PLANE DEFORMATION MONITORING NETWORK AND COMPUTATIONAL METHOD OF THE NSRL STORAGE RING

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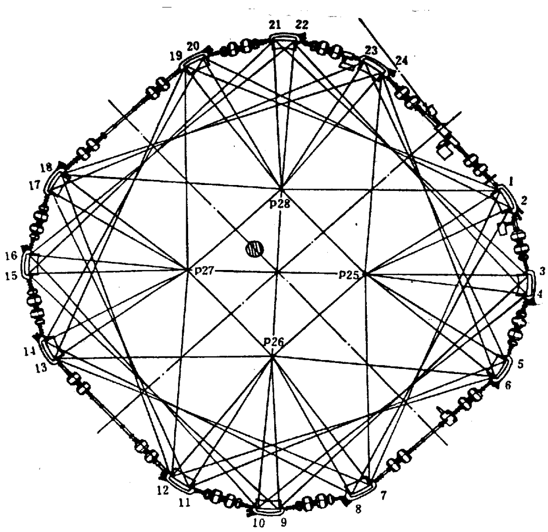
1. Introduction

The NSRL (National Synchrotron radiation Laboratory) accelerator consists of three major parts: the 800 *Mev* electron storage ring, the transport line and the 200 *Mev* electron linac. As the main parts of NSRL, the storage ring contains twelve dipoles, thirty-two quadrupoles, fourteen sextupoles, some kickers and septums etc. All these units are distributed separately along 66.13 meters circumference. During installation of the storage ring, an alignment network was established, which is called the Construction Control Network. When adopting the network, we accurately installed all these units into the storage ring in according to the designed position accuracy. Theoretically, this Construction Control Network can be used to monitor plane deformation as it has been adequately amended after the storage ring had been constructed. But in the several passed years, some new insertion units have been installed in the storage ring, some new photon beams and experiment stations have been established and some power cupboards have been put at the centre of storage ring. So some observation lines between two reference points of the Construction Control Network have been interrupted and the network can not be re-established. In order to survey the position changing condition of all the units in the storage ring, a new network must be established. In 1997, we designed a Plane Deformation Monitoring Network.

2. Comparison of Plane Construction Control Network and Deformation Monitoring Network

The Construction Control Network was a trilateration network. Its purpose was to align and install the units in the storage ring. Dipoles were chosen as the primary reference for the alignment and installation. All other components situated between two dipoles on straight-line sections are easily aligned from the two adjacent dipoles by means of optical instrumentation and other techniques.

It included 28 reference points. Each dipole had two reference points. The three dipoles in same quadrant have a coincident curvature centre, which was also set up as a reference point. Among these 28 knots, a same precision trilateration network consisting of 86 sides was established. This geodetic figure is composed of distance measurement only and is stiffened between magnets intentionally (see Figure1).



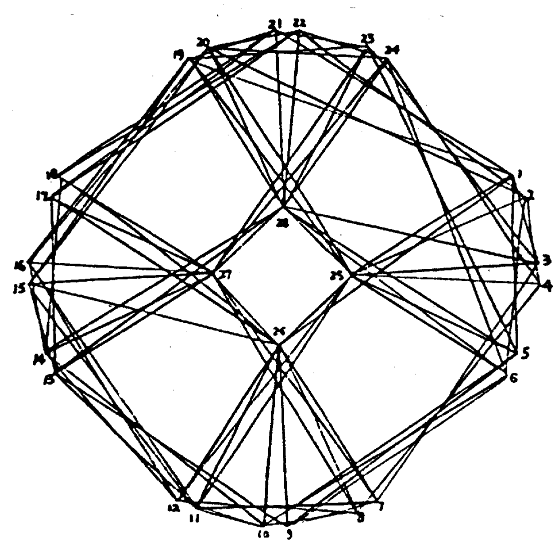


Figure 1-Construction Control Network   Figure 2-Deformation Monitoring Network

All the distance measurements were performed with DISTINVAR because the expansion coefficient of invar wire is very low, but its sensitivity to vibration can not be ignored. A laser interferometer was used and a secondary bench was set temporarily to calibrate invar wires at times during the measurement.

Among the 86 sides in the network, 48 sides are essential to guarantee the radial and tangential position precision of dipoles and the other 36 sides are redundant surveying sides in order to enhance the relative location accuracy of magnets. According to physical design, the allowable locating errors of dipoles are

$$\begin{cases} |\Delta X| \le 0.15, \\ |\Delta Y| \le 0.15. \end{cases} \quad [1]$$

Differing from that of the Construction Control Network, the purpose of Deformation Monitoring Network is to monitor the displacement of the components in the storage ring. In condition of common operation of the accelerator and not moving any instruments in the centre of the ring, it is required to monitor the displacement of units in the ring by adopting this monitoring network.

The monitoring network has the distinguishing features of high precision, being able to repeat observation and using accurate data processing methods. After optimally designing the accuracy target, datum, reliability standard, sensitivity target, separating capacity and cost target, we obtain a optimal network in 1997 see Figure 2.

Because angle measurement accuracy is influenced by error of centre, focusing error, bearing error and horizontal dioptre, the Deformation Monitoring Network is also a trilateration network.

There are 99 sides in the network. All the distance measurements are performed with DISTINVAR. For the calculation of the networks, the Construction Control Network had six initial sides. Adopting indirect adjustment principle, it was easy to obtain the theoretical coordinates of dipoles. After coordinate optimum fitting, the magnets were installed according to a new coordinate system in which twelve dipoles were adjusted to their ideal positions with minimum shift. But for the Deformation Monitoring Network, there are no initial sides. Rank $\underset{n\times u}{B}$, the coefficient matrix of observational equations, is not equal to the amount of unknown parameters t. There is

$$R(B) = t - 3, \qquad [2]$$

where the rank defect free network adjustment method is used.

3. Basal Principle of Rank Defect Free Network Adjustment Method.

Suppose in a free network, which has not any initials, observed values are $\underset{n\times 1}{L}$, which are the same accuracy and independent. If the essential observational number is t_0, the number of unknowns is u, and $u > t_0$, then

$$\hat{X} = X^0 + \delta\hat{X}, \qquad [3]$$

where $\hat{X}$ is value of assessment of unknown X, X^0 is the approximate value of $\hat{X}$ and $\delta\hat{X}$ is the correction values of X^0. If the linearized observation equation is

$$\underset{n\times 1}{L} = \underset{n\times u}{B}\,\underset{u\times 1}{X} + l^0 + \Delta, \qquad [4]$$

then the rank of coefficient matrix $\underset{n\times u}{B}$ is $R(B) = t < u$, and it is not a matrix of full column rank.

Now let $\hat{X}$, which is the value of assessment of unknown X, replace X, and $-V$ replace Δ, the true error. Then the error equation is

$$V = B\delta\hat{X} + BX^0 + l^0 - L. \qquad [5]$$

Let the constant term be $l = BX^0 + l^0 - L$, then

$$V = B\delta\hat{X} + l. \qquad [6]$$

In accordance with the principle of indirect adjustment, the normal equation is

$$B^T B\delta\hat{X} + B^T l = 0, \qquad [7]$$

where B^T is the transposed matrix of B.

Let

$$\begin{cases} B^T B = N, \\ B^T l = W, \end{cases} \qquad [8]$$

Then the normal equation is

$$N\delta\hat{X} + W = 0. \qquad [9]$$

If the B is a matrix of full column rank, then N is a full rank matrix, and equation [9] has a unique solution $\delta\hat{X}$. But now the rank of matrix B is smaller then the number of unknowns, so $R(N) = t < u$, and the determinant $|N| = 0$. N is a singular or rank defect matrix, and the rank defect is $d = u - t$. So equation [9] is a series of compatible equations, which have infinite resolutions.

To get the unique optimal estimation value of $\delta\hat{X}$, a new principle must be forwarded. That is, the estimation value obtained must satisfy the following condition from the principle of least squares:

$$\delta\hat{X}^T\delta\hat{X} = \min . \qquad [10]$$

The estimation value satisfying this requirement is marked as $\delta\hat{\hat{X}}$.

In accordance with linear algebra, the equations of [6], which are inconsistent simultaneous equations whose coefficient matrix has no full column rank, have a unique resolution of least squares:

$$\delta\hat{\hat{X}} = -B^+ l, \qquad [11]$$

where B^+ is the least norm inverse of matrix B.

The compatible simultaneous equations [9], whose coefficient matrix N is singular, also have a unique resolution of minimum norm:

$$\delta\hat{\hat{X}} = -N_m^- \mathrm{W} = -N_m^- B^T l, \qquad [12]$$

where N_m^- is the generalised inverse matrix of N, which is also called the least norm inverse matrix. N_m^- satisfies

$$\begin{cases} NN_m^- N = N, \\ (N_m^- N)^T = N_m^- N. \end{cases} \qquad [13]$$

Because N^+, the least norm inverse of N, also satisfies equation set [13], another unique minimum norm resolution is obtained.

$$\delta\hat{\hat{X}} = -N^+ W = -N^+ B^T l . \qquad [14]$$

Actually, the minimum norm resolution of the least squares not only satisfies the principle of the least squares but satisfies the equations [10] also.

The resolutions of equations [11], [12] and [14] are the same. They are all the resolution of equation set [4].

In order to get the resolution of $\delta\hat{\hat{X}}$, the generalised inverse matrix B^+ or N_m^- or N^+ must be obtained first.

What must be pointed is that, in accordance with equation set [10], the final result of X is changed as the approximate value X^0 changes. There are a few methods to solve the $\delta\hat{\hat{X}}$, but here just one method called the method of false observation value is discussed.

Two theorems about generalized inverse matrices N_m^- and N^+ can be proven using linear algebra. Let the ranks of B and N^- $(= B^T B)$ be $R(B) = R(N) = t < u$, and the rank defect be $d = u - t$, and suppose a matrix with full column rank is $\underset{t\times d}{G}(R(G) = d)$, and let

$$Q = (B^T B + GG^T)^{-1},$$

Then:

(1) If G satisfies

$$\underset{n\times t}{B}\;\underset{t\times d}{G} = \underset{n\times d}{0} \qquad [15]$$

then

$$\begin{cases} N_m^{-1} \ni Q = (B^T B + GG^T)^{-1}, \\ B^+ = QB^T = (B^T B + GG^T)^{-1} B^T. \end{cases} \qquad [16]$$

(2) If G satisfies the following equation in addition to equation [15] :

$$\underset{d\times 1}{G^T} \underset{t\times d}{G} = \underset{d\times d}{E}, \qquad [17]$$

then

$$N^+ = Q - GG^T. \qquad [18]$$

So, if the matrix $\underset{t\times d}{G}$ which satisfies equations [15] and [17] has been found, then N_m^-, B^+ and N^+ can be obtained.

Substituting equation [16] into [11] and [12], when G satisfies BG=0, gives

$$\delta\hat{\hat{X}} = -(QB^T)l = -(B^T B + GG^T)^{-1} B^T l, \qquad [19]$$

and

$$\delta\hat{\hat{X}} = -QW = -QB^T l = -(B^T B + GG^T)^{-1} B^T l, \qquad [20]$$

respectively.

When G satisfies BG=0 and $G^T G = E$ as well, substituting equation [18] into [14] gives

$$\delta\hat{\hat{X}} = -(Q - GG^T)B^T l = -QB^T l + GG^T B^T l$$
$$= -(B^T B + GG^T)^{-1} B^T l. \qquad [21]$$

It can be seen that the results of equations [19], [20] and [21] are the same. They all can be described as

$$(B^T B + GG^T)\delta\hat{\hat{X}} + B^T l = 0. \qquad [22]$$

Actually, equation [22] is norm equations satisfying the following equations:

$$\begin{cases} V = B\delta\hat{\hat{X}} + l, \\ V_g = G^T \delta\hat{\hat{X}}. \end{cases} \qquad [23]$$

The first equation is just the equation [6], the original error equation, and the second corresponds to the error equation which has a group of false observation values whose number is d (=u-t). The method used here is called the method of false observation values.

If the G has been obtained, the resolution of $\delta\hat{\hat{X}}$ can be obtained easily from equation [22].

For trilateration network, the matrix G is

$$\underset{3\times u}{G^T} = \begin{bmatrix} 1 & 0 & 1 & 0 & \cdots & 1 & 0 \\ 0 & 1 & 0 & 1 & \cdots & 0 & 1 \\ -y_1^0 & x_1^0 & -y_2^0 & x_2^0 & \cdots & -y_m^0 & x_m^0 \end{bmatrix}, \qquad [24]$$

where x_i^0, y_i^0 (i =1,2,...m) are the approximate coordinate values of the knots in the network. In the plane deformation monitoring network, $m = 28$, and $2m$=u.

4. Calculation of the Plane Deformation Monitoring Network

In the optimum Plane Deformation Monitoring Network, there are 28 reference points and 99 sides. The plane coordinate values of each point are chosen as unknowns, and the theoretical coordinate values as their approximate values, and the distance calculated from the theoretical coordinate values will be the approximate distance of every side.

Suppose the length of side between two undetermined points j and k is L_{jk}. Their adjustment coordinate values x_j, y_j, x_k, and y_k are unknowns. Their theoretical coordinates x^0_j, y^0_j, x^0_k and y^0_k are the approximate values of unknowns. Let δx_j, δy_j, δx_k and δy_k be the correction values of unknowns, then

$$\begin{cases} x_j = x_j^0 + \delta x_j, \\ y_j = y_j^0 + \delta y_j, \\ x_k = x_k^0 + \delta xk, \\ y_k = y_k^0 + \delta yk, \end{cases} \qquad [25]$$

and then the adjustment equation is

$$L_{jk} + V_{jk} = \sqrt{(x_k - x_j)^2 + (y_k - y_j)^2} \ . \qquad [26]$$

Substituting equation [25] into the above equation, and then expanding it in Taylor series and taking the linear term gives

$$L_{jk} + V_{jk} = \sqrt{(x_k^0 - x_j^0)^2 + (y_k^0 - y_j^0)^2} + \frac{(x_k^0 - x_j^0)}{\sqrt{(x_k^0 - x_j^0)^2 + (y_k^0 - y_j^0)^2}} \times (\delta x_k - \delta x_j)$$

$$+ \frac{(y_k^0 - y_j^0)}{\sqrt{(x_k^0 - x_j^0)^2 + (y_k^0 - y_j^0)^2}} \times (\delta y_k - \delta y_j).$$

Substituting $S_{jk}^0 = \sqrt{(x_k^0 - x_j^0)^2 + (y_k^0 - y_j^0)^2}$ into the above equation, the error equation is obtained:

$$V_{jk} = -\frac{\Delta x_{jk}^0}{S_{jk}^0}\delta x_j - \frac{\Delta y_{jk}}{S_{jk}^0}\delta y_j + \frac{\Delta x_{jk}^0}{S_{jk}^0}\delta x_k + \frac{\Delta y_{jk}^0}{S_{jk}^0}\delta y_k + S_{jk}^0 - L_{jk}, \qquad [27]$$

where Δx^0_{jk} , Δ^0_{jk} are the coordinate increments between j and k calculated from the approximate coordinate values . The sum of the first four terms on right side is the correction value of length of side led to by coordinate correction value, so the equation is also called the correction value equation.

If there are initials, for example, j is a known datum point, then $\delta x_j = 0, \delta y_j = 0$ and the result obtained is

$$V_{jk} = \frac{\Delta x_{jk}^0}{S_{jk}^0}\delta x_k + \frac{\Delta y_{jk}}{S_{jk}^0}\delta y_k + S_{jk}^0 - L_i \ . \qquad [28]$$

But as discussed above, the network has no initials, so the equation (27) is the error equation of the network, and the absolute values of coefficients of δx_j

and δx_k, δy_j and δy_k are equal. The consistent term is equal to the difference of approximate length of side minus the distance of observation.

For the network, there are 99 error equations having the same form as equation [27]. The theoretical coordinates of 28 datum points are shown in Table 1. The approximate length of 99 sides can be calculated from their coordinates.

Alternatively, equation [27] may be derived in matrix form as

$$\underset{99\times1}{V} = \underset{99\times56}{B}\ \underset{56\times1}{\delta X} + \underset{99\times1}{l}, \qquad [29]$$

where B is the coefficient matrix. This is the fundamental form of the error equation. $\underset{56\times1}{\delta X}$ represents the correction values of 56 approximate coordinates of the 28 datum points.

By performing the least square method, the normal equation from equation [29] is

$$\underset{56\times56}{N}\ \underset{56\times1}{\delta X} + \underset{56\times1}{U} = \underset{56\times1}{0}, \qquad [30]$$

where

$$\begin{cases} \underset{56\times56}{N} = \underset{56\times99}{B^T}\ \underset{99\times56}{B}, \\ \underset{56\times1}{U} = \underset{56\times99}{B^T}\ \underset{99\times1}{l}, \end{cases} \qquad [31]$$

respectively.

As discussed above, the coefficient matrix B is not a full rank matrix, $R(B)$=56-3=53, and $\underset{56\times56}{N}$ is a singular matrix, $R(N)$=53, rank defect d=3. So the normal equations have no unique solution. The rank defect free network adjustment method is used here.

Table 1 - Theoretical Coordinates of the 28 Datum Points

Point	*x*	*Y*	**point**	*x*	*y*
1	-9576.10	-4300.18	**15**	10848.91	450.00
2	-10026.10	-3520.76	**16**	10848.91	-450.00
3	-10848.91	-450.00	**17**	10026.10	-3520.76
4	-10848.91	450.00	**18**	9576.10	-4300.18
5	-10026.10	3520.76	**19**	4300.18	-9576.10
6	-9576.10	4300.18	**20**	3520.76	-10026.10
7	-4300.18	9576.10	**21**	450.00	-10848.91
8	-3520.76	10026.10	**22**	-450.00	-10848.91
9	-450.00	10848.91	**23**	-3520.76	-10026.10
10	450.00	10848.91	**24**	-4300.18	-9576.10
11	3520.76	10026.10	**25**	-3027.97	0.00
12	4300.18	9576.10	**26**	0.00	3027.97
13	9576.10	4300.18	**27**	3027.97	0.00
14	10026.10	3520.76	**28**	0.00	-3027.97

Consider a matrix $\underset{56\times3}{G}$, and let $Q=(B^T B+GG^T)^{-1}$. Adopting the false observation method, the normal equation is obtained:

$$(\underset{56\times99}{B^T}\,\underset{99\times56}{B}+\underset{56\times3}{G}\,\underset{3\times56}{G^T})\underset{56\times1}{\delta\hat{\hat{X}}}+\underset{56\times99}{B^T}\,\underset{99\times1}{l}=\underset{99\times1}{0}\ , \qquad [32]$$

where $\delta\hat{\hat{X}}$ is the estimated value of δX which satisfies $\delta x^T\delta x=\min$.
For a trilateration network, the matrix G is

$$\underset{3\times56}{G^T}=\begin{vmatrix} 1 & 0 & 1 & 0 & \cdots & 1 & 0 \\ 0 & 1 & 0 & 1 & \cdots & 0 & 1 \\ -y_1^0 & x_1^0 & -y_2^0 & x_2^0 & \cdots & -y_{28}^0 & x_{28}^0 \end{vmatrix}. \qquad [33]$$

Substituting G and G^T into equation [32] gives

$$\underset{56\times1}{\delta\hat{\hat{X}}}=-(B^T B+GG^T)^{-1}B^T l=QB^T l\,. \qquad [34]$$

Then substituting it into simultaneous equation [25], the adjustment value of coordinates of every datum point are obtained.

Substituting $\underset{56\times1}{\delta\hat{\hat{X}}}$ into equation [29], the correction values of length of observational sides can be obtained. Adding the respective length of observational sides, the adjustment values of length of every side $\underset{99\times1}{\hat{L}}$ can be obtained.

5. Conclusion

In 1997, we performed a survey for the network. Simultaneously we designed a computer program to calculate the Plane Deformation Monitoring Network adopting the method above. The result we obtained is most satisfactory.

6. References

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