Loose Ends in Strong-Field QED (Including Laser Acceleration)

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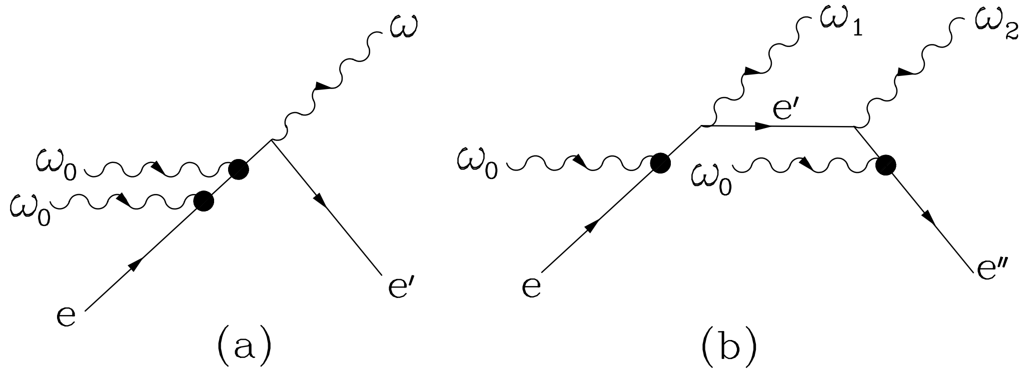
Nonlinear Compton Scattering

(a) $e + n\omega_0 \to e' + \omega$, (nonlinear Compton scattering).



(b) Repeated first-order Compton scattering.

Classical, dimensionless, invariant measure of field strength:

$$\eta = \frac{e\sqrt{\langle A_\mu A^\mu \rangle}}{mc^2} = \frac{eE}{m\omega_0 c}, \quad \text{where } A_\mu \text{ is the vector potential.}$$

$$\frac{v_\perp}{c} \approx \eta \qquad (\text{if } \eta \ll 1).$$

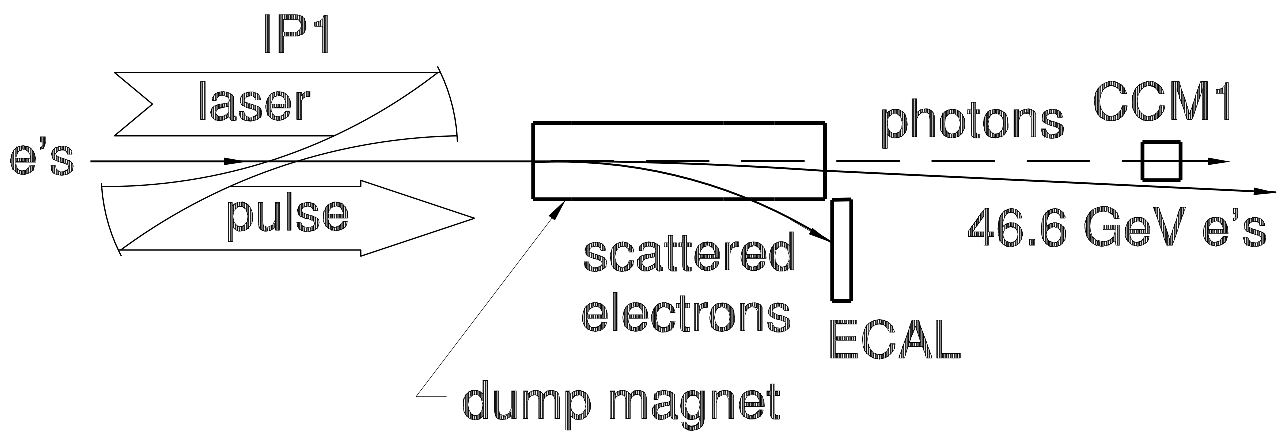
The accelerating electron emits multipole radiation.

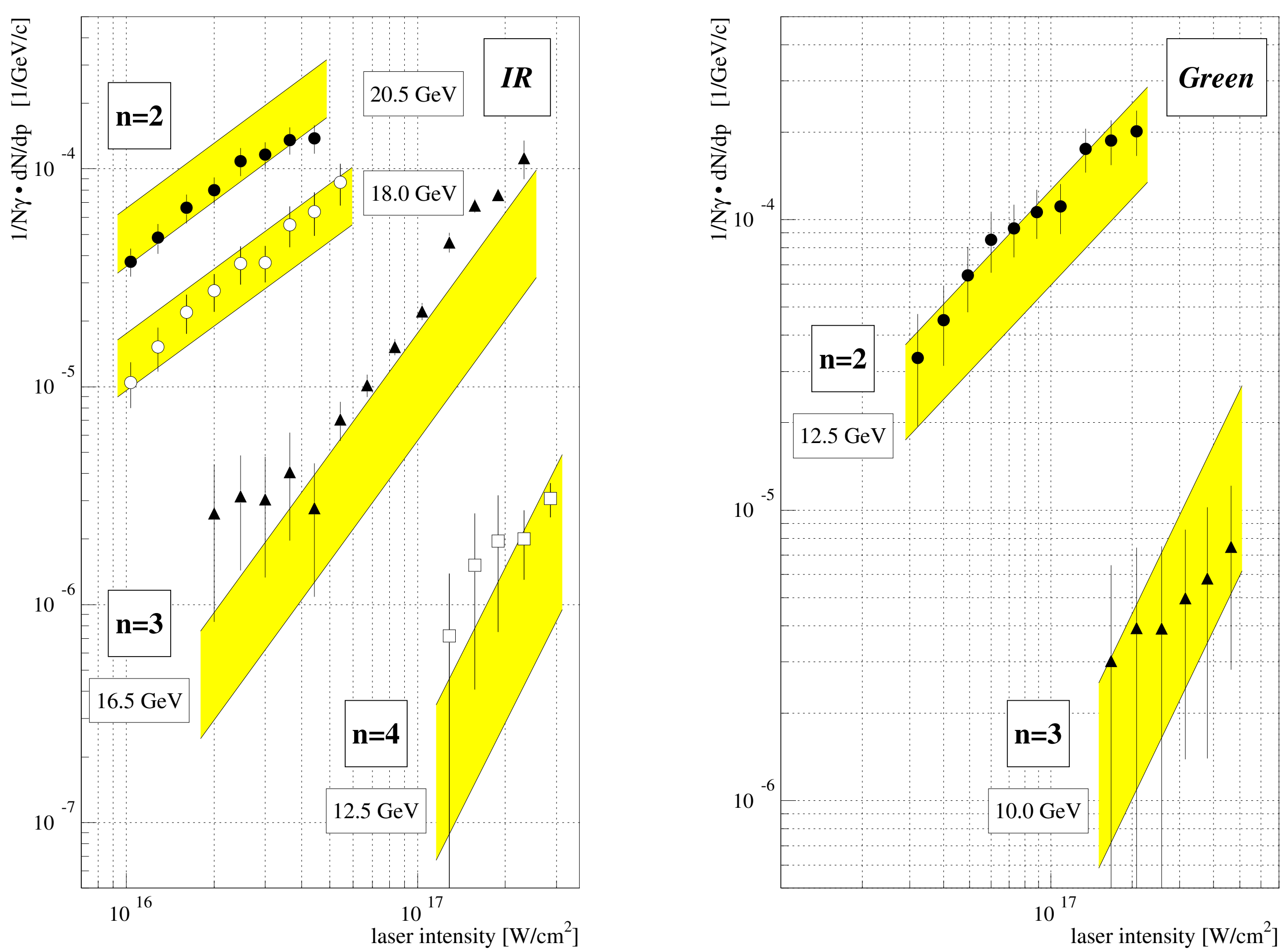
$\text{Rate}_n \propto \eta^{2n} \propto I^n$ for nth-order multipole ($\eta \lesssim 1$).

nth order $\Leftrightarrow$ absorption of n photons before emitting a single higher-energy photon $\Leftrightarrow$ nonlinear Compton scattering.

Observation of Nonlinear Compton Scattering

[C. Bula *et al.*, Phys. Rev. Lett. **76**, 3116 (1996)]





Normalize to scattered photon rate $\Rightarrow$ Rate(order n) $\propto I^{n-1}$.

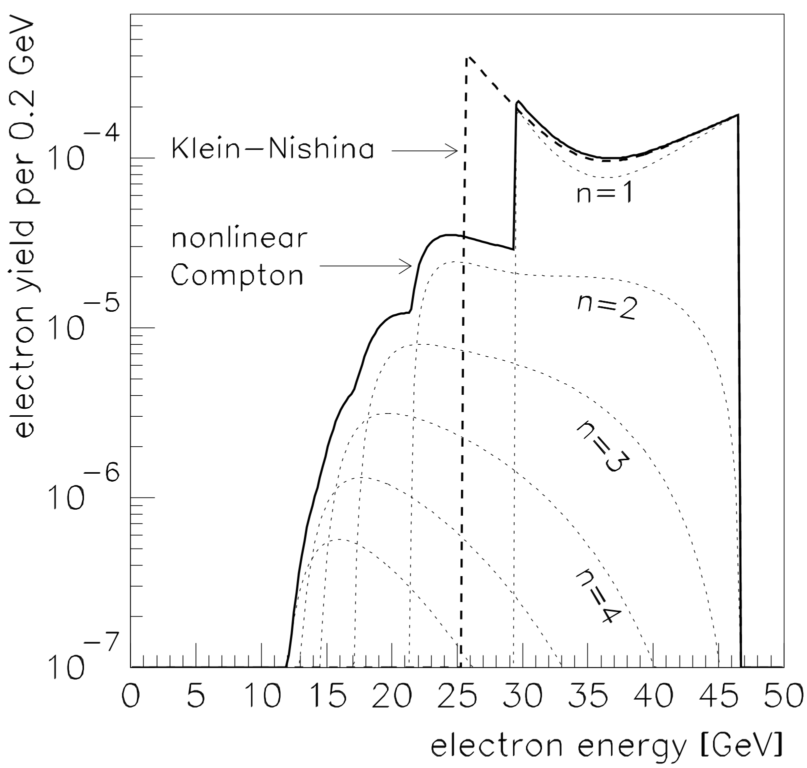
The Mass Shift Effect

An electron propagating in a (periodic) wave field of strength $\eta = eE/m\omega_0 c$ takes on an effective mass

$$\overline{m} = m\sqrt{1+\eta^2}.$$

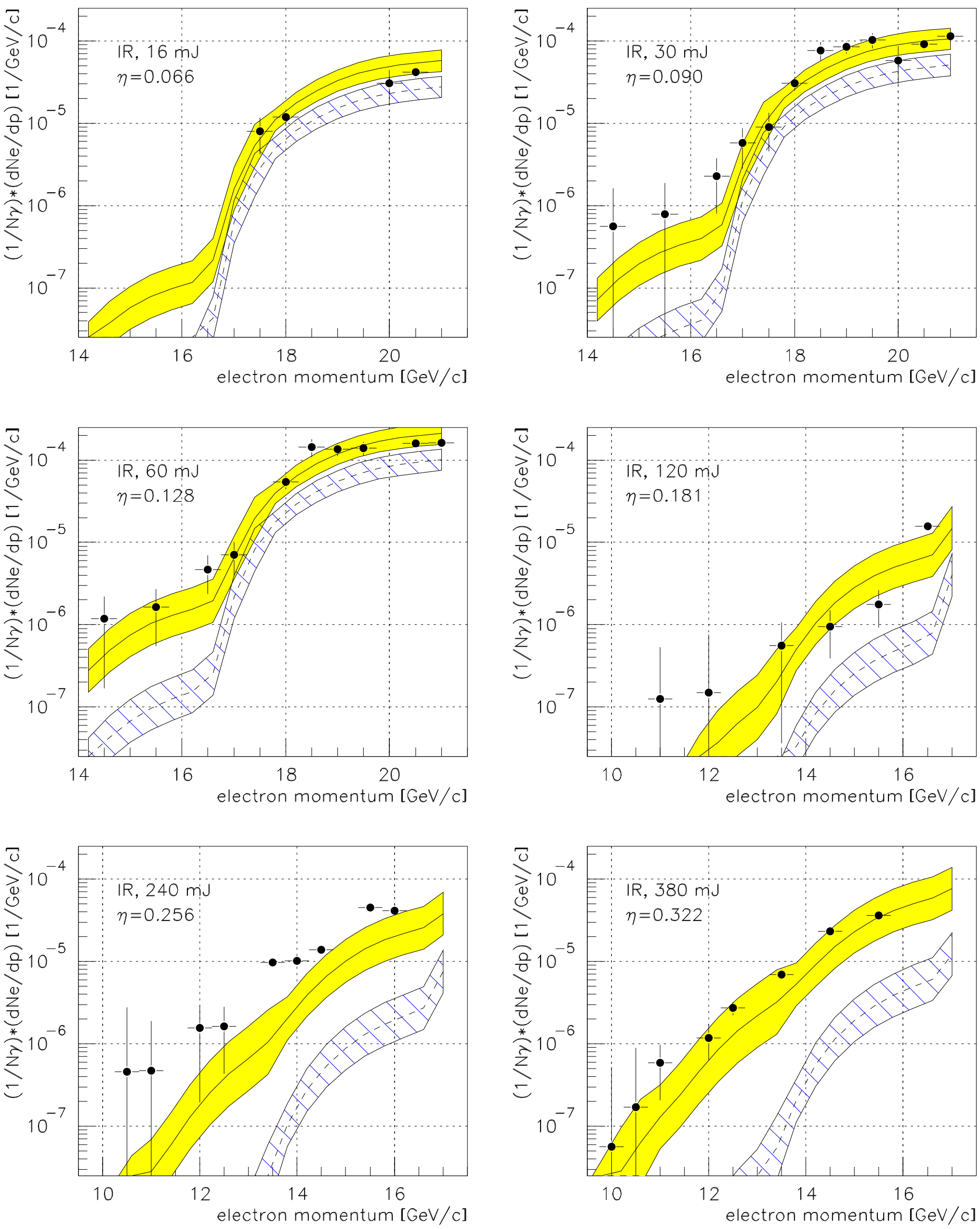
Classical view: the transverse oscillations of the electron are relativistic, so it becomes 'heavier'.

As a result the kinematic limits in nonlinear Compton scattering (and threshold for pair creation) are shifted.



$\eta = 0.64$

Observed Scattering Rates *vs.* Electron Energy



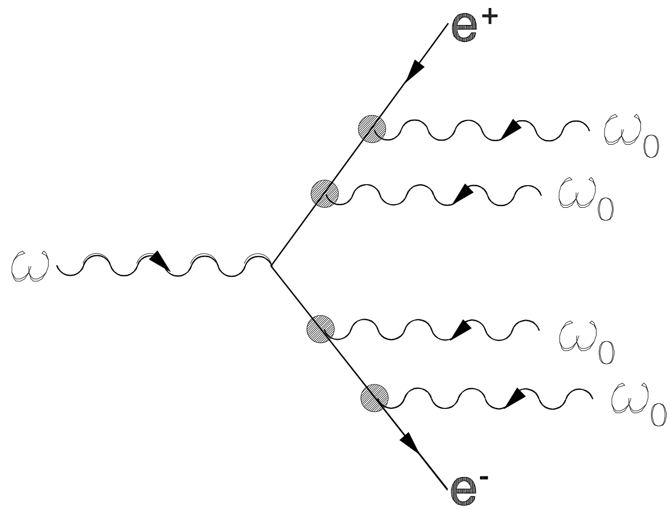
$n = 2$ edge at 17.6 GeV, $n = 3$ edge at 13.5 GeV.

Shaded = full simulation.

Striped = multiple-($n = 1$) scattering only.

Pair Creation by Light

Two step process: $e + \omega_0 \rightarrow e' + \omega$, then $\omega + n\omega_0 \rightarrow e^+e^-$.



Multiphoton pair creation is cross-channel process to nonlinear Compton scattering.

$\Rightarrow$ Similar theories [sums of Bessel functions whose arguments depend on η^2].

$\Rightarrow$ Breit-Wheeler cross section in weak-field limit.

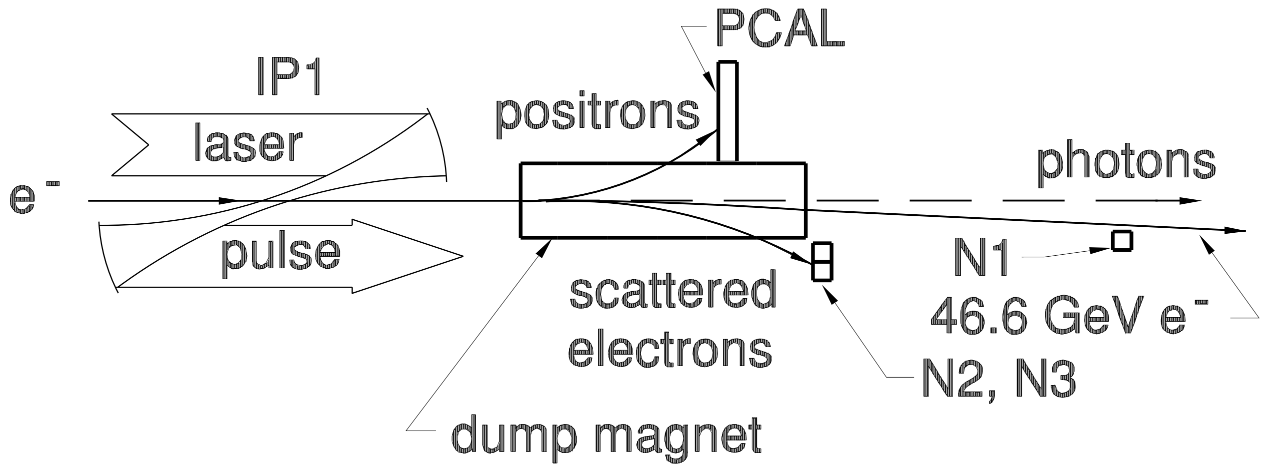
$\omega_{\mathrm{max}} \approx 29$ GeV for 46.6-GeV electrons + ($n = 1$) green laser.

Then need at least $n = 4$ laser photons to produce a pair.

$\Leftrightarrow$ Below threshold for 2-photon pair creation.

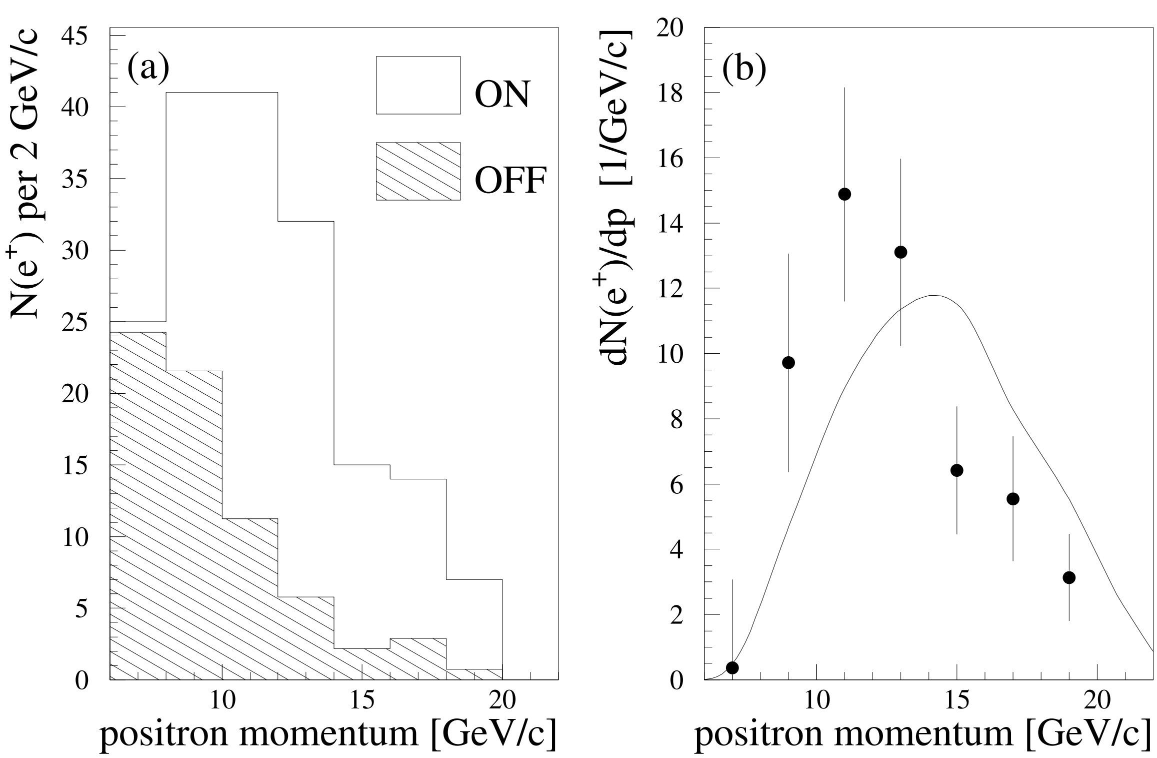
Positron Production by Laser Light

[D. Burke *et al.*, Phys. Rev. Lett. **79**, 1626 (1997)]



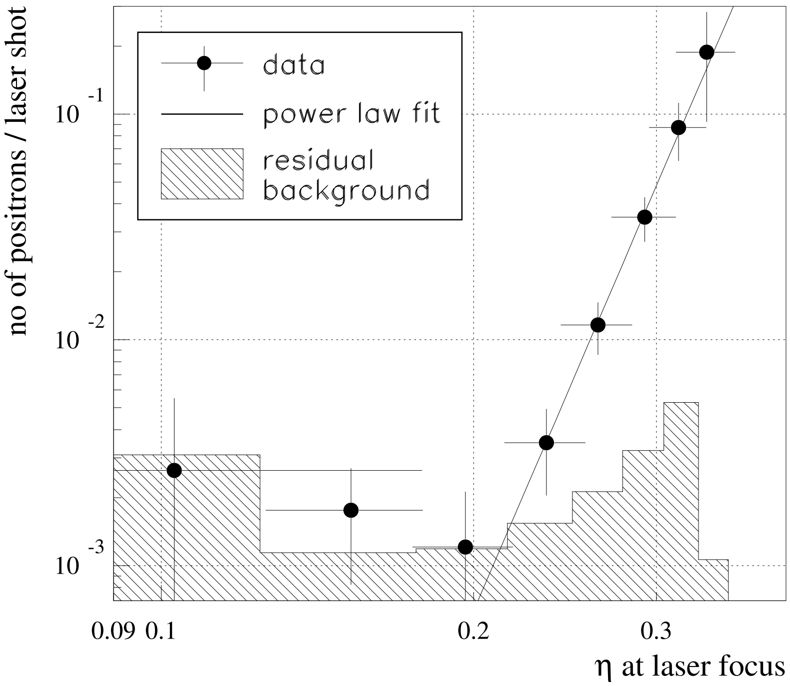
178 laser-on candidates $-0.175\times$ 398 laser-off candidates,

$\Rightarrow$ 106 $\pm$ 14 signal positrons (upper plots, no η cut)

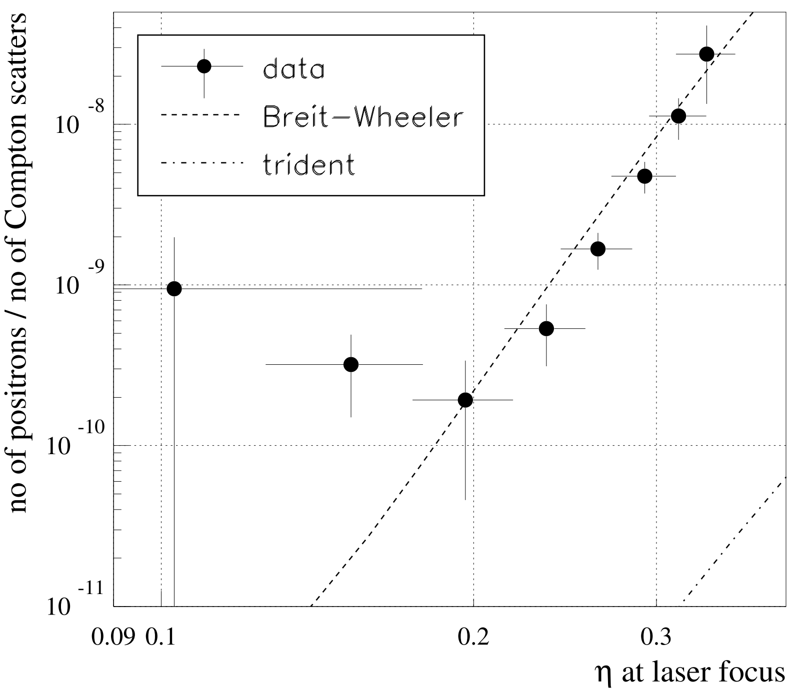


Positron Rate *vs.* η

Rate $\propto \eta^{2n}$ where $n = 5.1 \pm 0.2$ (stat.) $^{+0.5}_{-0.8}$ (syst.):



Normalized to Compton scattering rate:



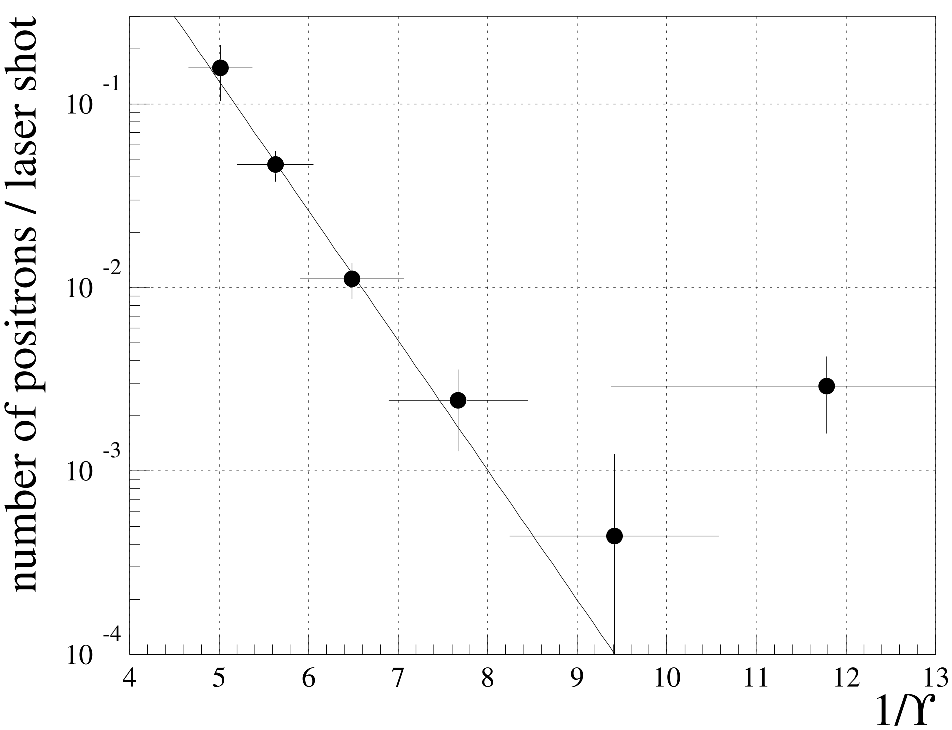
Strong-Field Pair Creation as Barrier Penetration

For a virtual e^+e^- pair to materialize in a field E, the electron and positron must separate by distance d sufficient to extract energy $2mc^2$ from the field:

$$eEd = 2mc^2.$$

The probability of a separation d arising as a quantum fluctuation is related to penetration through a barrier of thickness d:

$$P \propto \exp\left(-\frac{2d}{\lambda_C}\right) = \exp\left(-\frac{4m^2c^3}{e\hbar E}\right) = \exp\left(-\frac{4E_{\rm crit}}{E}\right) = \exp\left(-\frac{4}{\Upsilon}\right).$$



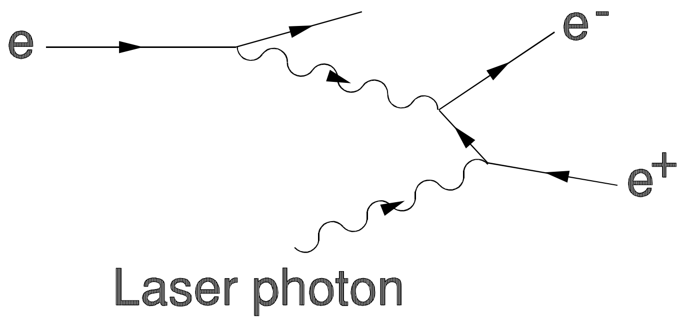
$$E_{\rm crit} = \frac{m^2c^3}{e\hbar},$$

$$\Upsilon = \frac{E}{E_{\rm crit}}.$$

$R_{e^+} \propto \exp[(-2.0 \pm 0.3)/\Upsilon]$.

Loose End #1: Trident Production

$$e + n\omega_0 \to e' e^+ e^-$$



Background when scattering occurs in presence of electron beam.

Theory only approximate:

Weizsäcker-Williams + multiphoton Breit-Wheeler.

$$e \to e' + \omega_1, \qquad \text{followed by} \qquad n\omega_0 + \omega_1 \to e^+ e^-.$$

$$\text{Rate}_{\text{Trident}} = \frac{2\alpha}{\pi} \sum_n \int_{4\overline{m}^2}^{s'_{n,max}} \frac{ds'_n}{s'_n} \ln \frac{s'_{n,max}}{s'_n} \text{Rate}_{\text{BW}}(s'_n, \eta),$$

$$s'_n = (n\omega_0 + \omega_1)^2, \qquad s'_{n,max} = (\sqrt{s_n} - \overline{m})^2,$$

$$s_n = (e + n\omega_0)^2 = \overline{m}^2 + 2n(e \cdot \omega_0).$$

Loose End #2:
Strong-Field Light-by-Light Scattering

$$n\omega_0 + \omega_1 \to \omega_2 + \omega_3$$

(Photon splitting by a laser)

No theory for laser fields with $\eta \gtrsim 1$.

$\eta < 1$ considered by I. Affleck and L. Kruglyak,
Phys. Rev. Lett. **59**, 1065 (1987).

Strong static magnetic field considered by S.L. Adler,
Ann. Phys. **67**, 599 (1971).

Loose End #2a: Vacuum Čerenkov Radiation

A strong wave creates an index of refraction via vacuum polarization.

$\Rightarrow$ Čerenkov radiation for an electron of high-enough energy in the wave (H. Mitter).

Loose End #3: QED Phase Transition

Does it exist at fields $E > E_{\text{crit}} = \dfrac{m^2c^3}{e\hbar} \approx 10^{16}$ V/cm?

Speculations renewed after claims of positron peaks in heavy-ion collisions at Darmstadt.

No strong theoretical argument in favor of a QED phase transition.

D.G. Caldi *et al.*, Phys. Rev. D **39**, 1432 (1989).

Bibliography at

`http://www.math.h.kyoto-u.ac.jp/~takasaki/bib-old/s_c_qed.ref`

Loose End #4: From Hawking to Unruh to ...

Hawking: An observer outside a black hole experiences a bath of thermal radiation of temperature

$$T = \frac{\hbar g}{2\pi c k} ,$$

where g is the local acceleration due to gravity.

Unruh: According to the equivalence principle an accelerated observer in a gravity-free region should also experience a thermal bath with:

$$T = \frac{\hbar a}{2\pi c k} ,$$

where a is the acceleration of the observer as measured in his instantaneous rest frame.

Unruh Radiation (?)

Suppose the observer is an electron, accelerated by a field E.

Thomson scattering off photons in the apparent thermal bath implies a radiation rate:

$$\frac{dU_{\rm Unruh}}{dtd\nu} = \text{thermal energy flux} \times \text{Thomson cross section}$$

$$= \frac{8\pi}{c^2}\frac{h\nu^3}{e^{h\nu/kT}-1} \times \frac{8\pi}{3}r_0^2.$$

On integrating over ν we find

$$\frac{dU_{\rm Unruh}}{dt} = \frac{8\pi^3\hbar r_0^2}{45c^2}\left(\frac{kT}{\hbar}\right)^4 = \frac{\hbar r_0^2 a^4}{90\pi c^6},$$

[Stefan-Boltzman: flux $\propto T^4$, Unruh: $T^4 \propto a^4$.]

This equals the Larmor radiation rate, $dU/dt = 2e^2a^2/3c^3$, when the acceleration $a = eE/m$ is about 10^{31} g, *i.e.*, when

$$E = \sqrt{\frac{60\pi}{\alpha}}\left(\frac{m^2c^3}{e\hbar}\right) \approx 3 \times 10^{18}\ \text{V/cm}.$$

Loose End #5: Laser Acceleration

`http://puhep1.princeton.edu/~mcdonald/accel/`

Can a laser pulse transfer energy to a free electron? *I.e.*, after the pulse has passed, the electron has gained energy.

Pulse $\Rightarrow$ fields vanish at $x_\mu = \pm\infty,\ \mu = 0, 1, 2, 3$

(Also, no nearby walls (transition radiation) or dielectric media (Čerenkov radiation).

Puzzle #1: A weak plane wave shakes a free electron transverse to the wave (momentum) vector; how is momentum conserved?

Conjecture: The field momentum carried in the **interference term** between the plane wave and the dipole radiation field is equal and opposite to the mechanical momentum of the oscillating electron.

This interference term corresponds to the **'dressing'** of the electron in the quantum view. Can one speak of quanta of the interference term?

Puzzle #2: A free electron cannot absorb real photons unless it (a) radiates, or (b) changes its mass. So how could a pulse of real photons add energy to a free electron?

a. The radiation (Compton scattering) and the radiation reaction induced by a laser pulse can indeed change an electron's energy. This is, however, not what is meant by laser acceleration.

b. An electron inside a wave has a shifted mass: $\overline{m} = m\sqrt{1+\eta^2}$, in the quantum view (Volkov solutions to the Dirac equation). This shift is due (?) to absorption without re-emission of wave photons. [The wave is classical in Volkov.]

More precisely, the electron continually absorbs photons from the wave and emits photons back into to wave, but there can be a net absorption (mass shift) while the electron is still in the wave.

$\Rightarrow$ **Temporary acceleration** of electrons.

[If such electrons hit ions they could radiate very energetic photons, $\Rightarrow$ γ-ray bursters?]

A focused laser pulse contains a spectrum of wave vector around a central value. If the laser absorbs photons of one wave number and emits (back into the pulse) at another, momentum is transferred to the electron.

This is a quantum view of the so-callled **ponderomotive force**:

$$\mathbf{F} = -\nabla \overline{m} c^2 \approx -\frac{mc^2}{2} \nabla \eta^2 \qquad \text{for} \qquad \eta \ll 1.$$

[This force has a potential, so $\Delta U = 0$ once outside wave.]

If the pulse is not monochromatic, the electron can absorb photons of one frequency and emit (back into the wave) at another, thereby transferring energy from the wave to the electron. If there is a net transfer of energy before the electron exits the pulse, we could say that the electron has been permanently accelerated by the pulse.

But, does this contradict the basic rule that a free electron can't absorb real photons without radiation?

Or, does the 'dressing' interaction of the electron with the wave count as a kind of radiation that permits net energy transfer?

Puzzle #3: A two-frequency wave.

[Yu and Takahashi, to appear in Phys. Rev. E]

A **semiclassical** analysis of the interaction of a free electron with a plane wave with two frequency components, ω_1 and ω_1 leads to a matrix element that suggests 4 possible interaction terms:

$$n_1\omega_1 + n_2\omega_2 + n_3(\omega_1 + \omega_2) + n_4(\omega_1 - \omega_2) + e \to e' + \omega.$$

The n_i are integers, some of which can be negative.

What is the meaning of terms 3 and 4?

[The Volkov solutions to the Dirac equation treat the background wave classically. Since these solutions provide most of our understanding of the 'dressing' of quantum electrons in a plane wave, that understanding does not yet incorporate photons as quanta of the background wave.]

Puzzle #4: An electron inside a (plane, monochromatic) wave is described by the Volkov solutions to the Dirac equation as having a **quasimomentum**:

$$q_\mu = p_\mu + \frac{\eta^2 m^2}{2(k \cdot p)} k_\mu, \qquad q^2 = \overline{m}^2,$$

where k = 4-momentum of wave photon,

and p = 4-momentum of electron prior to the pulse.

In quantum analyses, the kinematics of a process are deduced from **conservation of quasimomentum**, not ordinary momentum.

In particular, final electrons and positrons obtain a specified quasimomentum.

This contrasts with a possible classical view: final-state electrons and positrons are created in states described by ordinary momenta, and then evolve into the quasiparticle states over a time scale of one laser wave period.

In some cases, the size of the quasiparticle states (= amplitude of transverse shaking) is macroscopic (much greater than a laser wavelength).

Are these states really quantum states?

Puzzle #4a: If η is large, the threshold raised for pair creation by light because the final electrons and positrons must be born with mass $\overline{m}$, not mass m.

Puzzle #4b: Consider neutron decay in a strong wave field. Is the electron really created in a quasiparticle state of mass $\overline{m}$, which could be much larger than the neutron-proton mass difference?

Neutron decay in a strong wave includes the possibility of absorption of large numbers of wave photons by the final state electron. This can, in principle, solve the problem of creating the electron with mass $\overline{m}$.

It is not expected that the weak-interaction matrix element (*i.e.*, total decay rate) is changed unless the wave has the QED critical field strength:

$$E = \frac{m^2 c^3}{e\hbar}.$$