DEFORMATION ANALYSIS OF LEP

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1. INTRODUCTION

LEP (Large Electron Positron Collider) is in an underground accelerator, located in a tunnel of 27 km circumference and from 40 to 160 m deep. It is the largest accelerator in the world. The electrons and positrons circulate in opposite directions and hit each other in four points. The collisions are observed by means of detectors, housed in large underground caverns.

Due to the sensitivity of such accelerators to alignment errors (the initial requirement is 0.1 mm rms as relative accuracy along the beam lines), both physicists and surveyors must pay much attention to the stability of the accelerator (and therefore of the tunnel) after the installation of the elements. Maintenance surveys are made annually, and special methods have also developed for monitoring unstable areas. A complete levelling is made every year, followed by a "smoothing" process - i.e. an optimal refinement of successive positions - which makes that the accelerator is kept operational with respect to misalignments.

The installation of LEP has been completed in July 1989, and the first maintenance surveys were limited to some areas where instabilities were the most likely : straight sections each side of access pits, special equipment near large caverns, lowest part in the molasse, parts with high pressure water in the limestone. In 1991, a significant vertical movement was suspected in a non-surveyed sector, affecting the orbit control of the particle beams, and a subsidence was indeed discovered along a 300 m section in the molasse – on the supposed path of a fault. This fact, combined with the appearance of fully automatic levels with encoded staves, triggered the decision to make a full control every year.

The annual levelling of LEP can be characterised as follows:

- A quasi circle of 27 km circumference in tunnel
- Measurements with a LEICA NA3000 ($\sigma = \pm$ 0.4 mm/km, statistically $\pm$ 0.04 to $\pm$ 0.05 mm/station, at intervals of 39.5 m)
- Maximum height difference of 120 m between the highest point and the lowest point
- Measured points : alignment reference targets of the quadrupole magnets (entrance & exit points)

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- 800 quadrupole magnets, 1600 points to measure
- Cholesky method, with two independent traverses (forward/backward loops).

The data processing is made by least squares, according to a free network concept. In addition, a smoothing procedure (successive fits within a sliding window) is also carried out after each annual levelling measurement, in the purpose of refining the successive positions and finding the points being vertically too far (more than 0.3 mm in general) from the local smoothing curve. These points are then brought physically on their smoothed position (realignment) in order to keep the vertical configuration of LEP as optimal as possible. Tilt (transverse slope) measurements are also taken during this realignment process, thus putting the corrected element back to its right transverse position and reducing the correlated radial movement associated to this defect.

In this special study, the overall stability of the accelerator is checked. The deformation and the movement of the elements are analysed in four stages :

1) pre-processing of the data, data-filtering and data-restoring;
2) search for the technique which would precisely reflect the deformation of the accelerator without any influence of the adjustment datum or other uncertainty parameters;
3) distinction of the deformation of the tunnel from the movement of the elements;
4) description of this deformation and use of the results for predicting future perturbations of the alignment.

2. A GENERAL INVESTIGATION

In order to find a technique for the deformation and movement analysis, the dynamic features of the accelerator and the tunnel must be known. A general statistic investigation has been carried out on the vertical offsets, the inclination and deformation variation based on the F-Test theory.

Due to the realignment of some elements every year, the offsets obtained on these corrected points could not continuously reflect their vertical variation. The uncorrected offsets have been restored on the basis of the realignment data (annex 1). This restoration was also executed on the smoothing offsets.

Theoretically, the precision of the levelling made every year should be the same statistically. The precision of every section should be statistically identical from year to year and from section to section. More than 200 statistic tests were realised based on four kinds of offset differences. From the results, a comprehensive analysis had been made on its stability. Some conclusions are expressed in the last part of the paper.

3. DEFORMATION AND DISPLACEMENT ANALYSIS MODELS

Based on the features of vertical theoretical offsets, a very special technique was created in order to find the deformed or displaced zones.

3.1 Theoretical Deformation Model

The general model for a spatially curved object could be given as follows :

$$(\text{Model})_h = (\text{Model})_{tr} + (\text{Model})_{ro} + (\text{Model})_{\Delta h} \tag{1}$$

where :
$(\text{Model})_h$ ----- The real model of the curve shape
$(\text{Model})_{tr}$ ------The model for translation
$(\text{Model})_{ro}$ ------The model for rotation
$(\text{Model})_{\Delta h}$------The model for the inclined average plan.

One can express the model mathematically as follows :

$$D_h(T, R, P) = D_t(T) + D_r(R) + D_{\Delta h}(P) \tag{2}$$

where :
$D_h(T, R, P), D_t(T), D_r(R), D_\Delta(P)$ are correspondent to $(\text{Model})_h$, $(\text{Model})_{tr}$, $(\text{Model})_{ro}$, $(\text{Model})_{\Delta h}$.

T, R and P are the parameter sets of the different models.

Let us suppose this model describes a smoothing curve in two dimension (D, L). So the first differential of the model with respect to the distance could be written as :

$$\frac{\partial(D_h)}{\partial L} = \frac{\partial(D_t)}{\partial L} + \frac{\partial(D_r)}{\partial L} + \frac{\partial(D_{\Delta h})}{\partial L} \tag{3}$$

where $\frac{\partial(D_t)}{\partial L}$ is zero, and $\frac{\partial(D_r)}{\partial L}$ is a constant.

The first differential value at the random point i is :

$$\left.\frac{\partial(D_h)}{\partial L}\right|_{L=L_i} = \left.\frac{\partial(D_r)}{\partial L}\right|_{L=L_i} + \left.\frac{\partial(D_{\Delta h})}{\partial L}\right|_{L=L_i} \tag{4}$$

L_i is the accumulated distance of the point i to the original point. In fact, (4) is the vertical inclination of the curve at the point i.

From the same principle, the second differential of the model can be derived as :

$$\frac{\partial^2(D_h)}{\partial^2 L} = \frac{\partial^2(D_r)}{\partial^2 L} + \frac{\partial^2(D_{\Delta h})}{\partial^2 L} \tag{5}$$

where $\frac{\partial^2(D_r)}{\partial^2 L}$ would be zero.

For the point i, we could get the value :

$$\left.\frac{\partial^2(D_h)}{\partial^2 L}\right|_{L=L_i} = \left.\frac{\partial^2(D_{\Delta h})}{\partial^2 L}\right|_{L=L_i} \tag{6}$$

In fact, (6) is the vertical deformation of the curve at the point i.

From the differential definition, we have the first differential expression :

$$\left.\frac{\partial(D_h)}{\partial L}\right|_{L=L_i} = \lim_{\Delta L\to 0}\left.\frac{\Delta D_h}{\Delta L}\right|_{L=L_i} = \lim_{\Delta L\to 0}\frac{D_h(L_i+\Delta L)-D_h(L_i)}{\Delta L} \tag{7}$$

The second differential is :

$$\left.\frac{\partial^2(D_h)}{\partial^2 L}\right|_{L=L_i} = \lim_{\Delta L\to 0}\left.\frac{\Delta D'_h}{\Delta L}\right|_{L=L_i} = \lim_{\Delta L\to 0}\frac{D'_h(L_i+\Delta L)-D'_h(L_i)}{\Delta L} \tag{8}$$

where D'_h is the first differential.

For the practical realisation, the formulas (7) and (8) can be written as follows :

$$\left.\frac{\partial(D_h)}{\partial L}\right|_{L=L_i} \approx \left.\frac{\Delta D_h}{\Delta L}\right|_{L=L_i} = \frac{D_h(L_i+\Delta L)-D_h(L_i)}{\Delta L} \tag{9}$$

$$\left.\frac{\partial^2(D_h)}{\partial^2 L}\right|_{L=L_i} \approx \left.\frac{\Delta D'_h}{\Delta L}\right|_{L=L_i} = \frac{D'_h(L_i+\Delta L)-D'_h(L_i)}{\Delta L} \tag{10}$$

Theoretically speaking, the smaller ΔL would produce the better curve's description. But usually, there are some measuring errors and other random errors. For LEP there are two kinds of random factors : surveying errors and random movements of the elements. Their influence on the inclination and the deformation would be reduced with the ΔL's increasing. Therefore we should choose the minimum value of ΔL that could permit us to correctly get the desired signal from the observations with a minimum signal loss. This optimisation procedure is especially dependent to the inclination precision and the magnitude of the random movement of quadrupoles.

So the optimal distance (ΔL=4×Δ) was derived in the condition that the r.m.s of the measured inclination is identical with that of a random inclination. Its minimum interesting vertical displacement between two points at a distance ΔL is S_0 (S_0 = 0.21mm). The relation between them can be found from the following formula :

$$S_0 \geq 2m_{\Delta L} = \pm 2n \times 40 \times 1000 \times \sqrt{(\frac{1}{n} + \frac{1.77^2}{n^2})} \times 10^{-6} \text{ (mm)} \quad (11)$$

3.2 Theoretical Displacement Model

For each year the vertical offsets could be expressed with respect to an average plan. Let us suppose the plan equation of it is as follows:

$$A_i x_j + B_i y_j + C_i z_j = 0 \quad (12)$$

$i = 1,2,..., N$ (number of levelling measurements);

$j = 1,2,..., n$ (number of points);

A_i, B_i, C_i are parameters of the plan i.

x_j, y_j, z_j are the centralised co-ordinates of point j.

It can also be transformed into the following form if the co-ordinate z_j is considered as an observation :

$$\underset{n\times 1}{V} = \underset{n\times p}{D}\,\underset{p\times 1}{\beta} - \underset{n\times 1}{Z} \qquad P_j \quad (13)$$

where $V = \begin{pmatrix} v_1 \\ v_2 \\ . \\ . \\ v_n \end{pmatrix}$, $D = \begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \\ . & . \\ . & . \\ x_n & y_n \end{pmatrix}$, $\beta = \begin{pmatrix} a \\ b \end{pmatrix}$, P_j is the weight of point j.

The parameters can be estimated and F-Test could be used for the test of the parameter's significance. This test can offer us the information about whether the regression is effective.

If parameters are significant, the geometrical parameters of the plan, such as its normal direction in space, can be calculated (Figure 1). These parameters are then used for rotation.

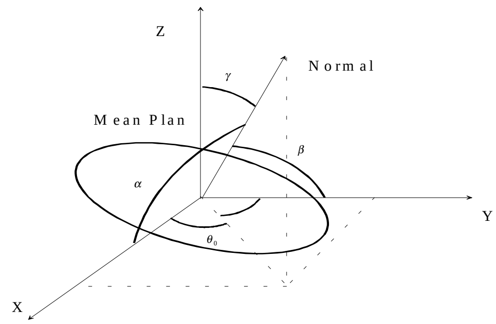

Figure 1 : Parameters of the reference plane

We hereby obtain a plan for vertical offsets each year. Then, its parameters could be used for rotating all the base points to the same horizontal or theoretical plan, in order to do the comparison of the same point in different years. So the rotated value of the vertical theoretical offsets could be written as :

$$\overline{z_{ij}} = z_{ij} + \sqrt{(x_j^2 + y_j^2)} \times \tan(\gamma_i) \times \cos(\theta_j) \tag{14}$$

For every point the statistic parameters of the $\overline{z_j}$, (variance, mean absolute error and range), are also computed for evaluating the chosen plan and for renewing the weight of the point in the next iteration during the point selection.

The zones of vertical displacement will be found and eliminated gradually during the process of establishment of the base plan. The weight function is considered as :

$$P \propto 1/\sigma_z^2 \tag{15}$$

It means that the bigger the scattering (or variance) of a point, the smaller its weight in the next iteration. There are many definitions about the iteration weight of the point, such as Huber and Hampel. But here three functions are used for experiment. For $(k+1)^{\text{th}}$ iteration, they are :

$$\text{A:} \quad P_j^{k+1} = \begin{cases} 0, & \sigma_{z_j}^k = \max\left\{\sigma_{z_1}^k, \sigma_{z_2}^k, \ldots, \sigma_{z_n}^k\right\} \\ 1, & else \end{cases} \tag{16}$$

$$\text{B:} \quad P_j^{k+1} = \begin{cases} 0, & \theta_{z_j}^k = \max\left\{\theta_{z_1}^k, \theta_{z_2}^k, \ldots, \theta_{z_n}^k\right\} \\ 1, & else \end{cases} \tag{17}$$

$$\text{C:} \quad P_j^{k+1} = \begin{cases} 0, & \phi_{z_j}^k = \max\left\{\phi_{z_1}^k, \phi_{z_2}^k, \ldots, \phi_{z_n}^k\right\} \\ 1, & else \end{cases} \tag{18}$$

Where $\sigma_{z_j}^k = \sqrt{\dfrac{\sum\left(\overline{z_{ij}^k} - \overline{\overline{z_j^k}}\right)^2}{N}}$, $\quad \overline{\overline{z_j^k}} = \dfrac{\sum_{i=1}^{N} \overline{z_{ij}^k}}{N}$; $\theta_{z_j}^k = \sqrt{\dfrac{\sum\left|\left(\overline{z_{ij}^k} - \overline{\overline{z_j^k}}\right)\right|}{N}}$, $\quad \overline{\overline{z_j^k}} = \dfrac{\sum_{i=1}^{N} \overline{z_{ij}^k}}{N}$

$$\phi_{z_j}^k = \overline{z_{\max,j}^k} - \overline{z_{\min,j}^k}, \quad \overline{z_{\max,j}^k} = \max\left\{\overline{z_{1j}^k}, \overline{z_{2j}^k}, \dots, \overline{z_{Nj}^k}\right\}, \quad \overline{z_{\min,j}^k} = \min\left\{\overline{z_{1j}^k}, \overline{z_{2j}^k}, \dots, \overline{z_{Nj}^k}\right\}$$

The principle for point selection is to keep the points of the average plan as many as possible under condition of meeting the stable point criterion. This is to get the result or conclusion with less risk. In this procedure the stable point criterion is considered to be 0.25 mm on scattering of the rotated vertical offset value during the years of surveillance.

The tests for significance include three aspects: tests for the significance of the estimated parameters of the plan, tests for statistical consistency on precision among the measurements every year and tests for the significance of the difference among the established average plans.
For the significance test of the estimated parameters, the following F-test has been used:

$$F = \frac{\hat{\beta}^T Q_{\hat{\beta}\hat{\beta}}^{-1} \hat{\beta}}{N1 \times \hat{\sigma}^2} \sim F(N1, N2 - N1) \qquad (19)$$

For evaluating the statistical consistency of the precision on the measurements made every year, the expression of the F-test becomes :

$$F = \frac{\hat{\sigma}_l^2}{\hat{\sigma}_m^2} \sim F(N2 - N1, N2 - N1) \qquad (l \neq m) \qquad (20)$$

$\hat{\sigma}_l, \hat{\sigma}_m$ are respectively the estimators of the unit weight variance for l^{th} and m^{th} measurement.

For testing the significance of the differences between the successive average plans, F-test is used according to :

$$F = \frac{\left(\hat{\beta}_l - \hat{\beta}_m\right)^T Q_{\hat{\beta}\hat{\beta}}^{-1} \left(\hat{\beta}_l - \hat{\beta}_m\right)}{N1 \times \hat{\sigma}^2} \sim F(N1, N2 - N1) \qquad (l \neq m) \qquad (21)$$

For analysing the deformation with a comparison of offsets, the offsets of all points must be rotated to the theoretical plan (horizontal plan) every year :

$$\overline{z_{ij}} = z_{ij} + \sqrt{(x_j^2 + y_j^2)} \times \tan(\gamma_i) \times \cos(\theta_j) \qquad (22)$$

4. DISPLACEMENT ANALYSIS

In this stage of the process, it is absolutely necessary to find a vertical reference, i.e. several points or groups of points which can be said more relatively stable than others. This then leads, conversely, to find the zones of deformation and displacement.

4.1 Zones of deformation

The zones of deformation have been found by the method described in the section 3.1 above. The inclinations and the deformations were calculated and presented graphically. The deformed zone could be very easily checked on the graph. Height deformed zones have been found and are listed in the table 1. After the elimination of the unstable points there are 864 points left. Those points are considered the original base points for the vertical base plan to be established. It is should be noted that there are probably some areas of vertical block displacement, which could not be found with their inclination and deformation change. This problem will be solved in the next step.

Table 1 : Deformed areas of LEP

Name	cumulated distances		Number of MQ		Inclination (significant)	Deformation (significant)
	From	To	From	To		
S1-A1-D1	1152.09	1587.73	186500	197500	93,98	93,98
S2-A1-D1	4997.98	6184.12	301500	331500	92-98	92-98
S45-A1-D3	11623.20	15075.17	499500	602500	93-98	92-98
S5-A2-D2	16306.28	16561.31	634500	644500	93-96,98	93-96,98
S6-A1-D2	17497.91	17736.05	679500	685500	93,94,96,98	93,94,96,98
S7-A1-D1	21027.74	22648.38	783500	825500	92-98	92-98
S8-A1-D2	23965.13	24321.77	874500	883500	92-98	92-98
S81-A1-D3	25742.63	599.09	120500	172500	92-98	92/98

4.2 Zones of vertical block displacement

The zones of vertical block displacement should be also eliminated in order to establish a relatively stable plan as the vertical reference.

The theory described in 3.2 was used in this procedure. This stage is gradually realised during the establishment procedure of the base plan. The points within a zone of vertical block displacement have their weight becoming smaller and smaller : at last, they become zero, equivalent to be eliminated.

This result was used to find the unstable zones listed in the table 2.

Table 2 : Unstable areas of LEP tunnels

Area	Model	Accum. Dist.		Number of MQ		Inclination (significant)	Deformation (significant)
		From	To	From	To		
1	No	25268	355	108500	165500	92-98	93-98
2	Yes	1270	1706	189500	201500	92,93,98	92,98
3	No	2574	7185	223500	370500	92-98	92-98
4	Yes	8646	15588	408500	615500	92,97	92,97
5	No	15666	16990	616500	665500	92,97	93,95,97
6	No	17418	17696	677500	684500	92,97	92,97
7	Yes	20672	25230	774500	107500	92-98	92-98

5. CONCLUSION

Both LEP machine and tunnel have been moving from its beginning. That has been bringing both vertical deformations and displacements on the configuration of the accelerator. Some zones are more and more stable, and some others more and more unstable. Even the average plan is not the same for every year.

The movement of the elements comes mainly from the movement and deformation of the tunnel. In addition to the annual levelling and smoothing, cross-section measurements have shown various kinds of deformation profiles of the concrete floor, with possible upward or downward effects on the machine. A punching effect on the single jack (the most loaded of the three under each magnet) is also possible.

The results obtained on LEP have proved that the methods and the statistics specially designed for such long lines are effective. They provide an excellent view of what happens with time (annex 2), and this will help in designing the survey process of the next accelerator (LHC) to be installed in the same tunnel.

Such a deformation and displacement analysis model, as developed in this paper, is a powerful tool for finding the deformed zones on any curvilinear spatial object. Other applications can be found for other kinds of long construction works : galleries, bridges, etc.

Acknowledgements

Many people from the CERN-SU Group (Positioning Metrology & Surveying) and from partner companies have contributed over years to the measurements, computations and realignments of the LEP machine, and we cannot give here all names. Among them, the authors are particularly thankful to M. Hublin, A. Mathieu, D. Missiaen and J. Schmitt for their help in extracting and processing some of the data and bringing useful information for the analysis.

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Restored Vertical Theoretical Offsets of LEP