Neutrino Mass and Oscillations

R. G. Hamish Robertson
Department of Physics,
University of Washington
Seattle, WA 98195

1 Introduction

The neutrino remains as exotic and challenging today as it was seventy years ago when first proposed by Pauli. What is known for certain about neutrinos is minimal indeed. They have spin $\frac{1}{2}$, charge 0, negative helicity, and exist in 3 flavors, electron, mu, and tau. Strictly speaking, only 2 flavors are certain: direct observation of the tau neutrino has not yet been achieved, but an experiment, DONUT, at Fermilab is in progress with this objective [1]. Limits on the masses from direct, kinematic experiments (ones that do not require assumptions about the non-conservation of lepton family numbers) have been steadily lowered by experiments of ever-increasing sophistication over the years, with the results given in Table 1. As will be discussed below, there are stronger limits on the mass of ν_e from tritium beta decay [2, 3], but the data show curious distortions near the endpoint that are not at present understood.

Electron Family	ν_e	< 15 eV
	e^-	510999.06 eV
Mu Family	ν_μ	< 170 keV
	μ^-	105658.389 keV
Tau Family	ν_τ	< 18 MeV
	τ^-	1777.1 MeV

Table 1: The kinematic mass limits for neutrinos, compared to the known masses of their charged-lepton partners [4]

There are strong theoretical and experimental motivations to search for neutrino mass. Presumably created in the early universe in numbers comparable to photons, neutrinos with a mass of only a few eV would contribute a significant fraction of the closure density. A species with a mass of $94h_0^2$ eV, where h_0 is the Hubble constant in

units of 100 km s^{-1} Mpc^{-1}, provides the closure density by itself, but the structure of the universe at small and intermediate scales is incompatible with the dominance of hot dark matter such as neutrinos. On the other hand, evidence has accumulated from surveys of galaxy distributions and of the cosmic microwave background that is consistent with the presence of some hot dark matter.

In gauge theories of the elementary particles, the fermion masses arise from the coupling of left- and right-handed fields. A Dirac mass for neutrinos is expected to be like the quark and charged-lepton masses, but experiment has shown that neutrino masses are tiny. For this reason, the Minimal Standard Model deprives neutrinos of right-handed fields, forcing them to be always relativistic and massless. A clear demonstration that neutrinos have mass forces a confrontation of our understanding of how mass is generated.

Sensitivity to very small neutrino masses is experimentally accessible if neutrino mass eigenstates are not flavor eigenstates. In that case, a state prepared by the weak interaction ($W^{\pm}$ decay) in a specific flavor projection consists of two or more physical mass components propagating slightly differently with time or distance. A remote detector with specific flavor sensitivity will then register 'a neutrino' with altered flavor projection as a result of the phase difference that accumulates. Because the neutrinos are in general highly relativistic, the phase difference increases not as the difference in the masses, but as the difference Δm^2 between the squares of the masses.

If neutrino oscillations occur, the kinematic limits in Table 1 must be understood to refer to appropriately weighted averages of mass eigenstates.

In general, the mass and flavor bases are related by a unitary transformation similar to the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix in the quark sector. In the neutrino sector, the matrix is called the Maki-Nakagawa-Sakata (MNS) matrix [5]. The customary 3-flavor version of the MNS matrix is given by Mann [6].

The CKM matrix is a 3-flavor matrix, and one of its most intriguing features is the presence of a free complex phase that provides a natural origin for CP violation within the Standard Model. The MNS matrix has at least this degree of freedom, but in the neutrino sector, neutrino-antineutrino mixing can occur as well, giving neutrinos both Dirac and Majorana properties. The MNS matrix is then at least a 6-dimensional one, and can presumably have additional free complex phases. CP violation in neutrinos is potentially a very interesting phenomenon, although experimentally challenging because, as Fisher *et al.* show [7], it is necessary to be able to track the oscillatory nature of three flavor components at once. Neutrinoless double beta decay is also a CP microscope, if the mass sensitivity needed can be reached.

There are now 3 experimental signals indicative of neutrino oscillations and mass. The implications, especially if all 3 are correct, are explored below. The reader is referred to the reviews in these proceedings by Suzuki [8], Mann [6], and DiLella [9] for a discussion of these epochal experiments.

2 Accelerator and reactor oscillation experiments

From the initial experiment discovering the neutrino in 1957, to the present day, reactors and accelerators have been a mainstay of research into the properties of neutrinos. Extensive and modern reviews of the subject are given by DiLella [9] and Fisher *et al.* [7]

Certain recent experiments stand out as particularly influential in shaping our present view of the properties of neutrinos:

- The Liquid Scintillator Neutrino Detector (LSND) experiment [10] at Los Alamos National Laboratory has found evidence for $\overline{\nu}_\mu \rightarrow \overline{\nu}_e$ with $0.2 \leq \Delta m^2 \leq 2.0$ eV^2 and $0.04 \geq \sin^2 2\theta \geq 0.0015$. Confirmatory evidence has been obtained in the charge-conjugate channel $\nu_\mu \rightarrow \nu_e$.

- The KARMEN Experiment [11] at Rutherford-Appleton Laboratory reports no signal in a region of parameter space overlapping much of that explored by LSND. The small remaining region defines the parameters just given for LSND, plus a small island at $\Delta m^2 = 4.5$ eV^2, $\sin^2 2\theta = 2.5 \times 10^{-3}$.

- The Brookhaven E776 Experiment [12] excludes (in the $\nu_\mu \rightarrow \nu_e$ channel) the small island at 4.5 eV^2.

- The Chooz long-baseline reactor antineutrino experiment [13] shows that $\overline{\nu}_e$ does not transform to anything for $\Delta m^2 \geq 10^{-3}$ eV^2, $\sin^2 2\theta \geq 0.1$. The Palo Verde experiment [11] confirms this, at somewhat lower significance. These negative results are remarkably decisive in ruling out substantial atmospheric $\nu_\mu \leftrightarrow \nu_e$ conversion and also in blocking any possibility of large-Δm^2, large-angle solutions to the solar neutrino problem.

Together with the results from atmospheric and solar neutrino measurements, these define the scenarios for neutrino mass and mixing that are most likely.

3 Atmospheric neutrinos

The experiments designed to search for proton decay, IMB [14] and Kamiokande [15], were obliged to deal quantitatively with atmospheric neutrino interactions as the major background to proton instability. It was known that the cosmic ray flux and resulting production rate of neutrinos in the upper atmosphere were uncertain at the level of about a factor of 2, but it gradually became apparent that the ratio of $\nu_\mu + \overline{\nu_\mu}$ to $\nu_e + \overline{\nu_e}$ was also not well predicted. The latter ratio, naively 2 (from pion and muon decay) and calculable to an accuracy of about 5%, was found to be low by a factor of typically 0.6. As this ratio depends little on the cosmic-ray flux and

normalization, the departure from the expected value was somewhat surprising, and neutrino oscillation solutions were proposed. Towards the end of the operation of Kamiokande, evidence was accumulating for a zenith-angle dependence of the ratio (equivalent to a path-length dependence).

Super-Kamiokande has now been in operation since April 1996, with a fiducial mass of approximately 22,500 tons acquiring events at 10 times the rate of Kamiokande. Other detectors, MACRO and Soudan II, have also been accumulating data. The present situation is summarized in the comprehensive review by Mann [6]. Evidence consistent with neutrino oscillation has emerged in the form of:

- An atmospheric neutrino flavor ratio (ν_μ/ν_e) equal to 0.68 of the expected magnitude,
- A zenith-angle dependence in the ν_μ flux that departs greatly from the no-oscillation expectation and agrees closely with an oscillation description, and,
- A zenith-angle dependence in the ν_e flux that agrees closely with the no-oscillation expectation.

In addition, the zenith-angle dependence significantly favors (by about 2σ) the $\nu_\mu \to \nu_\tau$ channel over the $\nu_\mu \to \nu_s$ channel when matter effects are taken into account. The best-fit oscillation parameters are,

$$\begin{aligned} \Delta m^2 &= 3.5 \times 10^{-3}\ \mathrm{eV}^2 \\ \sin^2 2\theta &= 1.0\ . \end{aligned}$$

There is no evidence for the subdominant oscillation $\nu_\mu \to \nu_e$ channel at the same mass difference, from which one can set a limit of about 0.05 on the square of the NMS matrix element U_{e3} in a 3-flavor description [6].

Are there any possible loopholes? The following points can be, and in some cases have been, made:

- The "R" value, the ratio of μ-flavor to e-flavor observed compared to that calculated, is smaller than expected for the oscillation parameters constrained by the zenith-angle dependence [16].
- The experimentally measured value of the rate for inclusive CC reactions of ν_μ on ^{12}C is about a factor 2 smaller than calculated, whereas the corresponding ν_e reaction has the expected rate [9].
- Recoil-order terms in the neutrino-nucleon cross sections, particularly the pseudoscalar form factor, have apparently been neglected. The pseudoscalar contribution has an effect of several percent in the ratio R (because it contains the

charged lepton mass), and may have a significantly larger role in the angular distributions where it appears as an interference term. The angular distributions are used to extract the zenith-angle dependence.

- The zenith-angle dependence, a very convincing aspect of the evidence for oscillation because it is so model-independent, does in fact depend on the extent to which pions and muons range out in the earth before decaying, and hence also on the altitude at which they are produced, the primary cosmic-ray spectrum, interaction cross sections, *etc.*

None of these points is thought to constitute a major concern, and neutrino oscillation appears to be the logical explanation for the results.

4 Solar neutrinos

The sun, it is believed, generates its energy by fusion reactions that can be summarized as

$$4\mathrm{p} + 2\mathrm{e}^- \rightarrow {}^4\mathrm{He} + 2\nu_e + 26.731 \text{ MeV}.$$

Each cycle through the hydrogen-burning process produces 2 electron neutrinos. It follows directly that the neutrino flux at the earth's surface is proportional to the thermal energy flux, which is an experimentally measured quantity. The electromagnetic solar constant (irradiance) $I = 0.1367$ W cm^{-2}. With a small correction (about 1%) for the energy carried away by the neutrinos themselves, the neutrino flux at the earth's surface is 6.44 x 10^{10} cm^{-2} s^{-1}, independent of detailed models of the sun. It is necessary only that the sun be in hydrostatic equilibrium over a period considerably longer than the photon migration time, 10,000 years.

In practice, no detector presently exists that can measure the total flux of solar neutrinos. Detectors have thresholds and strongly energy-dependent sensitivities. In hydrogen burning, a number of pathways lead to ^{4}He, and a complex spectrum of neutrinos from *pp*, *pep*, ^{7}Be, *hep*, ^{13}N, ^{15}O, ^{17}F, and ^{8}B results. The spectral shape of each individual component, whether line or continuum, is determined by laboratory measurement and/or electroweak theory. The relative intensity of each component, on the other hand, depends strongly on the temperature and composition, and therefore on astrophysical models of the sun. The flux component most easily detected, ^{8}B, is also the most temperature sensitive, varying as the 25th power of the central temperature, and it is moreover a component so minor (0.01%) that it is unconstrained by the sun's energy output.

Data from 5 experiments (3 different types of experiment) provide information on different combinations of the fluxes. The current results are summarized in Table 2. It is very surprising that, with only 3 independent types of measurement and 8 different neutrino sources in the sun, it is impossible to fit the data (well) without introducing

Cl-Ar	$2.56 \pm 0.16 \pm 0.16$ SNU	[17]
Kamiokande	$(2.80 \pm 0.19 \pm 0.33)$ x 10^6 ^{8}B ν_e cm^{-2} s^{-1}	[18]
SuperKamiokande	$(2.45 \pm 0.04 \pm 0.07)$ x 10^6 ^{8}B ν_e cm^{-2} s^{-1}	[8, 19]
SAGE	$67.2^{+7.2\ +3.5}_{-7.0\ -3.0}$ SNU	[20]
Gallex	$78 \pm 6 \pm 5$ SNU	[21]

Table 2: Results of the 5 solar neutrino experiments (1 SNU = 10^{-36} events per atom per second).

neutrino oscillations or some other non-standard-model physics! This comes about qualitatively as follows:

- The *hep* flux, as will be discussed, is negligibly small in the flux balance, and is known to be so from the high-energy part of the SK solar neutrino spectrum.
- The *pep* flux is tied to the *pp* flux in a way that depends very weakly (as the square root of the temperature) on models.
- The CNO and ^{7}Be neutrinos are detected both by Cl-Ar and by Ga-Ge, but not by Kamiokande and SK. While they cannot be individually disentangled in a model-independent analysis, that is not required to demonstrate the inconsistency of standard-physics solutions with the data.

Consequently, there are really only 3 relevant neutrino sources, namely *pp* + *pep*, ^{7}Be + CNO, and ^{8}B. There are also 3 independent experimental measurements, plus, if one elects to apply it, the luminosity constraint that relates the neutrino flux to solar energy output.

There is no combination of the fluxes with all fluxes non-negative [22, 23, 24] that fits the data. With the luminosity constraint applied to the total flux, the statistical significance of this conclusion is now at about the 3.5σ level. Of course, the conclusion depends on there not being large unknown systematic errors in the data or in the detectors' neutrino cross sections, but it does not depend on models of the sun. Even if the luminosity constraint is abandoned (equivalent to allowing variability of the solar core over times of order 30–10,000 years, or more exotic possibilities), there is no solution at about the 2σ level.

If the experimental errors have been properly estimated, then, this contradiction means that one of the assumptions made in fitting the data must be incorrect, and there are very few assumptions.

It must be concluded that the shape of the ^{8}B spectrum is not as expected, containing more strength at high energies and less at low, and/or the neutrino flavor content is not pure electron, which alters the relationship between the water-Čerenkov

Solution	$\nu_e \rightarrow$	Δm^2 eV^2	$\sin^2 2\theta$	$\chi^2_{\rm min}$
Large-angle	Active	1.8 x 10^{-5}	0.76	4.3
	Sterile	-	-	19
Small-angle	Active	5.4 x 10^{-6}	0.006	1.7
	Sterile	4.6 x 10^{-6}	0.007	1.7
Low	Active	7.9 x 10^{-8}	0.96	7.3
	Sterile	-	-	17
Vacuum	Active	8.0 x 10^{-11}	0.75	4.3
	Sterile	-	-	12

Table 3: Two-flavor neutrino oscillation fits to the solar neutrino data, in the framework of the Bahcall-Basu-Pinsonneault Standard Solar Model [24].

results and the radiochemical experiments (because elastic scattering, unlike inverse beta decay, can occur via the neutral-current interaction with neutrinos of all active flavors). These features, not permitted in the Minimal Standard Model, are characteristic of neutrino oscillation solutions [22]. In contrast to the standard-physics solution, such solutions can give an excellent account of all data.

The need for non-standard physics (presumably neutrino oscillations) is model-independent at the roughly 3.5-σ level, but the derivation of specific oscillation parameters is done in the context of astrophysical solar models [24] and experimental nuclear-physics inputs. Qualitatively, there are three 2-flavor solutions that describe the data reasonably well, and a fourth with a lower probability. These are termed the Large-Mixing-Angle [LMA], the Small-Mixing-Angle [SMA], the 'LOW', and the vacuum solutions; see Fig. 1. Table 3 summarizes the fit results within the framework of a standard solar model [24].

If neutrino oscillations are indeed the explanation of the solar neutrino problem, independent evidence for them might arise from:

- Spectral distortions in the ^{8}B flux,
- Day-night variations indicative of MSW regeneration in the earth,
- Yearly variations beyond those expected from the earth's orbital eccentricity, and
- Neutral-current interaction rates larger than expected from measured charged-current rates.

The SuperKamiokande collaboration is continuing a program of careful energy calibration and accumulation of high-statistics data in search of a distortion of the

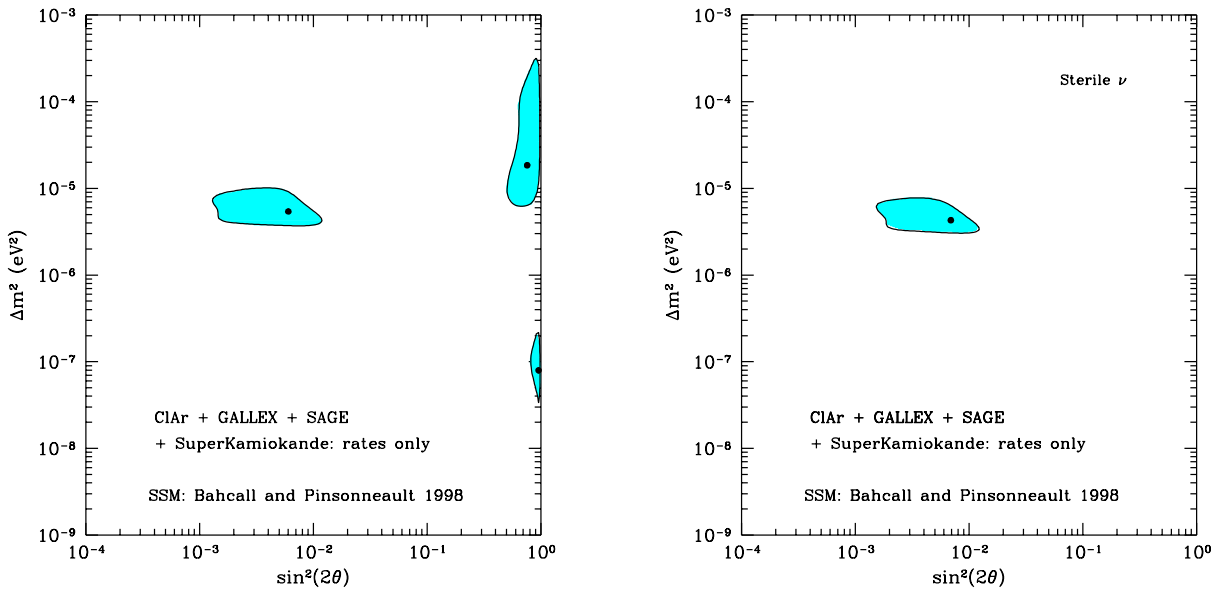

Figure 1: MSW solutions (rates only, 99% CL) for active neutrinos (left) and sterile neutrinos (right) [25].

spectrum. The expected effects are quite small at best and, at the upper end of the spectrum, vanishingly small for all but the vacuum solutions. There are, moreover, sources of distortion unrelated to neutrino physics:

- The possible presence of *hep* neutrinos (^{3}He + p $\rightarrow$ ^{3}H + e$^+$ + ν_e + 18.7 MeV) [26, 27]. The *hep* spectrum is considerably harder than the ^{8}B one, and contributes extra intensity in the vicinity of the ^{8}B endpoint at 15 MeV and beyond (to the 19 MeV endpoint of the *hep* spectrum). Calculation of the rate of the reaction is difficult because the lowest-order Gamow-Teller matrix element is small owing to the near orthogonality of the radial wavefunctions; forbidden terms dominate. Fortunately, comprehensive first-principles calculations of the rate, including the higher partial waves that contribute in the solar plasma, have been recently reported [28, 29]. Horowitz [28] made the first calculation of the continuum ^{3}P$_0$ axial-charge transition, finding the S-factor to be

$$S_{0,p}(E) \;=\; 1.7 \times 10^{-17}\ \text{eV b},$$

  which is almost as large as the 'standard' *s*-wave component in use hereto-

fore [30],

$$S_{0,s}(E) = 2.3 \times 10^{-17} \text{ eV b.}$$

Schiavilla [29] reports:

$$\begin{aligned} S_{0,s}(E) &= 3.9 \times 10^{-17} \text{ eV b} \\ S_{0,p}(E) &= 2.4 \times 10^{-17} \text{ eV b} \\ S_{0,d}(E) &\leq 1 \times 10^{-18} \text{ eV b,} \end{aligned}$$

for all the *s, p,* and *d* partial waves, respectively. The energy-dependence of $S_{0,i}$ is negligible for all partial waves, and thus

$$S_0(E) = 6.3 \times 10^{-17} \text{ eV b}$$

is an accurate value for this rate for the purposes of neutrino flux calculations. While 3 times larger than the value in previous use, it falls short of the 16.7 times needed to account fully for distortions being seen in SuperKamiokande [8].

- The shape of the neutrino spectrum from ^{8}B decay is not directly calculable because the final state (in ^{8}Be) is broad. The spectrum is inferred from the recoil alpha spectra in laboratory experiments having other objectives [31]. Preliminary results of measurements at Notre Dame University [32] specifically designed to address some possible systematic concerns indicate that the standard spectrum under predicts the intensity in the endpoint region by a fraction that peaks at about 14% at 2 MeV below the endpoint.

- The beta decay of ^{8}B to the ^{8}Be ground state is second-forbidden and has not been observed. With similar transitions (*e.g.*, ^{36}Cl) as a guide, it can be expected to have a branching ratio of order 10^{-3}. Its spectrum would in that case be similar in both magnitude and energy to the *hep* spectrum.

It is at the moment too soon to draw conclusions concerning neutrino oscillations from the shape of the spectrum at high energies, but the experimental and theoretical uncertainties are rapidly being reduced. One can expect before long to have a useful constraint on oscillation solutions from the spectral shape.

Day-night effects arise from matter regeneration in the varying path through the earth's core, and are less dependent on details. No certain evidence for time variations beyond statistical fluctuations has shown up to date, but the most recent data from SK [8] yield a two-standard deviation effect,

$$\frac{N-D}{N+D} = 0.065 \pm 0.031 \pm 0.013.$$

The absence of large day-night effects has already ruled out a large region of parameter space in the range $10^{-6} \leq \Delta m^2 \leq 10^{-5}$ eV^2 and $10^{-2} \leq \sin^2 2\theta$. As Suzuki [8] shows (for further discussion, see also Bahcall *et al.* [33]), the small but general night enhancement matches better with the LMA solution (the lower part, in the vicinity of $\sin^2 2\theta$=1.0, Δm^2= 1.9 x 10^{-5} eV^2) than it does the SMA solution, where effects are all negligible except when the sun is on the other side of the earth's core. The latter possibility seems disfavored at almost 3σ. The details of the LMA region are shown [33] in Fig. 2.

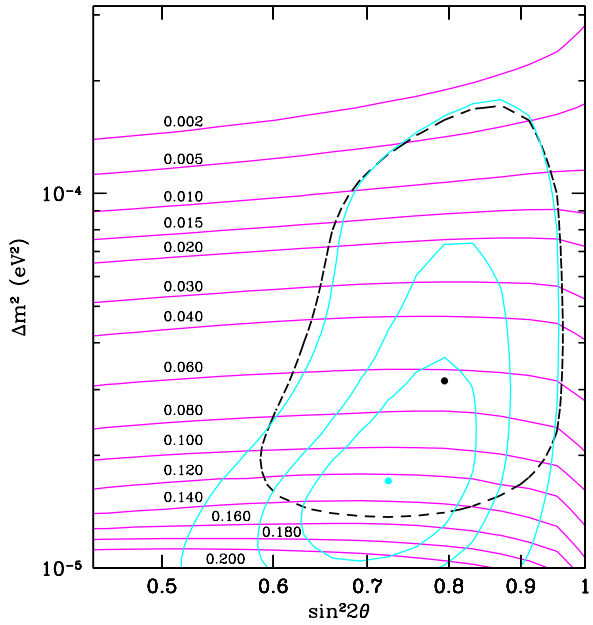

Figure 2: LMA solution, active neutrinos. Continuous closed contours: rates only (90, 95, 99% CL). Horizontal contours: night-day asymmetries. Dashed contour: rates plus night-day (99% CL) [33].

The fact that information on *solar* neutrino solutions was present in *atmospheric* neutrino data was evidently first noted by Giunti *et al.* [34]. The solar LMA solution is only valid for mixing with active, not sterile, neutrinos [25]. Hence this solution is in conflict with either the LSND experiment or the indications that the dominant atmospheric signal is $\nu_\mu - \nu_\tau$. If LMA were nevertheless the correct solution, upward-going electron neutrinos from the atmosphere would convert appreciably and equally to μ and/or τ neutrinos, potentially causing a deficit in ν_e and an increase in ν_μ. The

vacuum oscillation length is

$$L_0 = 2.47 \frac{E_\nu}{\Delta m^2}$$

when distances are in km, energies in GeV, and masses in eV. For the LMA solution, $\Delta m^2 \simeq 3 \times 10^{-5}$ and the oscillation length becomes one earth diameter (13,000 km) at a neutrino energy of 160 MeV. An inspection of Figs. 2 and 15 in Mann's paper [6], shows, if anything, a slight excess in the low-energy ν_e up-down asymmetry. Peres and Smirnov show [35], however, that when the calculation is done in detail and matter effects are taken into account, the excess is in fact expected for much of the atmospheric and LMA parameter space. The fact that the ν_μ flux is intrinsically about 2 times the ν_e flux and that both oscillation solutions are near maximal mixing conspire to produce effects that may be of either sign, depending on the specific parameter values.

The implication is that the LMA solution may be favored by the atmospheric and solar neutrino data. This is good news for the KamLAND experiment, a reactor $\overline{\nu}_e$ experiment that reaches the LMA parameter space, as DiLella has described [9].

Yearly time dependence is the hallmark of vacuum oscillation solutions as the earth's orbital eccentricity explores different oscillation phases. The eccentricity is small (a total 3.5% distance variation), and for the continuous neutrino spectrum emitted by ^{8}B and detected with detectors having relatively poor energy resolution, the effects are hard to see. It will be very different when high-statistics detectors primarily sensitive to ^{7}Be neutrinos (*e.g.* Borexino [36]) come on line, because the narrow line width of the source leads to striking time-dependence in the vacuum oscillation signal.

The question of the ratio of charged to neutral currents will be addressed in the Sudbury Neutrino Observatory (SNO) [37], a 1000-ton heavy-water Čerenkov detector now operating 2000 m underground in the INCO Creighton nickel mine near Sudbury, Ontario. SNO will permit observation of both the charged-current (CC) inverse beta decay of the deuteron:

$$\mathrm{d} + \nu_e \;\rightarrow\; \mathrm{p} + \mathrm{p} + e^- - 1.44 \text{ MeV} \; .$$

and the neutral-current (NC) neutrino breakup reaction:

$$\mathrm{d} + \nu_x \;\rightarrow\; \mathrm{p} + \mathrm{n} + \nu_x - 2.22 \text{ MeV}$$

The nuclear-physics uncertainties in the cross sections for these two processes arise mainly from the final states, which are members of the same isospin triplet. As a result, the ratio expected is known to a precision of order 1%, and significant departures (*e.g.*, a factor of 3, as present solar neutrino information would suggest) would point unequivocally to neutrino oscillations to an active species. If the oscillation is to sterile neutrinos, there are nevertheless spectroscopic and time-dependent signatures that may be measurable.

In addition to the NC/CC ratio, SNO will provide good information on the shape of the ^{8}B spectrum above 5 MeV because in the CC reaction on deuterium the energy of the incident neutrino is transferred largely to the electron. Sensitivity to day-night and yearly effects is similar to that of SuperKamiokande, but the spectroscopic resolution permits the energy-dependence of such effects to be investigated efficiently. The quasi-elastic CC cross section rises quadratically with energy, and the backgrounds at the 2000-m depth of SNO are low, so detection of *hep* neutrinos may be possible.

5 Interpreting the results

The width of the Z^0 permits the existence of 3 light neutrinos and their antineutrinos that couple universally to the weak interaction. If only three different mass eigenstates m_i, $i = 1, 2, 3$, exist, the mass splittings must satisfy

$$\sum_{\text{Splittings}} \Delta m_\nu^2 = (m_3^2 - m_2^2) + (m_2^2 - m_1^2) + (m_1^2 - m_3^2) = 0, \quad (1)$$

a trivial condition which is not met by any combination of the independent Δm_ν^2 from experimentally favored neutrino mass differences (Table 4).

How can this difficulty be evaded? One must either assume that an experimental datum is incorrect (or at least misinterpreted), or that a fourth neutrino type exists, one that does not couple to the weak interaction significantly. Specific remedies that have been proposed are,

- Accept a sterile neutrino that mixes with one or more active species.

- Reject the LSND result. Although there is evidence for the effect in both $\overline{\nu}_\mu \to \overline{\nu}_e$ and $\nu_\mu \to \nu_e$, the effect has not been seen in other experiments that explore similar, but not identical, regions [9]. The LSND result is very constraining if correct, which perhaps accounts for the eagerness to reject it.

- Reject the Cl-Ar result or the water-Čerenkov solar-neutrino result. These experiments are both primarily sensitive to the ^{8}B flux. Together they fragment the allowed oscillation parameter space into islands in which shape distortions of the ^{8}B spectrum and neutral-current contributions allow the two results to be reconciled. All of those islands lie at small Δm^2. Now, however, with the Chooz result [13], rejecting one or the other of those measurements no longer would permit a large-Δm^2, large-$\sin^2 2\theta$ solution to the solar-neutrino problem.

Experiment	Favored Channel	Δm^2 [eV^2]	$\sin^2 2\theta$
LSND	$\overline{\nu}_\mu \to \overline{\nu}_e$	0.2–2.0	$(1.5\text{–}40) \times 10^{-3}$
Atmospheric	$\nu_\mu \to \nu_\tau$	3.5×10^{-3}	1
	$\nu_\mu \to \nu_s$	Disfavored at $\sim 2\sigma$	
Solar			
MSW (large angle)	$\nu_e \to \nu_\mu$ or ν_τ	$(1.3\text{–}18) \times 10^{-5}$	0.6–0.95
MSW (small angle)	$\nu_e \to$ anything	$(0.4\text{–}1) \times 10^{-5}$	$(0.7\text{–}10) \times 10^{-3}$
Vacuum	$\nu_e \to \nu_\mu$ or ν_τ	$(0.05\text{–}5) \times 10^{-10}$	0.6–1

Table 4: Experimentally favored neutrino mass differences and mixing angles.

6 Sterile neutrinos

Heavy sterile neutrinos are a staple ingredient of most extensions to the Standard Model, but light sterile neutrinos have been regarded with distaste. The main reason is that the usual explanation invoked for the lightness of active neutrinos is the see-saw mechanism, realized with the aid of a sterile neutrino of GUT-scale mass. Having 'used up' that sterile Majorana neutrino component in producing light active neutrinos, one must then invoke complicated new mechanisms to generate and bring down other light sterile neutrinos.

Light sterile neutrinos are, however, just as natural as light active neutrinos if one does not start from the see-saw.

Our experience with the charged fermions is that they are described by the Dirac equation, to staggering precision. As the neutrino is a neutral fermion, it would be reasonable to suppose the Dirac equation should apply again. Dirac spinors are 4-component objects, with two spin states and distinct neutrinos and antineutrinos.

In deference to the handedness of the weak interaction, it is useful to project the four components, using the R/L and charge conjugation projection operators:

$$\psi_{R/L} = \tfrac{1}{2}(1 \pm \gamma_5)\psi$$
$$\mathrm{C}\psi_{R/L}\mathrm{C}^{-1} = \psi^c_{R/L}$$

With 3 active neutrino flavors, mass terms in the Lagrangian have the form

$$\mathcal{L}_m(x) \sim m_D \overline{\psi}(x)\psi(x) \Rightarrow M_D \overline{\Psi}(x)\Psi(x)$$

where m_D has been replaced by a nondiagonal 3×3 matrix M_D in flavor space and

$$\Psi = \begin{pmatrix} \psi^e \\ \psi^\mu \\ \psi^\tau \end{pmatrix} .$$

Mass terms in the Lagrangian must be Lorentz scalars, with no handedness. Following the development of Haxton and Stephenson [38] and Langacker *et al.* [39], the resulting mass matrix takes on the form:

$$(\overline{\Psi}^c_L, \overline{\Psi}_R, \overline{\Psi}_L, \overline{\Psi}^c_R) \begin{pmatrix} 0 & 0 & 0 & M_D^T \\ 0 & 0 & M_D & 0 \\ 0 & M_D^\dagger & 0 & 0 \\ M_D^* & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \Psi^c_L \\ \Psi_R \\ \Psi_L \\ \Psi^c_R \end{pmatrix}.$$

It allows for flavor oscillations if M_D is nondiagonal. If CP is conserved, the four mass submatrices are equal [38].

The mass matrix is comprised of equal-mass active and sterile neutrinos and their antineutrinos, a pair for each generation. This situation is what one would naively expect if neutrinos were exactly like electrons, for example.

The upper left and lower right quadrants of this matrix must be zero because the left- and right-handed projectors annihilate each other. For charged fermions, charge conservation assures the remaining elements other than the ones already specified must be zero. However, for *neutral* fermions, additional terms can be introduced elsewhere if we respect the requirement of hermiticity. Specifically,

$$\mathcal{L}_m(x) \Rightarrow M_D \overline{\Psi}(x)\Psi(x) + \overline{\Psi}^c_L M_L \Psi_L + \overline{\Psi}^c_R M_R \Psi_R$$

so that the mass matrix becomes

$$(\overline{\Psi}^c_L, \overline{\Psi}_R, \overline{\Psi}_L, \overline{\Psi}^c_R) \begin{pmatrix} 0 & 0 & M_L & M_D^T \\ 0 & 0 & M_D & M_R^\dagger \\ M_L^\dagger & M_D^\dagger & 0 & 0 \\ M_D^* & M_R & 0 & 0 \end{pmatrix} \begin{pmatrix} \Psi^c_L \\ \Psi_R \\ \Psi_L \\ \Psi^c_R \end{pmatrix}.$$

The new Majorana mass terms do not conserve lepton number. It is these non-Dirac mass terms that can generate the nonzero $\langle m_\nu^{Maj} \rangle$ that must be present if neutrinoless $\beta\beta$ decay occurs.

The see-saw arises from setting $m_L = 0$, $m_D \simeq m_u, m_c, m_t$, and $m_R \simeq M_{\rm GUT}$. After diagonalization, the eigenstates are a pair of light-heavy Majorana neutrinos in each generation. If, however, the Majorana mass terms are small (and so are the Dirac mass terms, although we do not understand why), then a different but equally interesting phenomenology results. The Majorana mass terms introduce a mixing between the active and sterile states, *viz,*

$$\psi_L \rightarrow \cos\theta \psi_L + \sin\theta \psi^c_R.$$

Such terms can be present within a generation and between generations, and lead to a complex 6-flavor neutrino mass, mixing, and charge-conjugation map. Gelb and

Rosen [40] have pursued this with a 4-flavor subset of neutrinos (3 active and one sterile), and show that not only can the observed mass splittings be reproduced, but the mixing angles are natural.

If the positive indications of neutrino oscillation from LSND, from atmospheric neutrinos, and from solar neutrinos are all correct, then either the atmospheric-neutrino or the solar-neutrino mixing must involve a sterile neutrino. There are two contradictory hints about which solution to choose. The zenith-angle distribution for partially-contained atmospheric neutrinos slightly favors active neutrino mixing. On the other hand, the indications of a possible day-night effect slightly favor the LMA solar solution, which only occurs with active neutrinos. In either case, the mass spectrum splits into a pair of doublets with the pair split by the LSND scale (≥ 0.5 eV) [41] (Fig. 3).

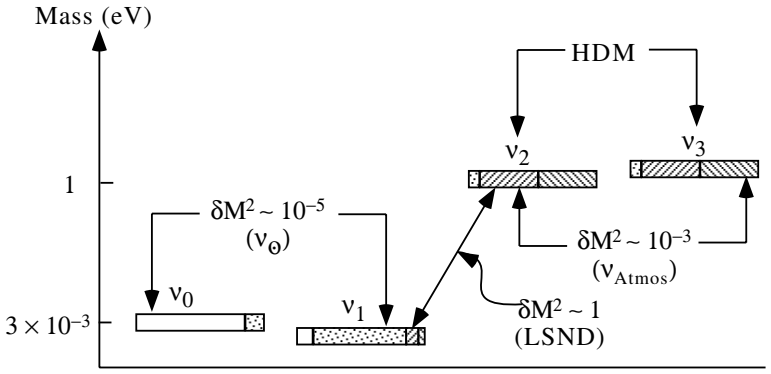

Figure 3: Four-neutrino mass and mixing scheme to accommodate all data [41]. An inverted order for the large splitting is also possible.

Either the standard hierarchical order or the inverted order (solar neutrinos heaviest) are possible, but the sterile partner of the electron neutrino is always the heavier in order to have resonant conversion in the sun. Three sterile neutrinos significantly mixed with the active ones may create conflicts with the pace of evolution of the early universe as determined from the helium abundance, although one such sterile neutrino is probably not ruled out. The same constraint also arises in the 'degenerate' scenario in which the mass splittings are as given by oscillation data, but all masses are shifted up (becoming nearly degenerate in the process). Such a scenario is appealing as a source of hot dark matter (HDM), and marginally acceptable in nucleosynthesis if only one sterile neutrino species is mixed.

An important and perhaps unfortunate consequence of this particular sterile neutrino picture is the suppression of neutrinoless double beta decay. Neutrinos retain their Dirac nature by and large, with relatively small Majorana components.

While this structure was forced by the need to avoid conflicts between experimental results, it is important to bear in mind that it may be true *even if one or more experiment is presently wrong.*

7 Direct methods – tritium and double beta decay

Oscillation experiments can never yield a value for *the mass* because such experiments are sensitive only to phase differences that arise from the differences in the squares of the masses.

Only two methods are presently known that have direct mass sensitivity that is at least roughly in the needed range, single beta decay (of tritium especially) and double beta decay. For many years, these difficult experiments have been laboriously pursued, but there was always a worry that one was looking at the "wrong" neutrino, because the natural prejudice is to suppose that the ν_e, ν_μ, ν_τ mass heirarchy probably looks like the corresponding charged leptons.

With the discovery of oscillations virtually certain now, this picture has completely changed. The small mass differences that are representative of oscillations, and the links they forge between mass eigenstates, mean that to measure the mass of one eigenstate is to measure them all. The highly sensitive techniques applicable to the electron neutrino bring all the neutrino masses into the laboratory.

There are two tritium beta decay experiments currently in operation, one in Troitsk [2], the other in Mainz [3]. Both make use of magnetic-electrostatic retarding-field analyzers. The Troitsk analyzer is connected to a gaseous T_2 source, the Mainz one to a solid frozen T_2 source. Steady progress has been made in both laboratories over the years in reducing backgrounds, improving stability and resolution, and checking for systematic effects. The sensitivity of both instruments is now in the range of 2 eV.

Initially, both experiments reported large negative values of the parameter m^2. This parameter, when positive, represents the weighted average of the square of the neutrino mass,

$$m^2 = \sum_i |U_{ei}|^2 m_i^2 \ ,$$

but when negative serves as an effective parameter to continue the functional form of the beta spectrum into the non-physical regime (more events near the endpoint instead of fewer) to allow for statistical fluctuations. In fact, all recent tritium experiments have reported $m^2 < 0$, in some cases well beyond the level expected from statistical fluctuations, indicative perhaps of systematic effects. The main effect seen in the Troitsk experiment was traced to electrons trapped in the source region and escaping to the spectrometer only after having suffered energy loss. In the Mainz experiment the main effect was a morphological change in the structure of the tritium film, which increased the energy loss.

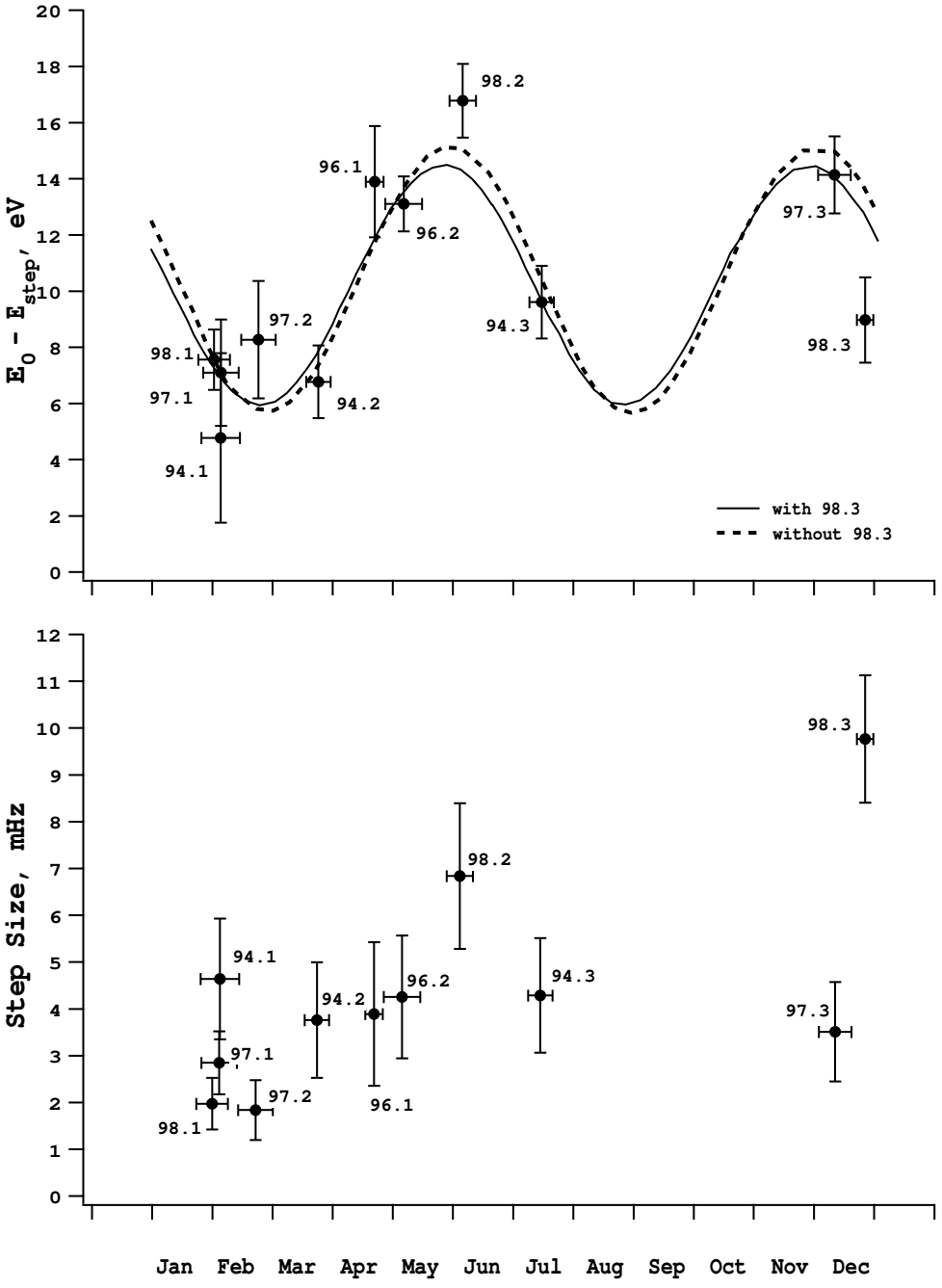

Figure 4: The step observed in the Troitsk integral tritium beta spectrum; top—position from endpoint; bottom—intensity. [2]

Once those problems were eliminated, a curious 'step' remained in much of the Troitsk data near the endpoint. (The spectrometer being an integral device, a step would correspond to a spike in the differential spectrum.) This step varied in intensity and position from run to run. In Fig. 4 the position of the step (which was always below the endpoint) and the intensity are shown. The position appeared to show a periodic motion with a period of 0.50 years. The last point, 98.3 is a run taken to test the prediction from the set of earlier data; it cannot be said either to agree or to disagree strongly with it.

In extracting a limit on the mass, the Troitsk group includes a step function in the fit. That is done consistently for all runs, with the step amplitude and position being fit parameters for each run. Unfortunately, this means that approximately half of the runs must be discarded because the step does not show up clearly in them and becomes excessively covariant with other fit parameters. When the step is fit, there remains no (other) non-standard contribution,

$$m_\nu^2 = -2.0 \pm 3.5 \pm 2.1 \text{ eV}^2$$

and a 95%-CL upper limit (Feldman-Cousins) on the neutrino mass is set at 2.5 eV. Fitting data without the step results in $-15 \leq m_\nu^2 \leq -12$ eV^2, unless the fit is restricted to the last 70 eV of the spectrum, in which case $m_\nu^2 = 5 \pm 5$ eV^2.

With a series of substantial technical improvements to their apparatus, the Mainz group succeeded in increasing the signal-to-background ratio tenfold and are able to take data at a sensitivity competitive with the Troitsk instrument. Out of 4 runs taken in 1997-8, one shows a step 12 eV from the endpoint at the same time as the Troitsk group measured one (shown as "98.2"in Fig. 4). At other times, 97.1, 98.1, and 98.3, no step was seen. The Mainz data fit without a step gives negative values for m_ν^2 very similar to the Troitsk ones ($-15 \leq m_\nu^2 \leq -12$ eV^2), but a different prescription for negative m_ν^2 is used, so direct comparison is not possible. When the fit is restricted to the last 15 eV of the spectrum,

$$m_\nu^2 = -0.1 \pm 3.8 \pm 1.8 \text{ eV}^2$$

and a 95%-CL upper limit (Feldman-Cousins) on the neutrino mass is set at 2.9 eV.

For positive ions, the spectrometers are Penning traps (in which charged particles can be confined by axial magnetic fields and electrostatic potentials) and the Mainz group has begun to explore the possibility that a significant density of ions may accumulate in them. A cloud of such ions would show no kinetic gas pressure, but could nevertheless cause inelastic collisions of the electrons being analyzed. One run has been carried out in which an oscillatory clearing electric field was applied at intervals to eject such ions, and in that run no negative m_ν^2 effect was seen. This is promising, but clearly a preliminary result. It must be remembered that the negative m_ν^2 effect was seen in the Mainz data without a step being present, and so they may not be observing the same phenomenon.

If a step really is present in the integral spectrum, it is a very exciting development. Capture of relic neutrinos produces a spike in the beta spectrum, because the decay energy is transferred entirely to the electron. There are no good laboratory limits on the local density of neutrinos near the earth. Perhaps this is an indication that it is very large, of order 10^{15} cm^{-3}. Stephenson *et al.* [42] present a model in which such a density can arise, and which appears not to conflict with any known facts.

If on the other hand, the step turns out to be instrumental and removable, it is already clear that the present generation of instruments has the capability of reaching a limit in the range of 1-2 eV, a remarkable achievement. Plans are afoot in both groups for next-generation devices that will attack the 1 eV level.

Many nuclei provide an opportunity to search for neutrinoless double beta decay when single beta decay is blocked by energy conservation. Because large high-resolution detectors can be made from Ge (enriched in ^{76}Ge), the current best limit comes from the Heidelberg-Moscow collaboration [43, 7]:

$$< m_\nu^{Maj} >= \sum_m \lambda_m |U_{em}|^2 m_m \leq 0.4\text{eV}^2$$

where m is an index summing over Majorana mass terms only and λ_m is the CP phase.

8 The future

With neutrino oscillations, and therefore mass, becoming more firmly established, the task of the experimentalist is clearer than at any time in the past – not necessarily easier, but clearer. No longer does the parameter ocean extend logarithmically to the horizon in every direction, but instead well-defined islands call out for close exploration.

Table 5, inspired by a similar approach in Fisher *et al.* [7], sets forth a list of key experiments and the implications of possible results of those experiments. The questions to be sorted out first include, what neutrino species are involved in the atmospheric and solar oscillation channels, can the LSND result be confirmed, what mixing parameters are responsible for the solar neutrino effects, and what is the magnitude of the mass itself? Answers to these questions will be hard-won, but they seem within reach. Even more difficult and subtle issues are the charge-conjugation properties of neutrinos and their transformation under CP. The next decade will be an exciting time in neutrino physics.

Information and help generously given by John Beacom, Wick Haxton, Boris Kayser, Vladimir Lobashev, Ernst Otten, Peter Rosen, and Robert Schrock, are most gratefully acknowledged.

Future Observation	Solar ν Oscillation	Atmospheric ν Oscillation	LSND ν Oscillation
~ 1 eV mass in tritium decay $\rightarrow$ HDM	A near-degenerate partner	Δm^2 yields masses of three eigenstates	
$0\nu\beta\beta$ $\rightarrow$ HDM	Majorana neutrinos, likely see-saw		
SNO NC/CC = 3	Active Flavor mixing	One must be sterile = conflict?	
SNO NC/CC = 1	Sterile	Active OK	
Deep Borexino ^{7}Be deficit, < NC	Sterile, Small-angle, Matter-enhanced		
Borexino yearly signal	Vacuum Oscillations		
A *pp* solar ν experiment, CC only	Define Δm^2, $\sin^2 2\theta$, NC/CC with Borexino		
KamLAND $\overline{\nu}_e$ disappearance	Large angle, active	One must be sterile = conflict?	
Results from K2K, MINOS, CERN		Measure $\nu_\mu \rightarrow \nu_\tau$	
Results from Boone, MINOS			Confirm LSND, measure $\nu_e \leftrightarrow \nu_\mu$

Table 5: Future neutrino program and possible outcomes.

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Discussion

Jon Thaler (University of Illinois): Is there any observable effect in electron capture rates?

Robertson: Yes, electron capture decay has been studied as a means for measuring the neutrino mass (as distinct from the antineutrino mass). Capture from higher-lying atomic subshells can be cut off or attenuated if the neutrino has mass. In practice the interpretation of the resulting X-ray spectra is made difficult by rearrangement processes analogous to shakeup and shakeoff.