# The Impact of Deformation on the Final Alignment of the Fermilab Main Injector 

George Wojcik, Stuart Lakanen, Particle Physics Division: Alignment and Metrology Group<br>Fermi National Accelerator, Illinois, USA

## 1. Abstract

The determination of deformation of the reference points is very important for the final alignment process and the future performance of the FMI machine. In case of Main Injector Tunnel practically all points of the network were subject to deformation. The method of minimizing the first norm of the displacement vector of the reference points was chosen as the method to analyze the deformation experienced within the project.
Basic principles of deformation determination and analysis, evaluations movement of the network relative of the beam position will be exhibited in this poster.

## 1. Introduction

In an ideal world deformation would be monitored by comparing a number of points within and outside of the area of deformation. This absolute network would result in the comparison of both the stable and unstable points. At Fermilab the FMI vertical control was established as an absolute network with the tunnel control being established from the outside monuments through ten site pipe locations. Once the control was established inside the tunnel it became necessary to view the network as a relative system as the site pipes were filled, for shielding purposes, eliminating any additional ties to the outside vertical control.
Procedures for vertical and horizontal deformation analysis of the FMI tunnel network used a part of the UNB Generalized Method (Chrzanowski et al. 1982 and 1986, Chen, 1983, Secord 1985) developed in University of New Brunswick Fredericton, N.B. Canada. This paper will discuss the basic principles of the method and a practical approach of deformation analysis for alignment of the machine.

## 2. Network adjustments from the 1996, 1997, 1998 campaigns.

The Alignment and Metrology Group of the Fermi National Laboratory completed two full underground tunnel networks, one in August of 1996 and one in July of 1998. The vertical network was measured in 1996, 1997, and 1998. The method and results of the first 1996 laser tracker network measurement and adjustment were discussed in paper presented on International Workshop on Accelerator Alignment in 1997. Observations of the second Laser tracker campaign were completed in 1998. Significantly different environmental conditions existed between the two campaigns. The effect of environmental factors was determined through additional double measurements in more stable sections of the tunnel and applied to observations. After this "modification", the new measurements were evaluated and adjusted like the first.


Vertical campaigns were adjusted independently, constrained on the more stable points and reinforced with laser tracker vertical measurements.
The two campaigns in the Fermi Main Injector Tunnel (286 floor monuments and 207 wall mounted tierods vertical) allow us to perform an analysis of horizontal and vertical deformation.

## 3. Strategy for analysis and monitoring deformation of the network

The deformation trend in the new construction of the Fermi Main Injector can be predicted by an examination of the physical location of the tunnel on the site. On Figure1 Indian Creek is shown with regard to the tunnel and the accompanying wet lands. The areas of creek intersections are considered to be more susceptible to deformation. Empirical data from the Main Ring tunnel has shown greater deformation with the vertical as compared to the horizontal components of the deformation. Recognizing this consideration, a strategy was developed to monitor the deformation at the FMI with two separate studies: a vertical study of one dimension and a horizontal study of two dimensions. This strategy allowed the monitoring effort to not only isolate on areas of greater concern; but also to schedule the frequency of the horizontal and vertical aspects at different frequencies as time and manpower dictated.


Figure1

## 4. Vertical adjustment strategies, the identification stabile points with network and an approach to estimate displacements.

The vertical observations from a minimum of two campaigns are adjusted using a minimally constrained least square adjustment by fixing on only one point. A comparison of the two campaigns leads to the computation of the vector of displacements for the surveyed points cofactor matrix.

$$
\begin{equation*}
\mathrm{d}=\mathrm{x}_{2}-\mathrm{x}_{1} \quad \mathrm{Q}_{\mathrm{d}}=\mathrm{Q}_{\mathrm{x} 1}+\mathrm{Q}_{\mathrm{x} 2} \tag{1}
\end{equation*}
$$

Where $\mathbf{x}_{\mathbf{2}}$ and $\mathbf{x}_{\mathbf{1}}$ are adjusted elevations from two campaigns
The poled variance factor $\partial \mathbf{d}^{2}$

$$
\begin{equation*}
\partial d^{2}=\left(\mathbf{d f}_{1} \partial_{01}+d f_{2} \partial_{02}\right) /\left(\mathbf{d f}_{1}+d f_{2}\right) \tag{2}
\end{equation*}
$$

Where
$\mathbf{d f}_{\mathbf{1}}$ is the degree of freedom of first adjustment
$\mathbf{d f}_{2}$ is the degree of freedom of second adjustment
$\partial_{01}$ is the variance factor of the first adjustment
$\partial_{02}$ is the variance factor of the second adjustment
Because variance factor of $\partial \mathbf{d}$ is a priori not available a posterior, variances $\partial_{\mathbf{0 1}}$ and $\partial_{\mathbf{0 2}}$ are use to check of compatibility of the weighting of the observations between the two adjustments by the statistical test of the null hypothesis $\mathbf{H}: \boldsymbol{\partial}_{\mathbf{0 1}}=\boldsymbol{\partial}_{\mathbf{0 2}}$ a with significance level $\boldsymbol{\alpha}$

$$
\begin{equation*}
\left[F \alpha / 2 ; \mathbf{d f}_{2}, \mathbf{d f}_{1}\right]^{-1}<\partial_{01}{ }^{2} / \partial_{01}{ }^{2}<\mathbf{F}\left(\alpha / 2 ; \mathbf{d f}_{1}, \mathbf{d f}_{2}\right) \tag{3}
\end{equation*}
$$

With the determination of proper weighting, the minimizing of the first normal of the deformation vector and the one-dimensional network the condition can be written as follows:

$$
\begin{equation*}
\min _{\mathbf{t}_{z}}\left(\sum_{i}\left|\mathbf{w}_{i}-\mathbf{t}_{\mathbf{z}}\right|\right) \tag{4}
\end{equation*}
$$

Where $\mathbf{w}_{\mathbf{i}}$ is the displacement of the point $\mathbf{P}_{\mathbf{i}}$ and $\mathbf{t}_{\mathbf{z}}$ is translation quantity in the vertical direction. The advantage is this method will find points minimally distorted by vertical deformation, which in turn will used to fix the adjustment of the new observations. The vertical displacement of a more recent survey is the difference between the old results (listed in the database) and newly adjusted points. Once a new elevation is established it may be used for all required deformation analysis as well as a vertical constraint for the horizontal networking.

Appendix A. Example of vertical deformation analysis, shows strategy of the identification of stable points. See Graphs: Figure $2 \& 3$

## 5. Estimation of the horizontal displacement of the deformed points.

The trend of deformation in X and Y directions were monitored using the minimally constrained adjustments of observations from the 1996 and 1998 laser tracker campaigns. From all appearances, the displacement vector as well the statistical calculations result in a one-dimensional network. The strategy of minimizing the first norm of the displacement vector of the surveyed points provides a datum, which readily identifies the unstable points and gives a less distorted resolution of the displacement. A transformation vector displacement to another datum is performed as follows:

$$
\begin{equation*}
d_{k+1}=S d_{k} \tag{5}
\end{equation*}
$$

with

$$
\begin{equation*}
\mathbf{S}=\mathbf{I}-\mathbf{H}\left(\mathbf{H}^{\mathrm{T}} \mathbf{W} \mathbf{H}\right)^{-1} \mathbf{H}^{\mathrm{T}} \mathbf{W} \tag{6}
\end{equation*}
$$

where $\mathbf{d}_{\mathbf{k}}, \mathbf{d}_{\mathbf{k}+1}$ are old and new displacement vectors and $\mathbf{H}$ datum defect matrix. The number of datum defects in the two dimensional network are four (two translation $\boldsymbol{\delta}_{\mathbf{x}}, \boldsymbol{\delta}_{\mathbf{y}}$, one rotation $\mathbf{H}_{\mathbf{z}}$ and scale factor $\mathbf{k}$ )

$$
\mathbf{H}=\left(\begin{array}{llll}
\delta_{x} & \delta_{y} & \mathbf{H}_{\mathrm{z}} & \mathbf{k}  \tag{7}\\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
1 & 0 & -\mathbf{y}_{\mathrm{i}}^{0} & \mathbf{x}_{\mathrm{i}}^{0} \\
0 & 1 & \mathbf{x}_{\mathbf{i}}^{0} & \mathbf{y}_{\mathbf{i}}^{0} \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot
\end{array}\right) \quad \mathbf{x}
$$

where $\mathbf{x}_{\mathbf{i}}^{\mathbf{0}}, \mathbf{y}_{\mathbf{i}}^{\mathbf{0}}$ are the coordinates of points $\mathbf{P}_{\mathbf{i}}$ with respect to the centroid of the network and is calculated as follows:

$$
\begin{equation*}
x_{0}=x_{i}-\left(\sum_{1}^{m} x_{j}\right) / m, y_{i}^{0}=y_{i}-\left(\sum_{1}^{m} y_{j}\right) / m \tag{8}
\end{equation*}
$$

In equation (6), $\mathbf{W}$ is the weight matrix and $\mathbf{I}$ identity matrix. The two-dimensional network minimizing vector for displacement is calculated using the method of iterative weight transformation with $\mathbf{W}=\mathbf{I}$ for the initial iteration as follows:

$$
\begin{equation*}
\mathbf{W}=\operatorname{diag}\left(\mathbf{1} /\left|\mathbf{d}_{\mathbf{i}}(\mathbf{k})\right|\right) \tag{9}
\end{equation*}
$$

where $\mathbf{d}_{\mathbf{i}}(\mathbf{k})$ is the $\mathbf{i}^{\text {th }}$ component of the vector $\mathbf{d}_{\mathbf{k}}$ after $\mathbf{k}^{\text {th }}$ iteration. The iteration procedures will continue until the absolute differences between the transformed displacement components $\left(\mathbf{d}_{\mathbf{k}+\mathbf{1}}-\mathbf{d}_{\mathbf{k}}\right)$ are smaller than a priori given tolerance $\boldsymbol{\delta}$.

The last iteration cofactor matrix is calculated as:

$$
\begin{equation*}
\mathbf{Q}_{\mathrm{k}+1}=\mathbf{S}_{\mathrm{k}} \mathbf{Q}_{\mathrm{d}} \mathbf{S}_{\mathbf{k}}^{\mathbf{T}} \tag{10}
\end{equation*}
$$

Using this method the deformation of 286 floor monuments was successfully calculated. An additional analysis of the deformation can now be performed.

## 6. Analysis of the deformation impact on the aligned components.

All components in Main Injector were aligned using precision optical tooling techniques or with a laser tracker. The alignment procedures began in 1997, using control established with a laser tracker network in 1996. During the alignment the vertical datum was updated as the vertical network was re-measured. Each campaign revealed changes in the vertical datum requiring additional deformation analysis. These additional studies defined which components were stable and which components would require realignment due to deformation. A rather interesting method was used during this process. If the vector of displacement implied changes in the position of any of the components, than a best fit to line was computed. Differences in elevations were computed between consecutive tierods throughout the ring for what had been the existing datum and the results from the new campaign. The differences between the two data sets were used to generate a best-fit line, which actually simulated a beam tuning. The residuals of this fit are extremely useful in determining if a re-alignment strategy should be considered. During the main injector project, the authors fitted both a line (i, $\mathbf{j}, \mathbf{k}$ ) beam coordinate system and a plane ( $\mathbf{X}, \mathbf{Y}, \mathbf{Z}$ ) tunnel coordinates system, see Figure 6. For deformation analysis connected directly to the beam position, calculations were performed as follows:


Figure 6

$$
\begin{equation*}
d x y=\left(d x^{2}+d y^{2}\right)^{1 / 2} \tag{11}
\end{equation*}
$$

where $\mathbf{d x}$ and dy are elements of point displacement in $X$ and $Y$ directions after minimization.
An important consideration for alignment is any movement of floor monuments relative to the beam (transverse and a long of beam directions) required calculations projection of dxy on $\mathbf{i}$ and $\mathbf{j}$ axis of beam coordinates system. Calculations were preformed as follows:

$$
\begin{equation*}
d i=d x y \sin A \quad \text { and } \quad d j=d x y \cos A \tag{12}
\end{equation*}
$$

where di, dj elements transverse and a long of beam directions and Az angle between direction of vector dxy and beam path direction. Result of this type of analysis showed in :
Appendix B. Example of horizontal deformation analysis. Showed strategy to calculation deformations value directly impacted alignment of components. See Graphs: Figure 4 \& 5

## 7. Conclusion and Recommendations

The determination of deformation of control points by this proposed method is very useful and can be applied for the economical planing of alignment surveys of the machine and monitoring deformation trends in sites like the Fermi Main Injector Tunnel. The method obtains the "best" displacement value, which can be obtained from field observations allowing for the combination and analysis of data from different methods of measurement.
To implement this procedure a software package has been developed and successively used to analyze the deformation from adjusted data resulting from the combined measurements with the Chesapeake Laser Tracker, the Kern Me5000 Mekometer, the Kern E2 Electronic theodolite and Leica NA3000 Electronic Level.

## References

Chen, Y.Q, A. Chrzanowski, J.M. Secord (1986). "A Strategy for the Analysis of the stability of Reference Points in deformation Survey". CISM JOURNAL AGSGC Vol. 44, No2 Summer 1990, pp. 141 to 149
Chrzanowski, A., Y.Q. Chen, J.M. Secord (1986). "Geometrical Analysis of Deformation Surveys" Proceedings of the Deformation Measurements Workshop, Modern Metrology in Precise Engineering and Deformation Surveys-II, Massachusettes Institute of Technology, October, pp 170-206
Secord, J.M. (1985). Implementation of a Generalized Method for the Analysis of Deformation Surveys, Dept. of Surveying Engineering Technical Report 117, University of New Brunswick, Fredericton, N.B., Canada, 161 pp.
Vanicek, P, E.J. Krakiwsky (1986). Geodesy The Concepts. 2nd Edition, North-Holland Pub. Co., Amsterdam, 697 pp.
Wilkins F. J. "Datum Definition for Deformation Analysis" SE 6910 Graduate Seminary Paper, University of New Brunswick, Fredericton, N.B., Canada, September 1989
Wojcik G. J., Lakanen S. A. "The Adjustment of the Fermilab Main Injector Underground Geodetic Survey" Proceedings of the $5^{\text {th }}$ International Workshops on Accelerator Alignment, Argonne National Laboratory, October 14-17, 1997

RIVER
$=\begin{aligned} & \text { RIVER } \\ & =\text { CROSSING }\end{aligned}$
Figure 2
CHANGES IN ELEVATIONS FOR TIERODS AND FLOOR PLUGS: 1997 Vs 1998



EXAMPLE OF VERTICAL DEFORMATION ANALISYS APPENDIX A
MORE STABLE POINTS CAMPAIGN 2 ADJUSTMENT MAY BE CONSTRAINED ON **

| NAME | DEFORM(m) | PROCESS |  | DEFORM(m) | DEFORM(mils) |  | SECTOR |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{d}=\mathrm{x} 2 \mathrm{-x} 1$ | ABS SUM |  | dmin-d |  |  |  |
| 186172 | -0.00147 | 0.50970 | 186172 | -0.00119 | -0.047 |  | SECTOR1 |
| 207610 | -0.00103 | 0.32696 | 207610 | -0.00075 | -0.030 |  | SECTOR1 |
| 207614 | -0.00031 | 0.10212 | 207614 | -0.00003 | -0.001 | ** | SECTOR1 |
| 207617 | -0.00032 | 0.10251 | 207617 | -0.00004 | -0.001 | ** | SECTOR1 |
| 207627 | 0.00006 | 0.16139 | 207627 | 0.00034 | 0.013 |  | SECTOR1 |
| 207633 | 0.00010 | 0.17401 | 207633 | 0.00038 | 0.015 |  | SECTOR1 |
| 207634 | 0.00024 | 0.22349 | 207634 | 0.00052 | 0.020 |  | SECTOR2 |
| 207107 | -0.00015 | 0.11096 | 207107 | 0.00013 | 0.003 | ** | SECTOR2 |
| 207108 | -0.00036 | 0.10571 | 207108 | -0.00008 | -0.002 | ** | SECTOR2 |
| 207109 | -0.00055 | 0.11096 | 207109 | -0.00027 | -0.011 |  | SECTOR2 |
| 207116 | -0.00037 | 0.10571 | 207116 | -0.00009 | -0.002 | ** | SECTOR2 |
| 186130 | -0.00013 | 0.14586 | 186130 | 0.00015 | 0.006 |  | SECTOR2 |
| 207117 | -0.00050 | 0.10698 | 207117 | -0.00022 | -0.009 |  | SECTOR3 |
| 207118 | -0.00036 | 0.11406 | 207118 | -0.00008 | -0.002 | ** | SECTOR3 |
| 207127 | -0.00028 | 0.10153 | 207127 | 0.00000 | 0.000 | ** | SECTOR3 |
| 207206 | -0.00026 | 0.10571 | 207206 | 0.00002 | 0.001 | ** | SECTOR3 |
| 207207 | -0.00028 | 0.10153 | 207207 | 0.00000 | 0.000 | ** | SECTOR3 |
| 186109 | 0.00000 | 0.10169 | 186109 | 0.00028 | 0.011 |  | SECTOR3 |
| 207208 | -0.00020 | 0.10517 | 207208 | 0.00008 | 0.002 | ** | SECTOR4 |
| 207220 | -0.00026 | 0.14411 | 207220 | 0.00002 | 0.001 | ** | SECTOR4 |
| 207221 | -0.00036 | 0.10517 | 207221 | -0.00008 | -0.002 | ** | SECTOR4 |
| 207231 | -0.00050 | 0.10169 | 207231 | -0.00022 | -0.009 |  | SECTOR4 |
| 207232 | -0.00050 | 0.10571 | 207232 | -0.00022 | -0.009 |  | SECTOR4 |
| 186174 | -0.00022 | 0.13233 | 186174 | 0.00006 | 0.002 | ** | SECTOR4 |
| 207301 | -0.00045 | 0.13233 | 207301 | -0.00017 | -0.007 |  | SECTOR5 |
| 207302 | -0.00036 | 0.10351 | 207302 | -0.00008 | -0.002 | ** | SECTOR5 |
| 207300 | -0.00018 | 0.12072 | 207300 | 0.00010 | 0.003 | ** | SECTOR5 |
| 207306 | -0.00028 | 0.10153 | 207306 | 0.00000 | 0.000 | ** | SECTOR5 |
| 207307 | -0.00046 | 0.10715 | 207307 | -0.00018 | -0.007 |  | SECTOR5 |
| 207308 | -0.00058 | 0.15501 | 207308 | -0.00030 | -0.012 |  | SECTOR5 |
| 186173 | -0.00045 | 0.12281 | 186173 | -0.00017 | -0.007 |  | SECTOR6 |
| 207315 | -0.00035 | 0.15501 | 207315 | -0.00007 | -0.002 | ** | SECTOR6 |
| 207316 | -0.00039 | 0.12072 | 207316 | -0.00011 | -0.003 | ** | SECTOR6 |
| 207328 | -0.00046 | 0.10466 | 207328 | -0.00018 | -0.007 |  | SECTOR6 |
| 186330 | -0.00028 | 0.10153 | 186330 | 0.00000 | 0.000 | ** | SECTOR6 |

EXAMPLE OF VERTICAL DEFORMATION ANALISYS APPENDIX A
MORE STABLE POINTS CAMPAIGN 2 ADJUSTMENT MAY BE CONSTRAINED ON **

| NAME | DEFORM(m) | PROCESS |  | DEFORM(m) | DEFORM(mils) |  | SECTOR |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{d}=\mathrm{x} 2-\mathrm{x} 1$ | ABS SUM |  | dmin-d |  |  |  |
| 186330 | -0.00028 | 0.10153 | 186330 | 0.00000 | 0.000 | ** | SECTOR6 |
| 207331 | -0.00052 | 0.12281 | 207331 | -0.00024 | -0.009 |  | SECTOR6 |
| 186066 | -0.00036 | 0.10571 | 186066 | -0.00008 | -0.002 | ** | SECTOR6 |
| 207333 | -0.00068 | 0.13749 | 207333 | -0.00040 | -0.016 |  | SECTOR7 |
| 207340 | -0.00078 | 0.10571 | 207340 | -0.00050 | -0.020 |  | SECTOR7 |
| 186056 | -0.00028 | 0.10153 | 186056 | 0.00000 | 0.000 | ** | SECTOR7 |
| 207408 | -0.00043 | 0.22675 | 207408 | -0.00015 | -0.004 | ** | SECTOR7 |
| 207412 | -0.00062 | 0.16833 | 207412 | -0.00034 | -0.013 |  | SECTOR7 |
| 207416 | -0.00048 | 0.11676 | 207416 | -0.00020 | -0.008 |  | SECTOR7 |
| 186044 | -0.00012 | 0.16833 | 186044 | 0.00016 | 0.006 |  | SECTOR8 |
| 207502 | -0.00047 | 0.12745 | 207502 | -0.00019 | -0.007 |  | SECTOR8 |
| 186028 | -0.00028 | 0.10153 | 186028 | 0.00000 | 0.000 | ** | SECTOR8 |
| 207504 | -0.00041 | 0.12508 | 207504 | -0.00013 | -0.003 | ** | SECTOR8 |
| 186026 | -0.00023 | 0.10280 | 186026 | 0.00005 | 0.001 | ** | SECTOR8 |
| 186023 | -0.00018 | 0.11316 | 186023 | 0.00010 | 0.003 | ** | SECTOR8 |
| 207508 | -0.00041 | 0.10280 | 207508 | -0.00013 | -0.003 | ** | SECTOR9 |
| 207509 | -0.00040 | 0.10715 | 207509 | -0.00012 | -0.003 | ** | SECTOR9 |
| 207515 | -0.00053 | 0.11316 | 207515 | -0.00025 | -0.010 |  | SECTOR9 |
| 186018 | -0.00028 | 0.10153 | 186018 | 0.00000 | 0.000 | ** | SECTOR9 |
| 207519 | -0.00054 | 0.14022 | 207519 | -0.00026 | -0.010 |  | SECTOR9 |
| 186001 | -0.00160 | 0.10153 | 186001 | -0.00132 | -0.052 |  | SECTOR9 |
| 207602 | -0.00054 | 0.14297 | 207602 | -0.00026 | -0.010 |  | SECTOR0 |
| 207604 | -0.00031 | 0.56433 | 207604 | -0.00003 | -0.001 | ** | SECTOR0 |
| 207605 | -0.00060 | 0.14297 | 207605 | -0.00032 | -0.013 |  | SECTOR0 |
| 207606 | -0.00050 | 0.10212 | 207606 | -0.00022 | -0.009 |  | SECTOR0 |
| 186248 | -0.00074 | 0.16155 | 186248 | -0.00046 | -0.018 |  | SECTOR0 |

dMIN $\quad-0.00028$

EXAMPLE OF HORIZONTAL DEFORMATION ANALYSIS APPENDIX B
DEFORMATION VECTOR d BEFORE AND AFTER TRANSFORMATION

| DP NAME | $\mathbf{D X}(\mathbf{m})$ | $\mathbf{D Y}(\mathbf{m})$ | $\mathbf{d x}(\mathbf{m})$ | $\mathbf{d y}(\mathbf{m})$ |
| :---: | :---: | :---: | :---: | :---: |
| 186175 | 0.00015 | 0.00110 | -0.00026 | 0.00105 |
| 186172 | -0.00014 | 0.00104 | -0.00055 | 0.00098 |
| 186151 | -0.00116 | 0.00049 | -0.00116 | 0.00019 |
| 186150 | -0.00099 | 0.00079 | -0.00097 | 0.00049 |
| 186130 | -0.00054 | 0.00082 | -0.00004 | 0.00062 |
| 186129 | -0.00023 | 0.00045 | 0.00029 | 0.00026 |
| 186109 | -0.00107 | -0.00026 | -0.00034 | -0.00006 |
| 186108 | -0.00092 | -0.00021 | -0.00018 | 0.00001 |
| 186174 | -0.00034 | -0.00044 | 0.00024 | 0.00023 |
| 186324 | -0.00023 | -0.00038 | 0.00035 | 0.00030 |
| 186173 | -0.00037 | -0.00058 | 0.00010 | 0.00023 |
| 186086 | -0.00046 | -0.00077 | 0.00001 | 0.00005 |
| 186066 | -0.00014 | -0.00180 | -0.00005 | -0.00074 |
| 186065 | -0.00018 | -0.00239 | -0.00012 | -0.00133 |
| 186044 | -0.00023 | -0.00111 | -0.00067 | -0.00014 |
| 186043 | -0.00005 | -0.00074 | -0.00051 | 0.00021 |
| 186023 | 0.00124 | -0.00085 | 0.00057 | -0.00028 |
| 186022 | 0.00119 | -0.00079 | 0.00051 | -0.00024 |
| 186000 | 0.00138 | 0.00119 | -0.00086 | 0.00128 |
| 186247 | -0.00005 | 0.00105 | -0.00048 | 0.00103 |

VECTORS dxy DEFORMATION OF SINGLE POINT OF THE NETWOR

| DP NAME | $\mathbf{d x}(\mathbf{m})$ | $\mathbf{d y}(\mathbf{m})$ | $\mathbf{d x y}(\mathbf{m})$ | $\mathbf{S}$ |
| :---: | :---: | :---: | :---: | :---: |
| 186175 | -0.00026 | 0.00105 | 0.00108 | SECTOR |
| 186172 | -0.00055 | 0.00098 | 0.00112 | SECTOR1 |
| 186151 | -0.00116 | 0.00019 | 0.00118 | SECTOR2 |
| 186150 | -0.00097 | 0.00049 | 0.00109 | SECTOR2 |
| 186130 | -0.00004 | 0.00062 | 0.00062 | SECTOR3 |
| 186129 | 0.00029 | 0.00026 | 0.00039 | SECTOR3 |
| 186109 | -0.00034 | -0.00006 | 0.00035 | SECTOR4 |
| 186108 | -0.00018 | 0.00001 | 0.00018 | SECTOR4 |
| 186174 | 0.00024 | 0.00023 | 0.00033 | SECTOR5 |
| 186324 | 0.00035 | 0.00030 | 0.00046 | SECTOR5 |
| 186173 | 0.00010 | 0.00023 | 0.00025 | SECTOR6 |
| 186086 | 0.00001 | 0.00005 | 0.00005 | SECTOR6 |
| 186066 | -0.00005 | -0.00074 | 0.00074 | SECTOR7 |
| 186065 | -0.00012 | -0.00133 | 0.00134 | SECTOR7 |
| 186044 | -0.00067 | -0.00014 | 0.00068 | SECTOR8 |
| 186043 | -0.00051 | 0.00021 | 0.00055 | SECTOR8 |
| 186023 | 0.00057 | -0.00028 | 0.00064 | SECTOR9 |
| 186022 | 0.00051 | -0.00024 | 0.00056 | SECTOR9 |
| 186000 | 0.00086 | 0.00128 | 0.00154 | SECTOR0 |
| 186247 | -0.00048 | 0.00103 | 0.00114 | SECTOR0 |

River
Crossing
Figure 4 Horizontal Deformation dxy Vector in meters


DEFORMATION OF POINTS di, dj TRANSVERSE AND A LONG THE BEAM

| DP NAME | $\mathbf{d x y}(\mathbf{m})$ | $\mathbf{d i}(\mathbf{m})$ | $\mathbf{d j}(\mathbf{m})$ | $\mathbf{S E C T O R}$ |
| :---: | :---: | :---: | :---: | :---: |
| 186175 | 0.00108 | 0.00091 | -0.00058 | SECTOR1 |
| 186172 | 0.00112 | 0.00089 | -0.00068 | SECTOR1 |
| 186151 | 0.00118 | 0.00067 | -0.00097 | SECTOR2 |
| 186150 | 0.00109 | 0.00087 | -0.00066 | SECTOR2 |
| 186130 | 0.00056 | 0.00056 | 0.00005 | SECTOR3 |
| 186129 | 0.00039 | 0.00033 | 0.00020 | SECTOR3 |
| 186109 | 0.00035 | 0.00035 | 0.00001 | SECTOR4 |
| 186108 | 0.00018 | 0.00018 | 0.00002 | SECTOR4 |
| 186174 | 0.00033 | 0.00033 | 0.00000 | SECTOR5 |
| 186324 | 0.00046 | 0.00046 | 0.00000 | SECTOR5 |
| 186173 | 0.00025 | 0.00025 | -0.00003 | SECTOR6 |
| 186086 | 0.00005 | 0.00005 | 0.00000 | SECTOR6 |
| 186066 | 0.00074 | 0.00072 | 0.00018 | SECTOR7 |
| 186065 | 0.00134 | 0.00129 | 0.00037 | SECTOR7 |
| 186044 | 0.00068 | 0.00026 | 0.00063 | SECTOR8 |
| 186043 | 0.00055 | 0.00026 | 0.00048 | SECTOR8 |
| 186023 | 0.00064 | 0.00041 | 0.00049 | SECTOR9 |
| 186022 | 0.00056 | 0.00035 | 0.00044 | SECTOR9 |
| 186000 | 0.00154 | 0.00151 | 0.00031 | SECTOR0 |
| 186247 | 0.00114 | 0.00095 | -0.00064 | SECTOR0 |



