Tau Physics

Antonio Pich
Departament de Física Teòrica, IFIC
Universitat de València — CSIC
Apt. Correus 2085
E-46071 València, Spain

1 Introduction

The τ lepton is a member of the third generation which decays into particles belonging to the first and second ones. Thus, τ physics could provide some clues to the puzzle of the recurring families of leptons and quarks. One naïvely expects the heavier fermions to be more sensitive to whatever dynamics is responsible for the fermion–mass generation. The pure leptonic or semileptonic character of τ decays provides a clean laboratory to test the structure of the weak currents and the universality of their couplings to the gauge bosons. Moreover, the τ is the only known lepton massive enough to decay into hadrons; its semileptonic decays are then an ideal tool for studying strong interaction effects in very clean conditions.

The last few years have witnessed a substantial change on our knowledge of the τ properties [1, 2]. The large (and clean) data samples collected by the most recent experiments have improved considerably the statistical accuracy and, moreover, have brought a new level of systematic understanding.

2 Charged–current universality

The decays $\tau^- \to e^- \overline{\nu}_e \nu_\tau$ and $\tau^- \to \mu^- \overline{\nu}_\mu \nu_\tau$ are theoretically understood at the level of the electroweak radiative corrections [3]. Within the Standard Model (SM),

$$\Gamma(\tau^- \to \nu_\tau l^- \overline{\nu}_l) \,=\, \frac{G_F^2 m_\tau^5}{192\pi^3}\, f(m_l^2/m_\tau^2)\, r_{EW}, \tag{1}$$

where $f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log x$. The factor $r_{EW} = 0.9960$ takes into account radiative corrections not included in the Fermi coupling constant G_F, and the non-local structure of the W propagator [3]. Using the value of G_F measured in

μ decay, $G_F = (1.16637 \pm 0.00001) \times 10^{-5}$ GeV^{-2} [3, 4], Eq. (1) provides a relation between the τ lifetime and the leptonic branching ratios $B_{\tau\to l} \equiv B(\tau^- \to \nu_\tau l^- \overline{\nu}_l)$:

$$B_{\tau\to e} = \frac{B_{\tau\to\mu}}{0.972564 \pm 0.000010} = \frac{\tau_\tau}{(1.6321 \pm 0.0014) \times 10^{-12}\,\mathrm{s}}\,. \qquad (2)$$

The errors reflect the present uncertainty of 0.3 MeV in the value of m_τ [5, 6].

m_τ	$(1777.05^{+0.29}_{-0.26})$ MeV
τ_τ	(290.77 ± 0.99) fs
$\mathrm{Br}(\tau^- \to \nu_\tau e^- \overline{\nu}_e)$	$(17.791 \pm 0.054)\%$
$\mathrm{Br}(\tau^- \to \nu_\tau \mu^- \overline{\nu}_\mu)$	$(17.333 \pm 0.054)\%$
$\mathrm{Br}(\tau^- \to \nu_\tau \pi^-)$	$(11.02 \pm 0.09)\%$
$\mathrm{Br}(\tau^- \to \nu_\tau K^-)$	$(0.690 \pm 0.025)\%$

Table 1: World average values for some basic τ parameters.

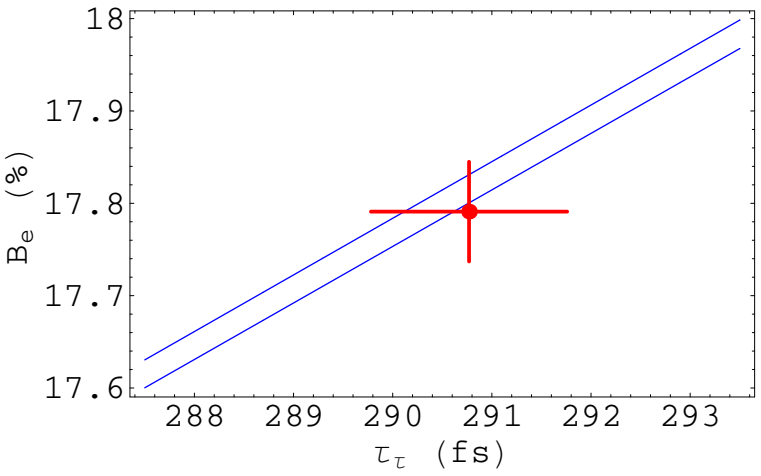


Figure 1: Relation between $B_{\tau\to e}$ and τ_τ. The band corresponds to Eq. (2).

The relevant experimental measurements are given in Table 1. The predicted $B_{\tau\to\mu}/B_{\tau\to e}$ ratio is in perfect agreement with the measured value $B_{\tau\to\mu}/B_{\tau\to e} = 0.974 \pm 0.004$. As shown in Fig. 1, the relation between $B_{\tau\to e}$ and τ_τ is also well satisfied by the present data. The experimental precision (0.3%) is already approaching the level where a possible non-zero ν_τ mass could become relevant; the present bound [7] $m_{\nu_\tau} < 18.2$ MeV (95% CL) only guarantees that such effect is below 0.08%.

These measurements can be used to test the universality of the W couplings to the leptonic charged currents. The ratio $B_{\tau\to\mu}/B_{\tau\to e}$ constrains $|g_\mu/g_e|$, while $B_{\tau\to e}/\tau_\tau$ and $B_{\tau\to\mu}/\tau_\tau$ provides information on $|g_\tau/g_\mu|$ and $|g_\tau/g_e|$. The present results are shown in Table 2, together with the values obtained from the ratios $\Gamma(\pi^-\to e^-\overline{\nu}_e)/\Gamma(\pi^-\to\mu^-\overline{\nu}_\mu)$ [8] and $\Gamma(\tau^-\to\nu_\tau P^-)/\Gamma(P^-\to\mu^-\overline{\nu}_\mu)$ $[P=\pi,K]$. Also shown are the constraints obtained from the $W^-\to l^-\overline{\nu}_l$ decay modes at the p-$\overline{p}$ colliders [6, 9] and LEP II [10]. The present data verify the universality of the leptonic charged–current couplings to the 0.15% (e/μ) and 0.23% (τ/μ, τ/e) level.

	$\lvert g_\mu/g_e\rvert$	$\lvert g_\tau/g_\mu\rvert$	$\lvert g_\tau/g_e\rvert$
$B_{\tau\to\mu}/B_{\tau\to e}$	1.0009 ± 0.0022	—	—
$B_{\tau\to e}\ \tau_\mu/\tau_\tau$	—	0.9993 ± 0.0023	—
$B_{\tau\to\mu}\ \tau_\mu/\tau_\tau$	—	—	1.0002 ± 0.0023
$B_{\pi\to e}/B_{\pi\to\mu}$	1.0017 ± 0.0015	—	—
$\Gamma_{\tau\to\pi}/\Gamma_{\pi\to\mu}$	—	1.005 ± 0.005	—
$\Gamma_{\tau\to K}/\Gamma_{K\to\mu}$	—	0.981 ± 0.018	—
$B_{W\to l}/B_{W\to l'}$ $(p\overline{p})$	0.98 ± 0.03	—	0.987 ± 0.025
$B_{W\to l}/B_{W\to l'}$ (LEP2)	1.002 ± 0.016	1.008 ± 0.019	1.010 ± 0.019

Table 2: Present constraints on charged–current lepton universality.

3 Neutral–current universality

In the SM, all leptons with equal electric charge have identical couplings to the Z boson. This has been tested at LEP and SLC [10], by measuring the total $e^+e^-\to Z\to l^+l^-$ cross–section, the forward–backward asymmetry, the (final) polarization asymmetry, the forward–backward (final) polarization asymmetry, and (at SLC) the left–right asymmetry between the cross–sections for initial left– and right–handed electrons and the left–right forward–backward asymmetry. $\Gamma_l\equiv\Gamma(Z\to l^+l^-)$ determines the sum $(v_l^2+a_l^2)$, where v_l and a_l are the effective vector and axial–vector Z couplings, while the ratio v_l/a_l is derived from the asymmetries which measure the average longitudinal polarization of the lepton l^-:

$$\mathcal{P}_l\ \equiv\ \frac{-2v_la_l}{v_l^2+a_l^2}\,. \tag{3}$$

The measurement of the final polarization asymmetries can (only) be done for $l=\tau$, because the spin polarization of the τ's is reflected in the distorted distribution of their decay products. Thus, $\mathcal{P}_\tau$ and $\mathcal{P}_e$ can be determined from a measurement of

	e	μ	τ	l
Γ_l (MeV)	83.90 ± 0.12	83.96 ± 0.18	84.05 ± 0.22	83.96 ± 0.09
$\mathcal{A}_{\rm FB}^{0,l}$ (%)	1.45 ± 0.24	1.67 ± 0.13	1.88 ± 0.17	1.701 ± 0.095

Table 3: Measured values [10, 12] of Γ_l and $\mathcal{A}_{\rm FB}^{0,l}$. The last column shows the combined result (for a massless lepton) assuming lepton universality.

$-\mathcal{A}_{\rm Pol}^{0,\tau} = -\mathcal{P}_\tau$	0.1425 ± 0.0044	$\frac{4}{3}\mathcal{A}_{\rm FB,LR}^{0,e} \Rightarrow -\mathcal{P}_e$	0.1558 ± 0.0064
$-\frac{4}{3}\mathcal{A}_{\rm FB,Pol}^{0,\tau} = -\mathcal{P}_e$	0.1483 ± 0.0051	$\frac{4}{3}\mathcal{A}_{\rm FB,LR}^{0,\mu} = -\mathcal{P}_\mu$	0.137 ± 0.016
$\mathcal{A}_{\rm LR}^{0} = -\mathcal{P}_e$	0.1511 ± 0.0022	$\frac{4}{3}\mathcal{A}_{\rm FB,LR}^{0,\tau} = -\mathcal{P}_\tau$	0.142 ± 0.016
$\{\frac{4}{3}\mathcal{A}_{\rm FB}^{0,l}\}^{1/2} = -P_l$	0.1506 ± 0.0042		

Table 4: $\mathcal{P}_l$ determinations from different asymmetry measurements [10, 12, 13].

the spectrum of the final charged particles in the decay of one τ, or by studying the correlated distributions between the final decay products of both $\tau's$ [11].

Tables 3 and 4 show the present experimental results. The data are in excellent agreement with the SM predictions and confirm the universality of the leptonic neutral couplings. The average of the two τ polarization measurements, $\mathcal{A}_{\rm Pol}^{0,\tau}$ and $\frac{4}{3}\mathcal{A}_{\rm FB,Pol}^{0,\tau}$, results in $\mathcal{P}_l = -0.1450 \pm 0.0033$ which deviates by $1.5\,\sigma$ from the $\mathcal{A}_{LR}^0$ measurement. Assuming lepton universality, the combined result from all leptonic asymmetries gives

$$\mathcal{P}_l = -0.1497 \pm 0.0016\,. \tag{4}$$

Figure 2 shows the 68% probability contours in the a_l–v_l plane, obtained from a combined analysis [10] of all leptonic observables. Lepton universality is now tested to the 0.15% level for the axial–vector neutral couplings, while only a few per cent precision has been achieved for the vector couplings [10, 12]:

$$\begin{aligned} a_\mu/a_e &= 1.0001 \pm 0.0014\,, & v_\mu/v_e &= 0.981 \pm 0.082\,, \\ a_\tau/a_e &= 1.0019 \pm 0.0015\,, & v_\tau/v_e &= 0.964 \pm 0.032\,. \end{aligned} \tag{5}$$

Assuming lepton universality, the measured leptonic asymmetries can be used to obtain the effective electroweak mixing angle in the charged–lepton sector [10, 12]:

$$\sin^2\theta_{\rm eff}^{\rm lept} \equiv \frac{1}{4}\left(1 - \frac{v_l}{a_l}\right) = 0.23119 \pm 0.00021 \qquad (\chi^2/\text{d.o.f.} = 3.4/4). \tag{6}$$

4 Lorentz structure

Let us consider the leptonic decay $l^- \to \nu_l l'^- \overline{\nu}_{l'}$. The most general, local, derivative–free, lepton–number conserving, four–lepton interaction Hamiltonian, consistent with

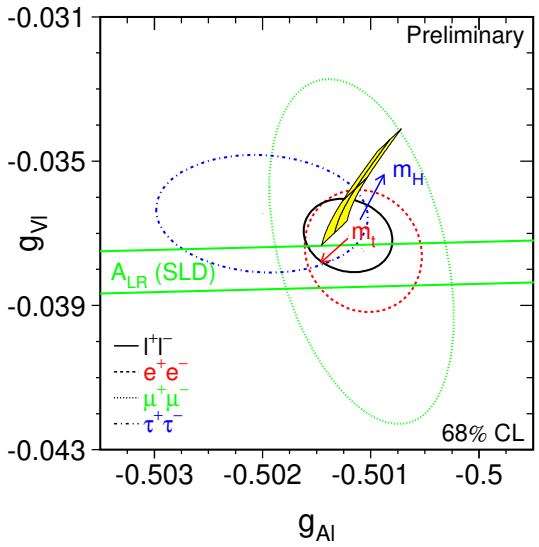


Figure 2: 68% probability contours in the a_l-v_l plane from LEP measurements [10]. The solid contour assumes lepton universality. Also shown is the 1σ band resulting from the $\mathcal{A}^0_{\rm LR}$ measurement at SLD. The shaded region corresponds to the SM prediction for $m_t = 174.3 \pm 5.1$ GeV and $m_H = 300^{+700}_{-210}$ GeV. The arrows point in the direction of increasing m_t and m_H values.

locality and Lorentz invariance [14, 15],

$$\mathcal{H} \;=\; 4\frac{G_{l'l}}{\sqrt{2}} \sum_{n,\epsilon,\omega} g^n_{\epsilon\omega} \left[\overline{l'_\epsilon}\Gamma^n(\nu_{l'})_\sigma\right] \left[\overline{(\nu_l)_\lambda}\Gamma_n l_\omega\right] \;, \tag{7}$$

contains ten complex coupling constants or, because a common phase is arbitrary, nineteen independent real parameters which could be different for each leptonic decay. The subindices $\epsilon, \omega, \sigma, \lambda$ label the chiralities (left–handed, right–handed) of the corresponding fermions, and n the type of interaction: scalar (I), vector (γ^μ), tensor ($\sigma^{\mu\nu}/\sqrt{2}$). For given n, ϵ, ω, the neutrino chiralities σ and λ are uniquely determined.

The total decay width is proportional to the following combination of couplings, which is usually normalized to one [15]:

$$\begin{aligned} 1 \;&=\; \frac{1}{4}\left(|g^S_{RR}|^2 + |g^S_{RL}|^2 + |g^S_{LR}|^2 + |g^S_{LL}|^2\right) + 3\left(|g^T_{RL}|^2 + |g^T_{LR}|^2\right) \qquad (8)\\ &\quad + \left(|g^V_{RR}|^2 + |g^V_{RL}|^2 + |g^V_{LR}|^2 + |g^V_{LL}|^2\right) \\ &\equiv\; Q_{LL} + Q_{LR} + Q_{RL} + Q_{RR}\,. \qquad (9) \end{aligned}$$

The universality tests mentioned before refer then to the global normalization $G_{l'l}$, while the $g^n_{\epsilon\omega}$ couplings parameterize the relative strength of different types of interaction. The sums $Q_{\epsilon\omega}$ of all factors with the same subindices give the probability of having a decay from an initial charged lepton with chirality ω to a final one with chirality ϵ. In the SM, $g^V_{LL} = 1$ and all other $g^n_{\epsilon\omega} = 0$.

The energy spectrum and angular distribution of the final charged lepton provides information on the couplings $g^n_{\epsilon\omega}$. For unpolarized leptons, the distribution is characterized by the so-called Michel [14] parameter ρ and the low–energy parameter η. Two more parameters, ξ and δ, can be determined when the initial lepton polarization is known. In the SM, $\rho = \delta = 3/4$, $\eta = 0$ and $\xi = 1$.

For μ decay, where precise measurements of the μ and e polarizations have been performed, there exist [15] upper bounds on Q_{RR}, Q_{LR}, and Q_{RL}, and a lower bound on Q_{LL}. They imply corresponding upper limits on the 8 couplings $|g^n_{RR}|$, $|g^n_{LR}|$, and $|g^n_{RL}|$. The measurements of the μ^- and the e^- do not allow to determine $|g^S_{LL}|$ and $|g^V_{LL}|$ separately; nevertheless, since the helicity of the ν_μ in pion decay is experimentally known to be -1, a lower limit on $|g^V_{LL}|$ is obtained from the inverse muon decay $\nu_\mu e^- \to \mu^- \nu_e$. These limits show nicely that the bulk of the μ decay transition amplitude is indeed of the predicted V$-$A type: $|g^V_{LL}| > 0.960$ (90% CL) [6].

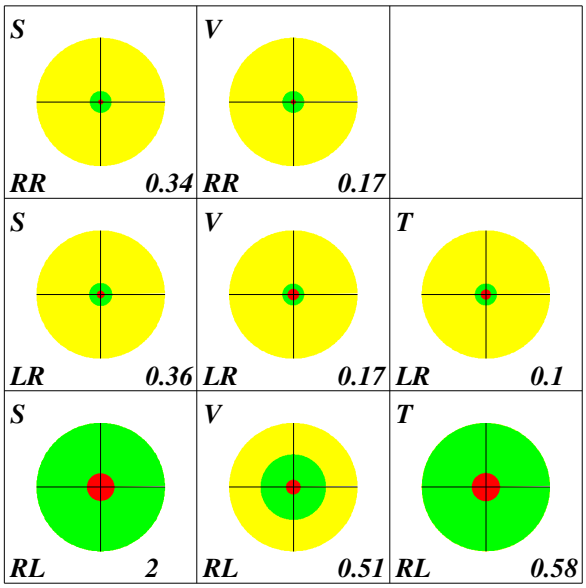


Figure 3: 90% CL experimental limits [16] for the normalized τ decay couplings $g'^n_{\epsilon\omega} \equiv g^n_{\epsilon\omega}/N^n$, where $N^n \equiv \max(|g^n_{\epsilon\omega}|) = 2, 1, 1/\sqrt{3}$ for $n = S, V, T$, assuming e/μ universality. The circles of unit area indicate the range allowed by the normalization constraint. The present experimental bounds are shown as shaded circles. For comparison, the (stronger) μ decay limits are also shown (darker circles).

	$\mu \to e$	$\tau \to \mu$	$\tau \to e$	$\tau \to l$
ρ	0.7518 ± 0.0026	0.771 ± 0.025	0.746 ± 0.011	0.752 ± 0.009
η	-0.007 ± 0.013	0.173 ± 0.122	—	0.035 ± 0.031
ξ	1.0027 ± 0.0085	1.053 ± 0.053	0.995 ± 0.042	0.978 ± 0.031
$\xi\delta$	0.7506 ± 0.0074	0.786 ± 0.039	0.733 ± 0.029	0.745 ± 0.021

Table 5: World average [16] Michel parameters. The last column ($\tau \to l$) assumes identical couplings for $l = e, \mu$.

The experimental analysis of the τ decay parameters is necessarily different from the one applied to the muon, because of the much shorter τ lifetime. The measurement of the τ polarization and the parameters ξ and δ is still possible due to the fact that the spins of the $\tau^+\tau^-$ pair produced in e^+e^- annihilation are strongly correlated [11]. Another possibility is to use the beam polarization, as done by SLD. However, the

polarization of the charged lepton emitted in the τ decay has never been measured. The measurement of the inverse decay $\nu_\tau l^- \to \tau^- \nu_l$ looks far out of reach.

The four LEP experiments, ARGUS, CLEO, and SLD have performed accurate measurements of the τ decay Michel parameters. The present experimental status [16] is shown in Table 5. The determination of the τ polarization parameters allows us to bound the total probability for the decay of a right–handed τ,

$$Q_{\tau_R} \equiv Q_{RR} + Q_{LR} = \frac{1}{2}\left[1 + \frac{\xi}{3} - \frac{16}{9}(\xi\delta)\right] . \tag{10}$$

At 90% CL, one finds (ignoring possible correlations among the measurements):

$$Q_{\tau_R}^{\tau\to\mu} < 0.047 , \qquad Q_{\tau_R}^{\tau\to e} < 0.054 , \qquad Q_{\tau_R}^{\tau\to l} < 0.032 , \tag{11}$$

where the last value refers to the τ decay into either $l = e$ or μ, assuming identical e/μ couplings. These probabilities imply corresponding limits on all $|g_{RR}^n|$ and $|g_{LR}^n|$ couplings. Including also the information from ρ and η, one gets the (90% CL) bounds on the τ decay couplings shown in Fig. 3, where e/μ universality has been assumed.

The probability $Q_{\tau\to l_R} \equiv Q_{RL} + Q_{RR}$ for the τ decay into a right–handed lepton could be investigated through the photon distribution [17] in the decays $\tau^- \to \nu_\tau l^- \overline{\nu}_l \gamma$. Owing to the large backgrounds, no useful limits can be extracted from the recent CLEO measurement of these radiative decays [18].

5 Lepton–number violation

In the minimal SM with massless neutrinos, there is a separately conserved additive lepton number for each generation. All present data are consistent with this conservation law. However, there are no strong theoretical reasons forbidding a mixing among the different leptons, in the same way as happens in the quark sector. Many models in fact predict lepton–flavor or even lepton–number violation at some level. Experimental searches for these processes can provide information on the scale at which the new physics begins to play a significant role.

Table 6 shows the most recent limits [19] on lepton–flavor and lepton–number violation decays of the τ. Although still far away from the impressive bounds [6] obtained in μ decay [$Br(\mu^- \to e^-\gamma) < 4.9 \times 10^{-11}, Br(\mu^- \to e^-e^+e^-) < 1.0 \times 10^{-12}, Br(\mu^- \to e^-\gamma\gamma) < 7.2 \times 10^{-11}$ (90% CL)], the τ decay limits start to put interesting constraints on possible new physics contributions.

6 The tau neutrino

All observed τ decays are supposed to be accompanied by neutrino emission, in order to fulfill energy—momentum conservation requirements. From a two—dimensional

X^-	U. L.	X^-	U. L.	X^-	U. L.	X^-	U. L.
$e^-\gamma$	2.7	$\mu^-\gamma$	1.1	$e^-\pi^+\pi^-$	2.2	$\mu^-\pi^+\pi^-$	8.2
$e^-e^+e^-$	2.9	$\mu^-\mu^+\mu^-$	1.9	$e^-\pi^+K^-$	6.4	$\mu^-\pi^+K^-$	7.5
$e^-e^+\mu^-$	1.7	$\mu^-\mu^+e^-$	1.8	$e^-K^+\pi^-$	3.8	$\mu^-K^+\pi^-$	7.4
$e^-\mu^+e^-$	1.5	$\mu^-e^+\mu^-$	1.5	$e^-K^+K^-$	6.0	$\mu^-K^+K^-$	15
$e^-\pi^0$	3.7	$\mu^-\pi^0$	4.0	$e^+\pi^-\pi^-$	1.9	$\mu^+\pi^-\pi^-$	3.4
$e^-\eta$	8.2	$\mu^-\eta$	9.6	$e^+\pi^-K^-$	2.1	$\mu^+\pi^-K^-$	7.0
$e^-\rho^0$	2.0	$\mu^-\rho^0$	6.3	$e^+K^-K^-$	3.8	$\mu^+K^-K^-$	6.0
e^-K^{*0}	5.1	μ^-K^{*0}	7.5	$e^-\pi^0\pi^0$	6.5	$\mu^-\pi^0\pi^0$	14
$e^-\overline{K}^{*0}$	7.4	$\mu^-\overline{K}^{*0}$	7.5	$e^-\pi^0\eta$	24	$\mu^-\pi^0\eta$	22
$e^-\phi$	6.9	$\mu^-\phi$	7.0	$e^-\eta\eta$	35	$\mu^-\eta\eta$	60

Table 6: 90% CL upper limits (in units of 10^{-6}) on $B(\tau^-\to X^-)$ [19]

X	ALEPH [7]	CLEO [20]	DELPHI [21]	OPAL [22]
3π	25.7	—	28	35.3
$3\pi\pi^0$	—	28	—	
5π	23.1	30	—	43.2
Combined	18.2	28	28	27.6

Table 7: 95% CL upper limits on m_{ν_τ} (in MeV), from $\tau^-\to\nu_\tau X^-$ events.

likelihood fit of the visible energy and the invariant–mass distribution of the final hadrons in $\tau^-\to\nu_\tau X^-$ events, it is possible to set a bound on the ν_τ mass. The best limits, shown in Table 7, are obtained from modes with a significant probability of populating the hadronic—mass end—point region.

The present data are consistent with the ν_τ being a conventional sequential neutrino. Since tau's are not produced by ν_e or ν_μ beams, we know that ν_τ is different from the electronic and muonic neutrinos. LEP and SLC have confirmed [10] the existence of three (and only three) different light neutrinos, with standard couplings to the Z. However, no direct observation of ν_τ, that is, interactions resulting from τ neutrinos, has been made so far. The DONUT experiment at Fermilab is expected to provide soon the first evidence of τ neutrinos (produced through $p+N\to D_s+\cdots$, followed by the decays $D_s\to\tau^-\overline{\nu}_\tau$ and $\tau^-\to\nu_\tau+\cdots$), through the detection of $\nu_\tau+N\to\tau+X$. This is an important goal in view of the recent SuperKamiokande results suggesting $\nu_\mu\to\nu_\tau$ oscillations with $m^2_{\nu_\tau}-m^2_{\nu_\mu}\sim(0.05\,\mathrm{eV})^2$. This hypothesis could be corroborated making a long–baseline neutrino experiment with a ν_μ beam pointing into a far (~ 700 Km) massive detector, able to detect the appearance of a τ. The possibility to perform such an experiment is presently being investigated.

7 Hadronic decays

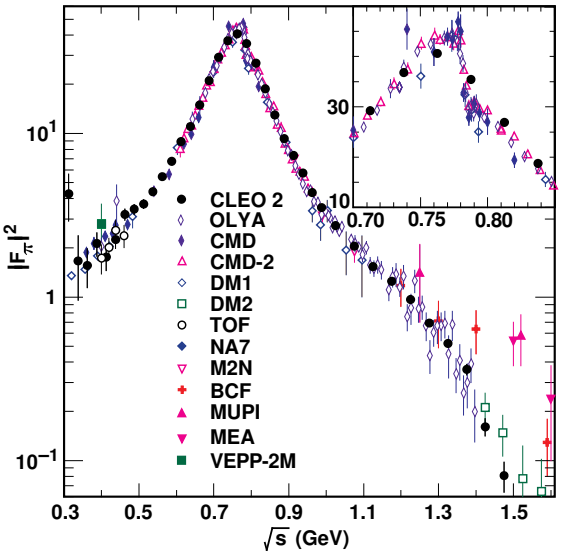


Figure 4: Pion form factor from $\tau^- \to \nu_\tau \pi^- \pi^0$ data [23] (filled circles), compared with $e^+e^- \to \pi^+\pi^-$ measurements.

The semileptonic decay modes $\tau^- \to \nu_\tau H^-$ probe the matrix element of the left–handed charged current between the vacuum and the final hadronic state H^-. For the two–pion final state, the hadronic matrix element is parameterized in terms of the so-called pion form factor:

$$\langle \pi^- \pi^0 | \overline{d} \gamma^\mu u | 0 \rangle \equiv \sqrt{2}\, F_\pi(s)\, (p_{\pi^-} - p_{\pi^0})^\mu \; . \tag{12}$$

Figure 4 shows the recent CLEO measurement [23] of $|F_\pi(s)|^2$ from $\tau \to \nu_\tau \pi^- \pi^0$ data (a similar analysis was done previously by ALEPH [24]). Also shown is the corresponding determination from $e^+e^- \to \pi^+\pi^-$ data. The precision achieved with τ decays is clearly better. There is a quite good agreement between both sets of data, although the τ points tend to be slightly higher.

The dynamical structure of other hadronic final states has been also investigated. CLEO has measured recently [25] the four $J^P = 1^+$ structure functions characterizing the decay $\tau^- \to \nu_\tau \pi^- 2\pi^0$, improving a previous OPAL analysis [26]. The interference between the two $\pi^-\pi^0$ systems generates a parity–violating angular asymmetry [27], which allows to determine the sign of the ν_τ helicity to be -1 (the modulus has been precisely measured through the study of correlated $\tau^+\tau^-$ decays into different final states: $|h_{\nu_\tau}| = 1.0000 \pm 0.0057$ [28]).

8 QCD tests

The inclusive character of the total τ hadronic width renders possible an accurate calculation of the ratio [29, 30]

$$R_\tau \equiv \frac{\Gamma[\tau^- \to \nu_\tau \,\text{hadrons}\,(\gamma)]}{\Gamma[\tau^- \to \nu_\tau e^- \overline{\nu}_e(\gamma)]} \;=\; R_{\tau,V} + R_{\tau,A} + R_{\tau,S}\,, \tag{13}$$

using analyticity constraints and the operator product expansion. One can separately compute the contributions associated with specific quark currents. Non-strange hadronic decays of the τ are resolved experimentally into vector ($R_{\tau,V}$) and axial-vector ($R_{\tau,A}$) contributions according to whether the hadronic final state includes an even or odd number of pions. Strange decays ($R_{\tau,S}$) are of course identified by the presence of an odd number of kaons in the final state.

The theoretical prediction for $R_{\tau,V+A}$ can be expressed as

$$R_{\tau,V+A} = N_C\,|V_{ud}|^2\,S_{EW}\,(1+\delta'_{EW}+\delta_P+\delta_{NP})\,, \tag{14}$$

with $N_C = 3$ the number of quark colors. The factors $S_{EW} = 1.0194$ and $\delta'_{EW} = 0.0010$ contain the known electroweak corrections at the leading [3] and next-to-leading [31] logarithm approximation. The dominant correction ($\sim 20\%$) is the purely perturbative QCD contribution [29, 30]

$$\delta_P = a_\tau + 5.2023\,a_\tau^2 + 26.366\,a_\tau^3 + \,\mathcal{O}(\alpha_s^4)\,. \tag{15}$$

This expansion in powers of $a_\tau \equiv \alpha_s(m_\tau^2)/\pi$ has rather large coefficients, which originate in the long running of the strong coupling along a contour integration in the complex plane; this running effect can be properly resummed to all orders in α_s by fully keeping [30] the known four–loop–level calculation of the contour integral.

The non-perturbative contributions can be shown to be suppressed by six powers of the τ mass [29], and are therefore very small. Their actual numerical size has been determined from the invariant–mass distribution of the final hadrons in τ decay, through the study of weighted integrals [32],

$$R^{kl}_{\tau,V+A} \equiv \int_0^{m_\tau^2} ds\,\left(1-\frac{s}{m_\tau^2}\right)^k\,\left(\frac{s}{m_\tau^2}\right)^l\,\frac{dR_{\tau,V+A}}{ds}\,, \tag{16}$$

which can be calculated theoretically in the same way as $R_{\tau,V+A}$. The predicted suppression [29] of the non-perturbative corrections has been confirmed by ALEPH [33], CLEO [34] and OPAL [35]. The most recent analyses [33, 35] give

$$\delta_{\rm NP} = -0.003 \pm 0.003\,. \tag{17}$$

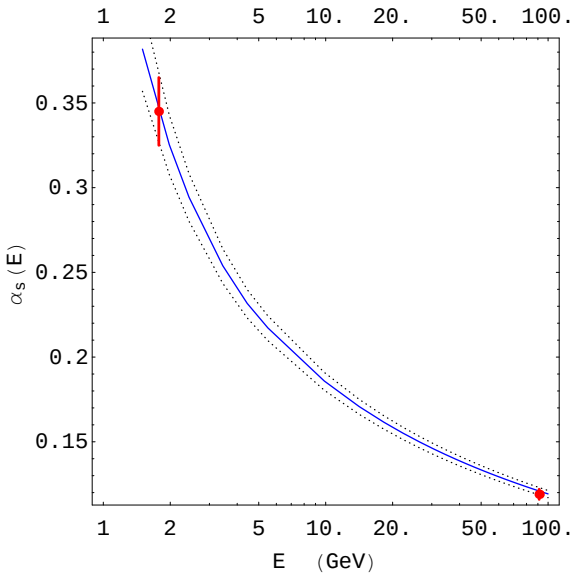


Figure 5: Measured values of α_s in τ and Z decays. The curves show the energy dependence predicted by QCD, using $\alpha_s(m_\tau)$ as input.

The QCD prediction for $R_{\tau,V+A}$ is then completely dominated by the perturbative contribution δ_P; non-perturbative effects being smaller than the perturbative uncertainties from uncalculated higher–order corrections. The result turns out to be very sensitive to the value of $\alpha_s(m_\tau^2)$, allowing for an accurate determination of the fundamental QCD coupling. The experimental measurement [33, 35] $R_{\tau,V+A} = 3.484 \pm 0.024$ implies $\delta_P = 0.200 \pm 0.013$, which corresponds (in the $\overline{\rm MS}$ scheme) to

$$\alpha_s(m_\tau^2) = 0.345 \pm 0.020\,. \tag{18}$$

The strong coupling measured at the τ mass scale is significatively different from the values obtained at higher energies. From the hadronic decays of the Z boson, one gets $\alpha_s(M_Z) = 0.119 \pm 0.003$, which differs from the τ decay measurement by eleven standard deviations! After evolution up to the scale M_Z [36], the strong coupling constant in (18) decreases to

$$\alpha_s(M_Z^2) = 0.1208 \pm 0.0025\,, \tag{19}$$

in excellent agreement with the direct measurements at the Z peak and with a similar accuracy. The comparison of these two determinations of α_s in two extreme energy regimes, m_τ and M_Z, provides a beautiful test of the predicted running of the QCD coupling; i.e. a very significant experimental verification of *asymptotic freedom*.

From a careful analysis of the hadronic invariant–mass distribution, ALEPH [24, 33] and OPAL [35] have measured the spectral functions associated with the vector and axial–vector quark currents. Their difference is a pure non-perturbative quantity, which carries important information on the QCD dynamics; it allows to determine low–energy parameters, such as the pion decay constant, the electromagnetic pion

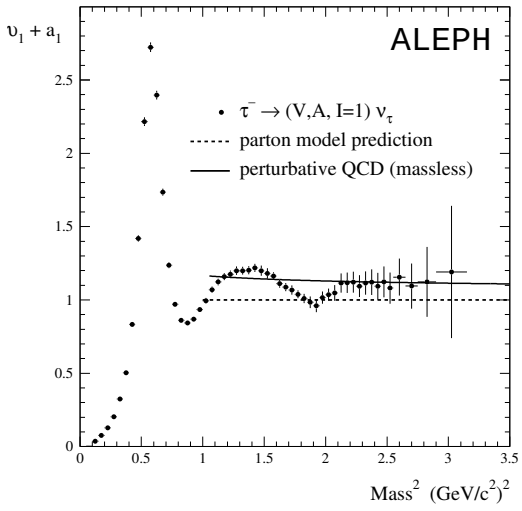


Figure 6: $V+A$ spectral function [33].

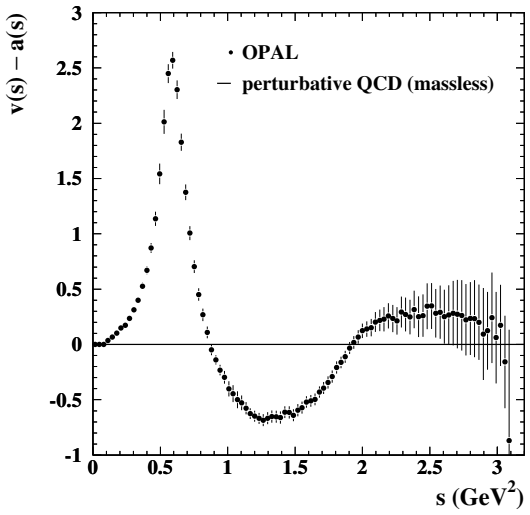


Figure 7: $V-A$ spectral function [35].

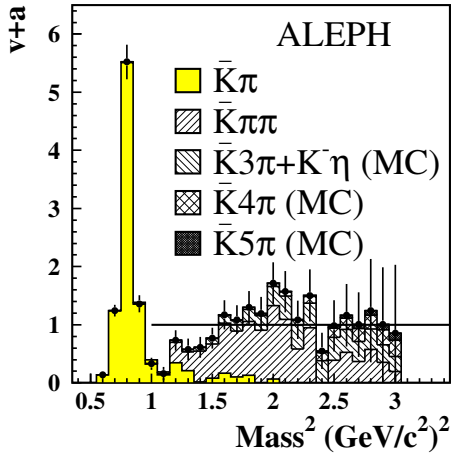
mass difference $m_{\pi^\pm} - m_{\pi^0}$, or the axial pion form factor, in good agreement with their direct measurements.

The vector spectral function has been also used to measure the hadronic vacuum polarization effects associated with the photon and, therefore, estimate how the electromagnetic fine structure constant gets modified at LEP energies. The uncertainty of this parameter is one of the main limitations in the extraction of the Higgs mass from global electroweak fits to the LEP/SLD data. From the ALEPH τ data [24], the Orsay group obtains [37] $\alpha^{-1}(M_Z) = 128.933 \pm 0.021$, which reduces the error of the fitted $\log(M_H)$ value by 30%. The same τ data allows to pin down the hadronic contribution to the anomalous magnetic moment of the muon a_μ^γ. The recent analyses [23, 37] have improved the theoretical prediction of a_μ^γ, setting a reference value to be compared with the forthcoming measurement of the BNL-E821 experiment, presently running at Brookhaven.

9 The strange quark mass

The LEP experiments and CLEO have performed an extensive investigation of kaon production in τ decays. ALEPH has determined the inclusive invariant mass distribution of the final hadrons in the Cabibbo–suppressed decays [38]. The separate measurement of the $|\Delta S| = 0$ and $|\Delta S| = 1$ decay widths allows us to pin down the SU(3) breaking effect induced by the strange quark mass, through the differences

$$\delta R_\tau^{kl} \equiv \frac{R_{\tau,V+A}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau,S}^{kl}}{|V_{us}|^2} \approx 24\,\frac{m_s^2(m_\tau)}{m_\tau^2}\,\Delta_{kl}(\alpha_s) - 48\pi^2\,\frac{\delta O_4}{m_\tau^4}\,Q_{kl}(\alpha_s)\,, \qquad (20)$$



(k,l)	δR_τ^{kl}	$m_s(m_\tau)$ (MeV)
$(0,0)$	0.394 ± 0.137	$143 \pm 31_{\rm exp} \pm 18_{\rm th}$
$(1,0)$	0.383 ± 0.078	$121 \pm 17_{\rm exp} \pm 18_{\rm th}$
$(2,0)$	0.373 ± 0.054	$106 \pm 12_{\rm exp} \pm 21_{\rm th}$

Table 8: Measured moments δR_τ^{kl} [38] and corresponding $m_s(m_\tau)$ values [39].

Figure 8: $|\Delta S| = 1$ spectral function [38].

where $\Delta_{kl}(\alpha_s)$ and $Q_{kl}(\alpha_s)$ are perturbative QCD corrections, which are known to $O(\alpha_s^3)$ and $O(\alpha_s^2)$, respectively [39]. The small non-perturbative contribution, $\delta O_4 \equiv \langle 0|m_s\bar{s}s - m_d\bar{d}d|0\rangle = -(1.5 \pm 0.4) \times 10^{-3}$ GeV^4, has been estimated with Chiral Perturbation Theory techniques [39]. Table 8 shows the measured [38] differences δR_τ^{kl} and the corresponding ($\overline{\rm MS}$) values [39] of $m_s(m_\tau)$. The theoretical errors are dominated by the large perturbative uncertainties of $\Delta_{kl}(\alpha_s)$.

Taking into account the information from the three moments, one finally gets [39]

$$m_s(m_\tau) = (119 \pm 12_{\rm exp} \pm 18_{\rm th} \pm 10_{V_{us}})\ {\rm MeV}\,, \tag{21}$$

where the additional error reflects the present uncertainty from $|V_{us}|$. This corresponds to $m_s(1\ {\rm GeV}^2) = (164 \pm 33)$ MeV.

10 Summary

The flavor structure of the SM is one of the main pending questions in our understanding of weak interactions. Although we do not know the reason of the observed family replication, we have learned experimentally that the number of SM fermion generations is just three (and no more). Therefore, we must study as precisely as possible the few existing flavors to get some hints on the dynamics responsible for their observed structure.

The τ turns out to be an ideal laboratory to test the SM. It is a lepton, which means clean physics, and moreover, it is heavy enough to produce a large variety of

decay modes. Naïvely, one would expect the τ to be much more sensitive than the e or the μ to new physics related to the flavor and mass–generation problems.

QCD studies can also benefit a lot from the existence of this heavy lepton, able to decay into hadrons. Owing to their semileptonic character, the hadronic τ decays provide a powerful tool to investigate the low–energy effects of the strong interactions in rather simple conditions.

Our knowledge of the τ properties has been considerably improved during the last few years. Lepton universality has been tested to rather good accuracy, both in the charged and neutral current sectors. The Lorentz structure of the leptonic τ decays is certainly not determined, but begins to be experimentally explored. An upper limit of 3.2% (90% CL) has been already set on the probability of having a (wrong) decay from a right–handed τ. The quality of the hadronic data has made possible to perform quantitative QCD tests and determine the strong coupling constant very accurately. Moreover, the Cabibbo–suppressed decay width of the τ provides a rather good measurement of the strange quark mass. Searches for non-standard phenomena have been pushed to the limits that the existing data samples allow to investigate.

At present, all experimental results on the τ lepton are consistent with the SM. There is, however, large room for improvements. Future τ experiments will probe the SM to a much deeper level of sensitivity and will explore the frontier of its possible extensions.

This work has been supported in part by the ECC, TMR Network EURODAPHNE (ERBFMX-CT98-0169), and by DGESIC (Spain) under grant No. PB97-1261.

References

[1] A. Pich and A. Ruiz (eds.), Proc. *Fifth Workshop on Tau Lepton Physics* (Santander, 14–17 September 1998), Nucl. Phys. B (Proc. Suppl.) **76** (1999).

[2] A. Pich, *Tau Physics*, in *Heavy Flavors II*, eds. A.J. Buras and M. Lindner, Advanced Series on Directions in High Energy Physics – Vol. 15 (World Scientific, Singapore, 1998), p. 453.

[3] W.J. Marciano and A. Sirlin, Phys. Rev. Lett. **61**, 1815 (1988).

[4] T. van Ritbergen and R.G. Stuart, Phys. Rev. Lett. **82**, 488 (1999); hep-ph/9904240; P. Malde and R.G. Stuart, Nucl. Phys. **B552**, 41 (1999); T. Seidensticker and M. Steinhauser, Phys. Lett. **B467**, 271 (1999).

[5] BES Collaboration, Phys. Rev. **D53**, 20 (1996); Phys. Rev. Lett. **69**, 3021 (1992).

[6] C. Caso *et al.* (Particle Data Group), *Review of Particle Properties*, Eur. Phys. J. **C3**, 1 (1998) and 1999 partial update for the edition 2000 [http://pdg.lbl.gov].

[7] ALEPH Collaboration, Eur. Phys. J. **C2**, 395 (1998).

[8] D.I. Britton *et al.*, Phys. Rev. Lett. **68**, 3000 (1992);
G. Czapek *et al.*, Phys. Rev. Lett. **70**, 17 (1993).

[9] J. Ellison, *W and Z Properties at the Tevatron*, Proc. EPS-HEP99 Conference (Tampere, July 1999) [hep-ex/9910037].

[10] The LEP Collaborations ALEPH, DELPHI, L3, OPAL, the LEP Electroweak Working Group and the SLD Heavy Flavor and Electroweak Groups, *A Combination of Preliminary Electroweak Measurements and Constraints on the Standard Model*, CERN-EP/99-15 (February 1999) and summer update [http://www.cern.ch/LEPEWWG/].

[11] R. Alemany *et al.*, Nucl. Phys. **B379**, 3 (1992);
M. Davier *et al.* Phys. Lett. **B306**, 411 (1993).

[12] M. Swartz, *Precision Electroweak Physics at the Z*, these proceedings.

[13] SLD Collaboration, hep-ex/9908006.

[14] L. Michel, Proc. Phys. Soc. **A63**, 514, 1371 (1950); C. Bouchiat and L. Michel, Phys. Rev. **106**, 170 (1957); T. Kinoshita and A. Sirlin, Phys. Rev. **107**, 593 (1957); **108**, 844 (1957).

[15] W. Fetscher, H.-J. Gerber and K.F. Johnson, Phys. Lett. **B173**, 102 (1986).

[16] I. Boyko, *Tests of lepton universality in tau decays*, Proc. EPS-HEP99 Conference (Tampere, July 1999).

[17] A. Stahl and H. Voss, Z. Phys. **C74**, 73 (1997).

[18] CLEO Collaboration, hep-ex/9909050.

[19] CLEO Collaboration, Phys. Rev. **D57**, 5903 (1998); **D55**, R3919 (1997); Phys. Rev. Lett. **79**, 1221 (1997); hep-ex/9910060.

[20] CLEO Collaboration, Phys. Lett. **B431**, 209 (1998); hep-ex/9906015.

[21] DELPHI Collaboration (preliminary), in [1].

[22] OPAL Collaboration, Eur. Phys. J. **C5**, 229 (1998); Z. Phys. **C72**, 231 (1996).

[23] CLEO Collaboration, hep-ex/9910046.

[24] ALEPH Collaboration, Z. Phys. **C76**, 15 (1997).

[25] CLEO Collaboration, hep-ex/9908030; Phys. Rev. **D61**, 012002 (2000).

[26] OPAL Collaboration, Z. Phys. **C75**, 593 (1997).

[27] J. Kühn and F. Wagner, Nucl. Phys. **B236**, 16 (1984).

[28] A. Stahl, in [1] p. 173.

[29] E. Braaten, S. Narison and A. Pich, Nucl. Phys. **B373**, 581 (1992);
E. Braaten, Phys. Rev. Lett. **60**, 1606 (1988); Phys. Rev. **D39**, 1458 (1989);
S. Narison and A. Pich, Phys. Lett. **B211**, 183 (1988).

[30] F. Le Diberder and A. Pich, Phys. Lett. **B286**, 147 (1992).

[31] E. Braaten and C.S. Li, Phys. Rev. **D42**, 3888 (1990).

[32] F. Le Diberder and A. Pich, Phys. Lett. **B289**, 165 (1992).

[33] ALEPH Collaboration, Eur. Phys. J. **C4**, 409 (1998); Phys. Lett. **B307**, 209 (1993).

[34] CLEO Collaboration, Phys. Lett. **B356**, 580 (1995).

[35] OPAL Collaboration, Eur. Phys. J. **C7**, 571 (1999).

[36] G. Rodrigo, A. Pich and A. Santamaria, Phys. Lett. **B424**, 367 (1998).

[37] M. Davier and A. Höcker, Phys. Lett. **B435**, 427 (1998); **B419**, 419 (1998);
R. Alemany *et al.*, Eur. Phys. J. **C2**, 123 (1998); M. Davier, in [1] p. 327.

[38] ALEPH Collaboration, hep-ex/9903015; Eur. Phys. J. **C10**, 1 (1999).

[39] A. Pich and J. Prades, JHEP **10**, 004 (1999); **06**, 013 (1998).

Discussion

Michel Davier (LAL, Orsay): The value you quoted for $m_s(m_\tau)$ from the ALEPH analysis is not the ALEPH result. Our analysis is more conservative and quotes considerably larger errors.

Pich: ALEPH [38] quotes two different results: $(149^{+24_{\rm exp}+21_{\rm th}}_{-30_{\rm exp}-25_{\rm th}} \pm 6_{\rm fit})$ MeV and $(176^{+37_{\rm exp}+24_{\rm th}}_{-48_{\rm exp}-28_{\rm th}} \pm 8_{\rm fit} \pm 11_{J=0})$ MeV. The first number is obtained truncating the longitudinal perturbative series $\Delta^L_{kl}(\alpha_s)$ at $O(\alpha_s)$. The known $O(\alpha_s^2)$ and $O(\alpha_s^3)$ contributions are positive; thus, you took a smaller value of $\Delta_{kl}(\alpha_s)$ and, therefore, got a larger result for m_s. Your error is larger to account for the missing $O(\alpha_s^2)$ and $O(\alpha_s^3)$ corrections.

The second number has been obtained subtracting the $J = 0$ contribution; unfortunately, only the pion and kaon contributions are known. Since the longitudinal spectral functions are positive definite, this procedure gives an upper bound on m_s [39]. ALEPH makes a tiny ad-hoc correction to account for the remaining unknown $J = L$ contribution, and quotes the resulting number as a $m_s(m_\tau)$ determination. Since you added a generous uncertainty, your number does not disagree with ours. However, it is actually an upper bound on $m_s(m_\tau)$ and not a determination of this parameter.