Wheeler cross-sections for $w \to 0$ (positron production at the threshold):

$$\begin{aligned}
\sigma_{Unp}(w \to 0) &= 4\pi r_0^2 \frac{m^2}{\tilde{s}} \left(w + 2w^3 + O(w^4) \right) , \\
\sigma_p(w \to 0) &= 4\pi r_0^2 \frac{m^2}{\tilde{s}} \left(2w + \frac{4}{3}w^3 + O(w^4) \right) , \\
\sigma_n(w \to 0) &= 4\pi r_0^2 \frac{m^2}{\tilde{s}} \left(\frac{8}{3}w^3 + O(w^4) \right) .
\end{aligned}$$

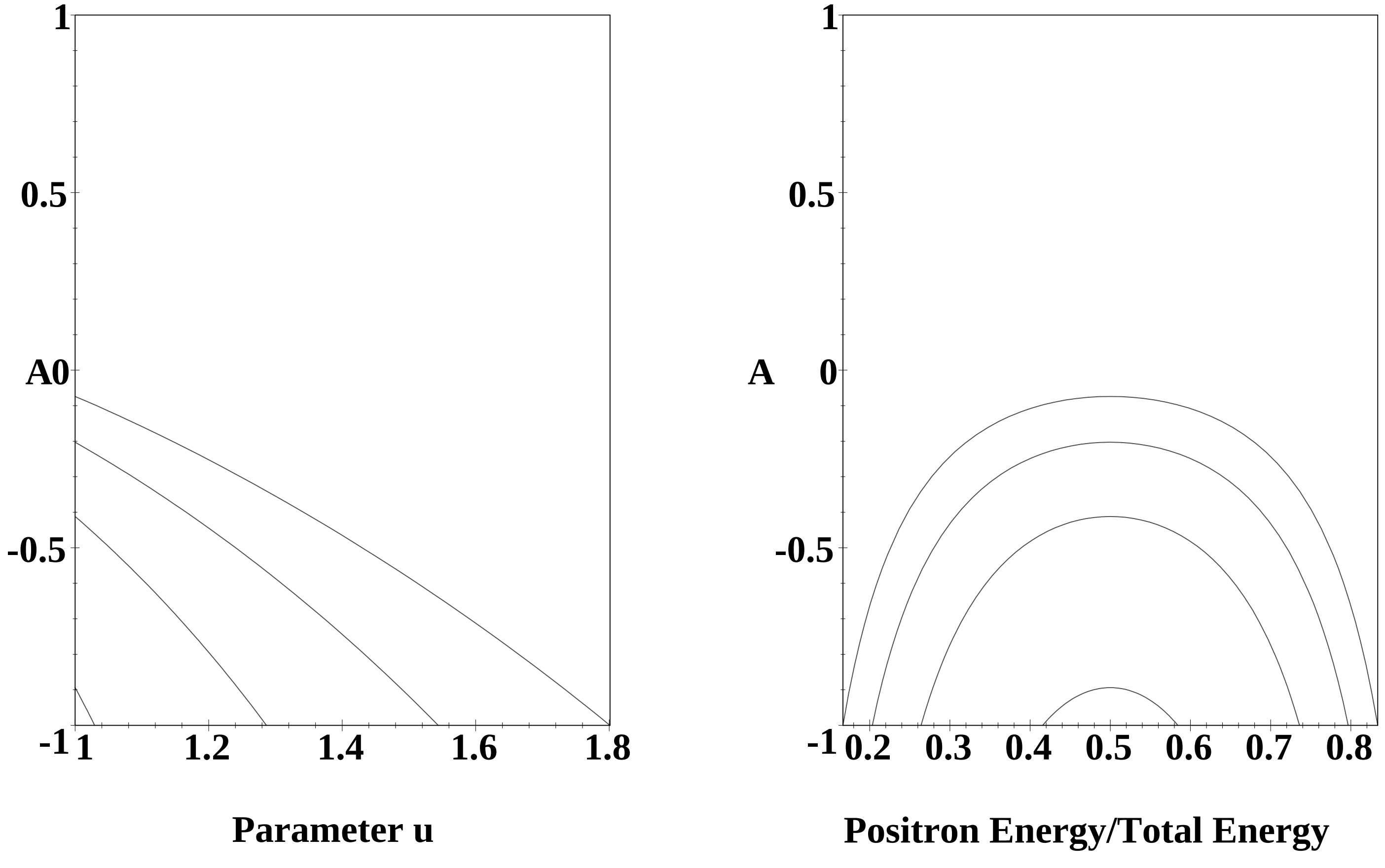

Figure 15: s=4,5,6,7 asymmetry versus u and fractional positron energy for E144 Green circular polarized laser and $E_{tot} = 29$ GeV photon. Higher orders s have higher $u_{max} = u_s$ and higher maximum positron energy.

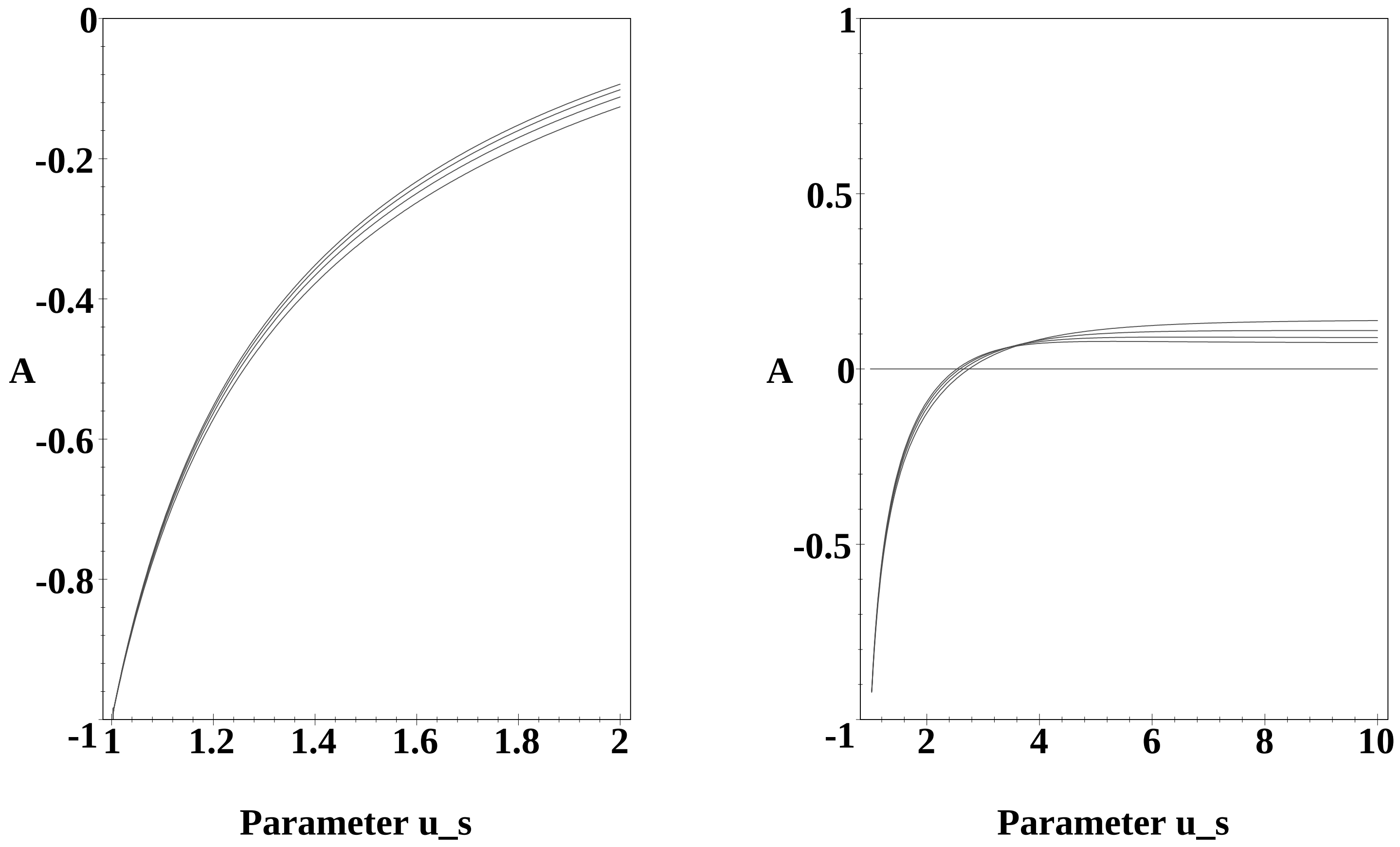

Figure 16: s=4,5,6,7 total asymmetry A_{tot} in pair production versus u_s for circular polarized laser. (These plots are for arbitrary laser frequency, photon energy and field intensity; s=4,5,6,7 are shown because of the relevance to E144.)

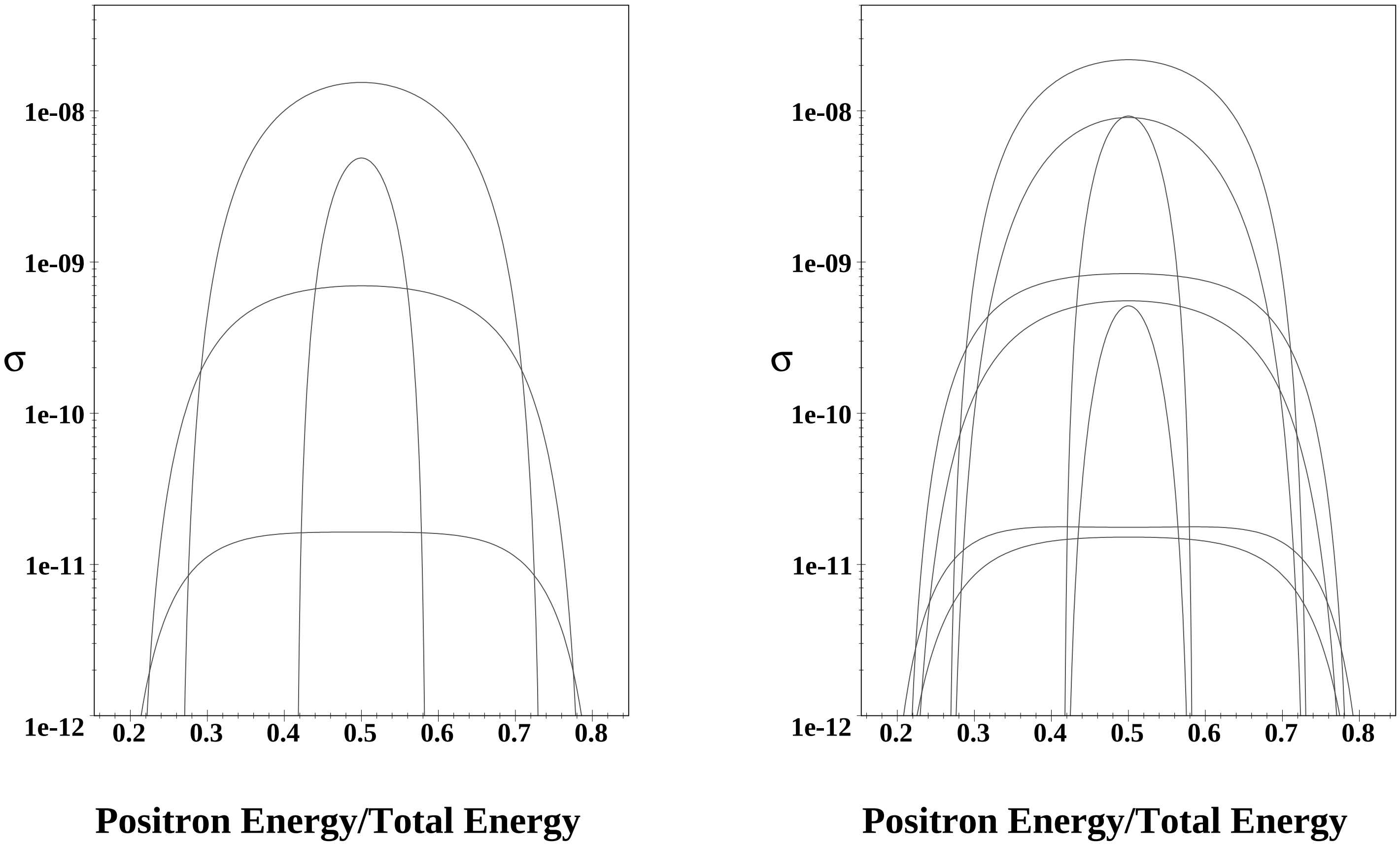

Figure 17: s=4,5,6,7 unpolarized (left plot) and polarized (right plot) differential cross-sections versus the fractional positron energy E_{e^+}/E_{tot} for circular polarized E144 Green laser, $E_{tot} = 29$ GeV photons and $\eta = 0.1$. Higher orders s have higher $E_{e^+,max}$ and lower cross-sections (scaled as $1/\eta^{(2s-2)}$). Cross-sections are in $2r_0^2m^2/\tilde{s}$ units.

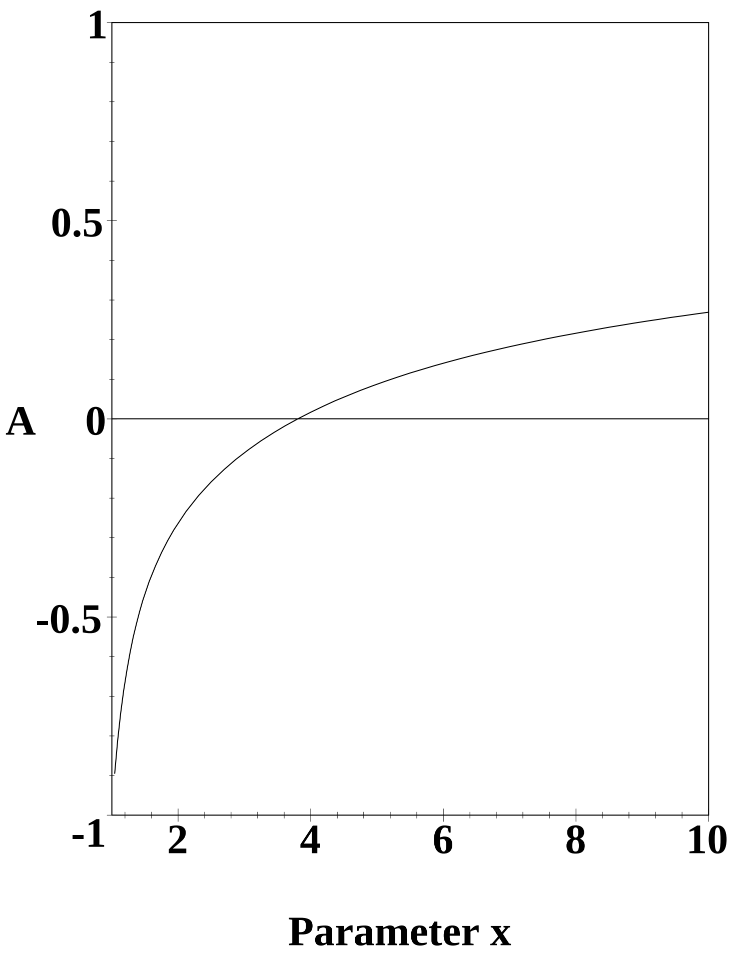

Figure 18: Total asymmetry in s=1 positron production in the focus of circular polarized laser versus the parameter x for arbitrary laser frequency.

6 E144 experiment: Discussion of polarization effects for positrons produced below the threshold for s=1.

In E144 experiment 46.6 GeV electron beam collides with Green ($\omega = 2.34$ eV) or IR ($\omega = 1.17$ eV) laser at 17°, that corresponds to the parameter x for Compton scattering equals 1.63 for Green laser and 0.82 for IR laser.

As an example we calculate the maximum energy of backscattered photons:

$$\begin{aligned}
\omega_s^{max} &= E_e \frac{sx}{1+\eta^2+sx}\,, \\
\omega_s^{max} &= 28.9\,(s=1) \quad 35.7\,(s=2) \text{ GeV for green laser, } \eta = 0\,; \\
\omega_s^{max} &= 21.0\,(s=1) \quad 28.9\,(s=2) \text{ GeV for IR laser, } \eta = 0\,.
\end{aligned}$$

Maximum energy of backscattered photons will be lower for higher field intensities η.

In general case if high energy photon was produced through multiphoton Compton scattering of an electron involving s_0 laser photons in a region with the field η_0 and e^+e^- pair was produced through interaction of this photon with s laser photons in a region with the field η then the above parameter x should be:

$$x \geq \frac{2}{s}\left(1+\sqrt{1+\frac{s}{s_0}\frac{1+\eta_0^2}{1+\eta^2}}\right)(1+\eta^2)$$

Value of x equals to the right side of the above equation corresponds to the threshold when e^+e^- pair can be produced, we will call this value $x(s, s_0)$. Condition for pair production can be written then as:

$$\begin{aligned}
x(s,s_0) &\leq x\,, \\
x(s,s_0) &= \frac{2}{s}\left(1+\sqrt{1+\frac{s}{s_0}\frac{1+\eta_0^2}{1+\eta^2}}\right)(1+\eta^2)\,.
\end{aligned}$$

Below we calculated parameter $x(s, s_0)$ at $\eta = \eta_0 = 0$ for different s, s_0:

$$\begin{aligned}
x(1,1) &= 4.83 \quad x(1,2) = 4.45 \quad x(1,3) = 4.31 \quad x(1,4) = 4.24 \\
x(2,1) &= 2.73 \quad x(2,2) = 2.41 \quad x(2,3) = 2.29 \quad x(2,4) = 2.22 \\
x(3,1) &= 2.00 \quad x(3,2) = 1.72 \quad x(3,3) = 1.61 \quad x(3,4) = 1.55 \\
x(4,1) &= 1.62 \quad x(4,2) = 1.37 \quad x(4,3) = 1.26 \quad x(4,4) = 1.21 \\
x(5,1) &= 1.38 \quad x(5,2) = 1.15 \quad x(5,3) = 1.05 \quad x(5,4) = 1.00 \\
x(6,1) &= 1.22 \quad x(6,2) = 1.00 \quad x(6,3) = 0.91 \quad x(6,4) = 0.86 \\
x(7,1) &= 1.09 \quad x(7,2) = 0.89 \quad x(7,3) = 0.81 \quad x(7,4) = 0.76 \\
x(8,1) &= 1.00 \quad x(8,2) = 0.81 \quad x(8,3) = 0.73 \\
x(9,1) &= 0.93 \quad x(9,2) = 0.74 \\
x(10,1) &= 0.86
\end{aligned}$$

Because $x(s, s_0)$ should be less than $x = 1.63$ for Green laser and $x = 0.82$ for IR laser the total number of photons $N = s + s_0$ involved in positron production should be at least 5 ($s = 4, s_0 = 1$) for Green laser and 10 ($s = 8, s_0 = 2$ or $s = 7, s_0 = 3$) for IR laser.

By minimization of the expression for $x(s, s_0)$ it is easy to show that for a given x the minimum number of laser photons required to produce a pair $N = s + s_0$ and values of s and s_0 are:

$$N = \frac{8}{x}, \quad s_0 = \frac{1}{4}N, \quad s_0 = \frac{3}{4}N.$$

For instance $x = 0.82$ (IR laser) corresponds to $N = 9.7$, $s_0 = 2.4$ and $s = 7.3$. The closest integer numbers are $N = 10$, $s_0 = 3$, $s = 7$ and $N = 10$, $s_0 = 2$, $s = 8$ in agreement with the above result.

Positrons were produced in E144 using Green laser. The above arguments suggest that the minimum configuration to produce a pair with green laser is $N = 5$, $s_0 = 1$ and $s = 4$. But $N = 5, s = 4$ process is only possible at very small field intensities $\eta \to 0$ ($x(4,1) = 1.62$ vs $x = 1.63$) when positron rate is also negligible. At higher field intensities value of $x(4,1)$ increases - for instance $x(4,1) = 1.66$ for $\eta = 0.2$, $\eta_0 = 0$ and positron production is forbidden kinematically. For positron production cross-sections to be large enough field intensity should be high $\eta > 0.3 - 0.4$ and hence $N = 5$ channel is forbidden kinematically. Positrons are produced in E144 through $N = 6$ process with s=4 ($x(4,2)_{min} = 1.37$) or s=5 ($x(5,1)_{min} = 1.38$). Processes with $s = 3, 2, 1$ are negligible because they have larger $x(s, s_0)$ (for example $x(3,3)_{min} = 1.61$). For $s = 5$ $A_{tot} = -1$ at $u_s = 1$ but rapidly increases to the level $A_{tot} = -0.3 \div -0.4$ at $u_s > 1.05$; for $s = 4$ $A_{tot} = 0$ at $u_s = 1$ and also rapidly decreases to the level $A_{tot} = -0.3 \div -0.4$ (Figure 12). Involvement of higher orders N reduces the cross-section proportional to η^{-2N} but increases the "gap" of allowed positron energies.

We will assume that the main contribution to the pair production is from the $N = 6$ processes: ($s = 5, s_0 = 1$) or ($s = 4, s_0 = 2$). Then:

$$N_{tot}^{(s=5,s_0=1)} \approx N_{tot,Unp}^{(s=5,s_0=1)} \left(1 + P(u_1)\, A_{tot}^{(s=5)}\right),$$
$$N_{tot}^{(s=4,s_0=2)} \approx N_{tot,Unp}^{(s=4,s_0=2)} \left(1 + P(u_2)\, A_{tot}^{(s=4)}\right).$$

For linear polarized laser: Taking $P(u_1) \approx 0.67$ $P(u_2) \approx 0$ (Figure 1) and $A_{tot}^{(s=4,5)} \approx -0.3 \div -0.4$ (Figure 12) (average of fast oscillating total asymmetry functions versus u_s for $s = 4, 5$) we get:

$$\sigma_{tot}^{(s=5,s_0=1)} \approx 0.75 \div 0.80\, \sigma_{Unp}^{(s=5,s_0=1)}, \quad \sigma_{tot}^{(s=4,s_0=2)} \approx 1.00\, \sigma_{Unp}^{(s=4,s_0=2)}.$$

In average correction less than $-20 \div -25\%$ should be applied to calculations with unpolarized cross-sections to get the correct positron rate.

More accurate estimate should involve different weights for different processes and different orders N.

Exact calculations with numerical integration method gave the overall effect of polarization for E144 equal to factor 0.75 at $\eta = 0.2$, 0.77 at $\eta = 0.32$ and 0.82 at $\eta = 1$ in good agreement with the above estimate.

For circular polarized laser: Taking $P(u_1) = P(u_2) \approx 1$ (Figure 6) and

$$\begin{array}{lcll} A_{tot}^{(s=4)} & \approx & -1 & (u_s \approx 1)\,, \\ A_{tot}^{(s=5)} & \approx & -0.5 & (u_s \approx 5/4)\,, \\ A_{tot}^{(s=6)} & \approx & -0.3 & (u_s \approx 6/4)\,, \\ A_{tot}^{(s=7)} & \approx & -0.2 & (u_s \approx 7/4)\,. \end{array}$$

(Figure 16) we get:

$$\begin{array}{lcl} \sigma_{tot}^{(s=4)} & \approx & 0.0\,\sigma_{Unp}\,, \\ \sigma_{tot}^{(s=5)} & \approx & 0.5\,\sigma_{Unp}\,, \\ \sigma_{tot}^{(s=6)} & \approx & 0.7\,\sigma_{Unp}\,, \\ \sigma_{tot}^{(s=7)} & \approx & 0.8\,\sigma_{Unp}\,. \end{array}$$

Assuming again that the main contribution to the positron production for circular polarized laser is from the processes $N = 6$ $(s = 5, s_0 = 1)$ or $(s = 4, s_0 = 2)$ we conclude:

$$\sigma_{tot}^{(s=5,s_0=1)} \approx 0.50\,\sigma_{Unp}^{(s=5,s_0=1)}\,, \quad \sigma_{tot}^{(s=4,s_0=2)} \approx 0.00\,\sigma_{Unp}^{(s=4,s_0=2)}\,.$$

$$\sigma_{tot} < 0.5\,\sigma_{Unp}$$

In average it should be more than -50% correction to calculations with unpolarized cross-sections.

More accurate estimate should involve different weights for different processes and different orders N. No calculations using numerical integration method has been done so far for circular polarized laser.

From the above estimates we can conclude that for positron production in E144 linear polarization have $\approx$ factor 2 advantage compared to circular polarization due to effects of polarization of intermediate photon. This is *in addition* to higher unpolarized cross-section for linear polarization compared to circular one (Figures 13,17).

7 NLC positron source example: Discussion of polarization effects for positron production above the threshold for s=1.

Consider positrons produced above the threshold for $s = 1$ through two-step process $s_0 = 1$, $s = 1$ in a relatively weak field; all orders $s > 1$ are insignificant.

There are two effects of polarization of intermediate photon: it affects the total positron rate and the rate of positrons with maximum energy. Positrons with maximum energy are produced by maximum energy photons ($u = u_1$ for Compton scattering) interacting with laser photons at $u = u_1$; hence $A(u_1)$ and $P(u_1)$ are important. For total positron rate A_{tot} and $< P >$ are important.

Polarization effects in total positron rate can be estimated by formulae:

$$\begin{aligned} N_{tot} &= N_{tot,Unp}\,(1+ < P > A_{tot})\ , \\ N_{E_{e^+,max}} &= N_{E_{e^+,max,Unp}}\,(1 + P(u_1)A_{u_1})\ . \end{aligned}$$

Consider example of 250 GeV electrons interacting with 5 eV laser. Parameter $x = u_1 = 19.1$ for Compton scattering, maximum energy of backscattered photons $\omega_{max} = 238$ GeV (most of the positrons are still produced by highest energy photons), parameter $x = u_1 = 4.6$ for pair creation.

Using plots (2), (7), (14), (18) one can estimate $< P >^{(lin)} = 0.1$, $A_{tot}^{(lin)} = -0.1$ for linear polarization and $< P >^{(circ)} = -0.55$, $A_{tot}^{(circ)} = +0.05$ for circular polarization. Effect of polarization on the total positron rate is small (a few percent) for both circular and linear polarized laser.

Positron spectrum is modified significantly though.

Using asymptotic formulae for $P(u_1)$ (sections 2,3) and $A(u_1)$ (sections 4,5) one can calculate $P(u_1)^{(lin)} = 0.10$, $A(u_1)^{(lin)} = -0.12$ for linear polarization and $P(u_1)^{(circ)} = 1$, $A(u_1)^{(circ)} = -1$ for circular polarization. Effect of polarization is small for linear polarized laser and **huge** for circular polarized laser. For circular polarized laser positrons near the high end of the spectrum (maximum energy) are strongly suppressed compared to unpolarized calculations. As we discussed earlier this effect can be explained by the total angular momentum conservation.