Quark Mixing Matrix Studies and Lepton Flavor Violation Searches Using Rare Decays of Kaons

William Molzon
Department of Physics and Astronomy
University of California
Irvine, CA 92697-4575

1 Introduction

Despite the successes of the Standard Model (SM) of particle physics, a fundamental understanding of the family structure of the quarks and leptons remains elusive. Studies of the kaon system have long contributed to the limited understanding we do have and experiments continue to have an impact on our empirical knowledge, despite the maturity of this field. In this paper I review recent results on studies of rare decays of kaons that contribute in this area, and discuss future prospects in the field.

Studies of rare decays are set in the context of other recent results relevant to understanding the family structure of matter. In the quark sector, two kaon experiments now give consistent results for the ratio of the level of direct CP violation ($\Delta S = 1$ transitions of $K^0_L \to \pi^+\pi^-$ and $\pi^0\pi^0$) to indirect CP violation ($\Delta S = 2$ transitions in $K^0 - \bar{K}^0$ mixing), quantified in the value of $\epsilon'/\epsilon$. The measured ratio is larger than most SM predictions, but a definitive test of the model is complicated by hadronic physics uncertainties in the SM prediction. It is unclear if the measured value can result from a phase in the quark mixing (CKM) matrix as the SM predicts, and much theoretical effort is devoted to answering this important question.

In the lepton sector, compelling evidence for neutrino mass and mixing has been found, the strongest being the results on atmospheric neutrinos, pointing to very large mixing between $\nu_\mu$ and some other neutrino, probably $\nu_\tau$. Neutrino oscillations also appear to be the only consistent explanation for the low flux of solar neutrinos observed by a number of experiments. A tremendous experimental effort is beginning with the goal of measuring fully the lepton mass mixing matrix.

The study of rare decays of kaons potentially contributes to both these areas. Measurements of $K \to \pi l\bar{l}$ decay rates would allow a precise determination of the
parameter that quantifies CP violation in the SM and an unambiguous test of the consistency of the SM explanation of CP violation. Measurements in the B sector would be compared to those in the K sector to further test the SM. In the leptonic sector, experiments to detect $K \rightarrow (\pi)\mu e$ provide a means to search for lepton flavor violation (LFV) due to new physics processes other than neutrino mass and mixing.

Measurements of rare decay rates, in particular the small observed rate of effective flavor changing neutral current (FCNC) decays and the absence of LFV decays, have been important in restricting new physics models and that continues to be true. The theoretical motivation for the experiments has grown recently with the realization that unified supersymmetric models can be constructed that have observable effects in both these areas and that can be tested in new experiments.

In the remainder of this paper I will discuss recent experimental progress in LFV searches and CKM matrix studies.

## 2 Lepton Flavor Violation

Experiments to search directly for LFV in the charged sector have been performed for many years, all with null results. Stringent limits have resulted from searches for $K_L^0 \rightarrow \mu^\pm e^\mp$ [1, 2], $K_L^0 \rightarrow \pi^0 \mu^\pm e^\mp$ [3], $K^+ \rightarrow \pi^+ \mu^+ e^-$ [4], $\mu^+ \rightarrow e^+ \gamma$ [5, 6], $\mu^+ \rightarrow e^+ e^+ e^-$ [7], and $\mu^- N \rightarrow e^- N$ [8]. The sensitivity of these processes to mechanisms that allow LFV varies. In general, kaon decay experiments are most sensitive for models that relate lepton and quark family numbers; examples are models with leptoquarks, which carry both quark and lepton number.

Two experiments have reported new limits on LFV processes in the kaon sector: final results of a search for $K_L^0 \rightarrow \mu^\pm e^\mp$ from BNL E871 [9] and preliminary results of a search for $K^+ \rightarrow \pi^+ \mu^+ e^-$ from BNL E865 [10]. They are the culmination of rare K decay programs that began about 15 years ago, and no experiments that would improve on their sensitivities are currently proposed or are likely to be proposed soon.

The two modes are related, and differ in the Lorentz structure of the couplings to which they are sensitive; the $K_L^0$ mode is pseudo-scalar or axial-vector, while the $K^+$ mode is scalar or vector. For a $V \pm A$ interaction, the $K^+ \rightarrow \pi^+ \mu^+ e^-$ branching fraction would be smaller by a factor of more than 10 by virtue of the larger $K^+$ total decay rate and the phase space suppression of the 3-body final state. The mass scale to which this type of experiment is sensitive is easily determined, for example by comparing the rate for $K^+ \rightarrow \mu^+ \nu_\mu$ to that for $K^0 \rightarrow \mu^+ e^-$ (with the exchange of a hypothetical X boson of mass $M_X$, coupling g and mixing factors $\lambda_{sd}$ and $\lambda_{\mu e}$). Forming the ratio of these rates results in the relationship
For example, if \( B(K^0_L \to \mu^\pm e^\mp) = 10^{-12} \), \( M_X = 200 \text{ GeV}/c^2 \) and \( g = g_W \), one would infer that \( \lambda_{sd} \lambda_{\mu e} \simeq 10^{-6} \).

The experimental difficulties of a search with sensitivity \( 10^{-12} \) include producing enough kaons, building an apparatus with sufficient acceptance and rate handling capability, and rejecting processes that could mimic a signal. A beam line and experiment to reach this level would have the following parameters: a beam with \( 10^8 K^0_L \) per second, a region with 8% decay probability, a detector with 1.5% acceptance, and 3000 hours data collection. The performance is relatively insensitive to the kaon beam energy, with the exception that low energy (\(<10 \text{ GeV}\)) allows the use of threshold Cerenkov counters in particle identification. Because well defined, clean beams can be made, backgrounds result primarily from other \( K \) decays, and excellent kinematic measurement and particle identification are required to reject them. Coincidences of two kaon decays, each giving a lepton, are also a potential source of background. The detectors must run at rates up to 1 MHz per detector element and provide good timing signals to reject them.

The E871 search for \( K^0_L \to \mu^\pm e^\mp \) was done at Brookhaven National Laboratory (BNL); the beam-line and apparatus are described in the literature [9, 11, 12]. E871 used a \( K^0_L \) beam produced with a 24 GeV proton beam from the Alternating Gradient Synchrotron (AGS). The AGS delivered \( \sim 1.5 \times 10^{13} \) protons per 1.5 s spill each 3.6 s, producing a 65 \( \mu \text{str} \) beam of \( 2 \times 10^8 K^0_L \) per pulse in the momentum interval \( 2 < p_K < 8 \text{ GeV}/c \). Photons in the beam were attenuated using lead foils in a sweeping magnet immediately following the target. The beam was produced at an angle of 3.75° to minimize the neutron flux; the n/K ratio was about 10. This is the most intense neutral kaon beam ever made.

Kaons decayed in an 11 m long evacuated tank, and the decay products were detected in a magnetic spectrometer and system of particle identification counters. The apparatus is shown in Fig. 1; it differed from that of earlier experiments in the use of a beam-stop [12] in the first of two analyzing magnets. This reduced rates in downstream detectors and allowed them to be essentially continuous across the projected beam-line. Important features of the spectrometer were the use of two sequential analyzing magnets to measure redundantly the momenta of charged decay particles and the use of small diameter straw chambers where rates were highest. Redundant means of identifying both electrons and muons were used.

To ensure that selection criteria were free of bias from knowledge of potential signal events, they were chosen by studying \( K^0_L \to \mu^\pm e^\mp \) candidates outside an exclusion region larger than one that would contain signal events (see Fig. 2).
Selection criteria included requirements on the quality of the tracks measured in the spectrometer and the existence of appropriate signals in the particle identification counters. Large errors in $M_{\mu e}$ were shown by Monte Carlo simulation to come from $K_0^0 \rightarrow \pi e\nu$ decays in which an electron scattered at large angles at or before the first tracking detector and the pion decayed to a muon before the spectrometer. Accidental coincidences of two semi-leptonic decays were calculated to be a potential background; events with three or more fully reconstructed tracks in the spectrometer were rejected, reducing this background by an order of magnitude. The number and kinematic distributions of events with $M_{\mu e} > 493$ MeV/$c^2$ were well reproduced by a Monte Carlo simulation of these processes; an example is shown in Fig. 2.

Final selection criteria (including the choice of the signal region) were optimized to reduce the expected background to 0.1 events and only then were events
inside the exclusion region analyzed. Figure 2 shows the final distribution in $p_T^2$ versus $M_{\mu e}$. There are no events in the signal region and the number of events in the exclusion region is consistent with the Monte Carlo prediction. Based on no observed events, a limit was set, $B(K_0^0 \rightarrow \mu^\pm e^\mp) < 4.7 \times 10^{-12}$ [90\% CL]. This is the smallest limit ever set on a branching fraction for a hadron and results in a lower limit $M_X > 190 \, TeV/c^2$ assuming weak interaction strength $V_A$ coupling with maximal mixing.

A search for the corresponding charged mode, $K^+ \rightarrow \pi^+ \mu^+ e^-$, was done by BNL E865, which used an intense, unseparated, 6 GeV/c positive beam. In many respects the apparatus and detection techniques were similar to those of E871. $K^+$ mesons decayed in an evacuated decay region. The decay products were first separated in a magnet, and then momentum analyzed using a proportional wire chamber spectrometer surrounding a second magnet. Cerenkov counters and a shashlik calorimeter identified $e^-$ on one side of the apparatus, and gas Cerenkov counters vetoed electrons on the other side, where $\mu^+$ were also identified in a range stack. A series of selection criteria on kinematic quantities was imposed, as shown in Fig. 3. The probability of a good event having a particular value of the kinematic quantity was used to form a likelihood with which events were further selected. A scatter plot of the $\pi^+ \mu^+ e^-$ mass vs. this likelihood is shown in Fig. 3 for data and Monte Carlo generated $K^+ \rightarrow \pi^+ \mu^+ e^-$ events. Three events satisfied individual selection criteria, but failed a likelihood requirement that was set (without prior knowledge of the distribution) at a value that resulted in a 20\% loss of sensitivity. Based on the absence of events satisfying all selection criteria, a limit $B(K^+ \rightarrow \pi^+ \mu^+ e^-) < 4.0 \times 10^{-11}$ was set [10].

These experiments may be the last to search for LFV using K decays. E871 has demonstrated the existence of a background at a sensitivity of order $10^{-13}$, resulting from $K_L^0 \rightarrow \pi e \nu$ decay with Mott scattering of the electron plus $\pi$ decay conspiring to produce background indistinguishable from $K_L^0 \rightarrow \mu^\pm e^\mp$ events; a dedicated experiment with a long data collection time would be required to reach that level. E865 is already at the point of being limited by background, and given the inherently lower sensitivity of $K^+ \rightarrow \pi^+ \mu^+ e^-$ in comparison with $K_L^0 \rightarrow \mu^\pm e^\mp$, it is unlikely a new effort will be undertaken. Much more sensitive probes of LFV are now being proposed [13, 14, 15] with muon processes, and this is likely where progress will be made.

3 Quark Mixing Matrix Measurements

The K system is one in which particularly incisive studies of the quark mixing (CKM) matrix can be made. Because FCNC decays are not allowed at tree level in the SM, many processes are dominated by one loop diagrams containing the top
quark. The best example of this is $K^{0}_L \rightarrow \pi^{0}\nu\nu$ it is pure CP violating (and predominantly direct CP violating), and the rate can be unambiguously related to the parameter $\eta$ in the Wolfenstein parameterization of the CKM matrix. There are also experimental advantages to doing experiments with kaons. Because of their long lifetime, clean, intense beams can be produced, often with little contamination from other particles. Experiments can be done far from the environment in which the kaons are produced, allowing for experiments with a branching fraction sensitivity that cannot be contemplated in a collider environment.

The experiments to study the CKM matrix focus on decays of charged or neutral kaons to pairs of leptons, with or without an additional pion. The most important are $K^{0}_L \rightarrow \pi^{0}\nu\nu$, $K^{+} \rightarrow \pi^{+}\nu\nu$, $K^{0}_L \rightarrow \mu^{+}\mu^{-}$, $K^{0}_L \rightarrow \pi^{0}e^{+}e^{-}$, and $K^{0}_L \rightarrow \pi^{0}\mu^{+}\mu^{-}$. Other modes (for example $K^{0}_L \rightarrow e^{+}e^{-}$ and $K^{+} \rightarrow \pi^{+}l^{+}l^{-}$) are dominated by radiative decay processes and will not have much input on CKM measurements. In the remainder of this section, I will briefly review the phenomenology of K decays and the CKM matrix, and then discuss recent results.

### 3.1 Phenomenology of CKM Matrix and $K \rightarrow (\pi)l\bar{l}$

Within the context of the SM, the rates for these rare decays can be calculated with small theoretical uncertainty. They proceed through box and penguin diagrams of the type shown in Fig. 4. They are dominated by the top quark con-
tribution and depend on the CKM matrix elements $V_{ts}$ and $V_{td}$. The short distance contributions to the decay amplitudes are proportional to $\text{Re}(V_{td}V_{ts}^*) = A^2\lambda^5(1-\rho)$ for $K_L^0 \rightarrow \mu^+\mu^-$, to $\text{Im}(V_{td}V_{ts}^*) = A^2\lambda^5\eta$ for $K_L^0 \rightarrow \pi^0\nu\bar{\nu}$, and to $V_{td}V_{ts}^* = A^2\lambda^5(1-\rho-i\eta)$ for $K^+ \rightarrow \pi^+\nu\bar{\nu}$. For the latter two, any additional complications in the calculation of the decay rate are negligible; hadronic effects are incorporated by comparing to corresponding semi-leptonic decays, and contributions from charm quark diagrams are small (and only relevant for $K^+ \rightarrow \pi^+\nu\bar{\nu}$, because $K_L^0 \rightarrow \pi^0\nu\bar{\nu}$ is purely CP violating). In the case of $K_L^0 \rightarrow \mu^+\mu^-$, there are significant long distance effects (primarily from $2\gamma$ intermediate states) that very much complicate the extraction of short distance physics.

As was originally pointed out by Jarlskog [16] in the context of the CKM matrix, all triangles in the complex plane formed by the inner product of pairs of rows or columns of a unitary matrix have the same area; this area is proportional to $\eta$ and hence to the degree of CP violation in the SM. To order $\lambda^6$, the area is given by $A = J/2 = \frac{1}{2} \times \lambda(1-\lambda^2/2) \times A^2\lambda^5\eta$; $J$ is referred to as the Jarlskog invariant. The triangle formed by the inner product of the first and third columns is often referred to as the unitarity triangle. As has been stressed recently by a number of authors, any triangle is equally good (from a theoretical viewpoint) in quantifying CP violation in the CKM matrix. K decays are particularly powerful in measuring $J$ using the first and second columns to form a triangle. The base then depends only on the Cabibbo angle, and the height is proportional to $B(K^0_L \rightarrow \pi^0\nu\bar{\nu})$. Measurements of $B(K^+ \rightarrow \pi^+\nu\bar{\nu})$ and $B(K_L^0 \rightarrow \mu^+\mu^-)$ provide constraints on the closure of the triangle and might indicate deviations from the SM prediction. It is more difficult to determine $J$ from B decays, but eventually the value determined in that system could be compared with that derived from K decays to again test the SM.

There has recently been renewed interest in K decay measurements from two sources. First, E787 at BNL [17] reported the first observation of the decay $K^+ \rightarrow \pi^+\nu\bar{\nu}$ with one event, corresponding to a branching fraction of $4.2^{+9.7}_{-3.5} \times 10^{-10}$.

![Figure 4](image-url)  

**Figure 4:** The three diagrams on the left are SM box and penguin diagrams contributing to $K \rightarrow (\pi)\ell\bar{\ell}$ decays, for example $K^0 \rightarrow \mu^+\mu^-$. The two diagrams on the right are typical of those that arises in supersymmetry models and that may also contribute to these decays and to indirect CP violation.
This is about a factor of five above the SM prediction. Second, the KTeV [18] and NA48 [19] collaborations have presented first results of their new measurements of $\varepsilon'/\varepsilon$. They are in good agreement with an earlier result [20] from NA31 at CERN and somewhat higher than the result [21] from E731 at Fermilab. The value is significantly different from zero and also significantly higher than most expectations. Although the recent results on $K^+ \to \pi^+ \nu\bar{\nu}$ and $\varepsilon'/\varepsilon$ hint at the possibility of new physics contributions, neither requires it. The SM prediction of the $K^+ \to \pi^+ \nu\bar{\nu}$ rate is theoretically unambiguous, but the measurement is not statistically inconsistent with it. On the other hand, the average of the $\varepsilon'/\varepsilon$ measurements is significantly different from most predictions, but the model calculations are theoretically ambiguous.

Given these hints, three questions arise in understanding the extent to which experiments can make a definitive statement about the need for non-SM physics. First, is the SM prediction for $\varepsilon'/\varepsilon$ currently inconsistent with experiment and can it be made reliable? Second, what new physics could contribute to $\varepsilon'/\varepsilon$ and the decay $K^+ \to \pi^+ \nu\bar{\nu}$? Third, do measurements of rare decays of kaons limit possible contributions from non-SM sources and can they provide unambiguous evidence for deviations from the SM?

The first question remains controversial. Bosch et al. [22] have varied all parameters entering into the SM prediction for $\varepsilon'/\varepsilon$ and contend that the experimental result can be accommodated within the SM only by simultaneously stretching many of the values entering into the prediction, including the values of matrix elements derived from either lattice calculations or phenomenological models. The second question has been studied in the context of effective $Zds$ couplings of the type shown in Fig. 4. These arise naturally in supersymmetric theories and could easily give large contributions to both $\varepsilon'/\varepsilon$ and rates for rare K decays. For example, Bosch et al. [22] infer a SM prediction of $1.6 \times 10^{-11} < B(K^0_L \to \pi^0 \nu\bar{\nu}) < 3.9 \times 10^{-11}$ and show that $\varepsilon'/\varepsilon < 28 \times 10^{-4}$ implies $B(K^0_L \to \pi^0 \nu\bar{\nu}) < 48 \times 10^{-11}$ if the enhanced value of $\varepsilon'/\varepsilon$ is due to a new effective $Zds$ coupling. Within this context, effective $Zds$ couplings with large contributions to $B(K^0_L \to \pi^0 \nu\bar{\nu})$ are not excluded. The advantage of a measurement of this decay rate is that the SM prediction is unambiguous within a rather narrow window and deviations would provide definitive evidence for non-SM physics that may not obtain from even a very precise measurement of $\varepsilon'/\varepsilon$. Similarly, SM physics and current measurements imply [23] $B(K^+ \to \pi^+ \nu\bar{\nu}) = 8.2 \times 10^{-11}$, whereas the above limit on $\varepsilon'/\varepsilon$ plus the measured branching fraction for $K^0_L \to \mu^+ \mu^-$ implies $B(K^+ \to \pi^+ \nu\bar{\nu}) < 29 \times 10^{-11}$ even with arbitrary effective $Zds$ couplings. Again, current measurements allow significant contributions to the $K^+ \to \pi^+ \nu\bar{\nu}$ decay rate from non-SM physics, and a precise measurement of this rate will restrict this possibility. I next discuss recent experimental results that bear on these questions.
3.2 Measurement of $B(K_L^0 \to l^+l^-)$

The decay $K_L^0 \to \mu^+\mu^-$ is by far the best measured of the modes discussed. Recently, E871 at BNL has reported final results [11, 24] of measurements of $B(K_L^0 \to e^+e^-)$ and $B(K_L^0 \to \mu^+\mu^-)$. The experiment used the same apparatus as that used for the $K_L^0 \to \mu^\pm e^\mp$ search discussed earlier and data were collected during the same period. In the analysis, the emphasis was not on eliminating background but rather on understanding the relative acceptance for $K_L^0 \to \mu^+\mu^-$ decays and the $K_L^0 \to \pi^+\pi^-$ decays to which the experiment was normalized. To avoid biases in the analysis, an unknown prescale factor was applied to the normalization sample and only revealed and corrected for after the analysis was completed.

Figure 5 shows distributions in the $\mu^+\mu^-$ invariant mass and square of the transverse momentum for events satisfying appropriate selection criteria. A small ($\sim 1.2\%$) background in the $K_L^0 \to \mu^+\mu^-$ candidate event sample (primarily from $K_L^0 \to \pi^0\nu\bar{\nu}$ events) was subtracted by linearly extrapolating the level of events at large $\mu^+\mu^-$ transverse momentum under the $K_L^0 \to \mu^+\mu^-$ peak. The $K_L^0 \to \mu^+\mu^-$ sample contains $\sim 6200$ events. Appropriate correction factors (for example for trigger and particle identification efficiencies) were applied to the relative $\mu^+\mu^-$ and $\pi^+\pi^-$ acceptances; these were typically derived from ancillary measurements and were typically a few percent. The largest of these were for pion absorption in the spectrometer ($5.2\%$), the online $\mu^+\mu^-$ trigger efficiency ($\sim 3\%$), and the off-line $\mu^+\mu^-$ particle identification efficiency ($\sim 6\%$). The resulting branching ratio is $\Gamma(K_L^0 \to \mu^+\mu^-)/\Gamma(K_L^0 \to \pi^+\pi^-) = (3.474 \pm 0.057) \times 10^{-6}$. Multiplying by the measured value of $B(K_L^0 \to \pi^+\pi^-)$ results in $B(K_L^0 \to \mu^+\mu^-) = (7.18 \pm 0.17) \times 10^{-9}$. Statistical and systematic errors are added in quadrature.
the largest of these are the statistical uncertainty on the number of $K^0_L \rightarrow \mu^+ \mu^-$ events (1.32%) and the systematic uncertainties on pion absorption (0.50%) and the relative geometrical and reconstruction efficiencies (0.57%).

An estimate of the short distance contribution to this decay is derived by first subtracting the “unitarity bound” due to the absorptive $2\gamma$ contribution to the decay rate. This is derived from the measured $K^0_L \rightarrow \gamma \gamma$ decay rate and a QED calculation and results in a measured dispersive part of the branching fraction of $(0.11 \pm 0.18) \times 10^{-9}$. Then, $|ReA_{exp}|^2 < 0.37 \times 10^{-9}$ [90% CL]. Proceeding beyond this requires a model for calculating the long distance dispersive amplitude, a controversial procedure [25]. Using one recent estimate [26], the Wolfenstein parameter $\rho$ is found to be bounded from below at $-0.33$, consistent with existing constraints.

E871 has also measured [11] the branching fraction for the decay $K^0_L \rightarrow e^+ e^-$ for the first time. This decay rate is suppressed with respect to that of $K^0_L \rightarrow \mu^+ \mu^-$ due to helicity considerations and is expected to be completely dominated by long distance effects. Unlike the case of $K^0_L \rightarrow \mu^+ \mu^-$, these can be reliably calculated [25, 27] and the branching fraction is expected to be $\sim 10^{-11}$. The analysis proceeded in much the same way as for $K^0_L \rightarrow \mu^+ \mu^-$. Backgrounds resulted primarily from physics sources: $K^0_L \rightarrow e^+ e^- \gamma$ and $K^0_L \rightarrow e^+ e^- e^+ e^-$. The analysis proceeded in much the same way as for $K^0_L \rightarrow e^+ e^-$. Backgrounds resulted primarily from physics sources: $K^0_L \rightarrow e^+ e^- \gamma$ and $K^0_L \rightarrow e^+ e^- e^+ e^-$. The fit yields 4 $K^0_L \rightarrow e^+ e^-$ events and a corresponding value of $B(K^0_L \rightarrow e^+ e^-) = (8.9^{+3.5}_{-2.8}) \times 10^{-12}$. This is in good agreement with predictions of chiral perturbation theory calculations and is the smallest branching fraction ever measured.

These measurements of $B(K^0_L \rightarrow e^+ e^-)$ and $B(K^0_L \rightarrow \mu^+ \mu^-)$ are unlikely to be improved upon in the foreseeable future. For the former, little can be learned about short distance physics from improved measurements. For the latter, extracting a value for $\rho$ is now limited by model assumptions in the calculation of the long distance dispersive contributions, and until they can be improved, better precision on this branching fraction measurement is not useful.

### 3.3 Measurement of $B(K^+ \rightarrow \pi^+ \nu \bar{\nu})$

The measurement of $B(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ allows a direct determination of the magnitude of $V_{td}V_{ts}^*$ (after a small correction for c quark contribution); hence it measures directly the magnitude of one side of the triangle formed by $V_{td}V_{ts}^*$. Experimental difficulties arise from the facts that the 2 neutrinos are unobservable and that many $\pi^+$ are also produced from the copious decay mode $K^+ \rightarrow \pi^+ \pi^0$. E787 at BNL detected this decay for the first time [17], observing one event with
a branching fraction consistent with the SM prediction. The event was found first in a data sample corresponding to a sensitivity of about 0.2 events at the SM level, raising the hope that there might be a non-SM contribution, despite the high probability of finding an event at that sensitivity. Nonetheless, this hope motivated some useful theoretical discussion of what might be responsible for a rate larger than that predicted by the SM.

Figure 6: The figure shows a scatter plot of range vs. energy for candidate $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ events following event selection on the left and an event display of the one signal event on the right.

E787 stops $K^+$ mesons in an active target and then measures all conceivable final state properties: the $\pi^+$ kinetic energy (determined from the energy deposited in the target and in a plastic scintillator range stack), the momentum (measured in a magnetic spectrometer), the range (determined from the penetration distance in the range stack), and the decay chain $\pi \rightarrow \mu \rightarrow e$. An event display of the one candidate is shown in Fig. 6. The figure also shows a scatter plot of the range vs. energy for events satisfying a set of selection criteria on decay chain, momentum, energy in photon veto counters, and tracking quality. About 70% of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ events satisfying these criteria would fall in the rectangular box. Background from $K^+ \rightarrow \mu^+ \nu \bar{\nu}$ would be at a larger range and energy.

E787 recently presented preliminary results [28] from a larger data set, with no additional events seen. Based on the total sensitivity currently reported, the branching fraction is given by $B(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (1.5^{+3.5}_{-1.3}) \times 10^{-10}$ [90% CL].

Improvements in the precision of this measurement will come in stages. Data currently exists to improve the sensitivity by about a factor of two, with analysis expected to be completed in about 1 year. A modestly upgraded experiment, E949 [29], was recently approved at BNL, and the U.S. DOE has agreed to operate
the accelerator for this experiment for about 5000 additional hours of running. It is expected to get 7–14 events at the SM level. Further improvement in sensitivity may come from the CKM experiment [30] proposed at Fermilab to detect about 100 events at the SM branching fraction. This experiment uses a technique completely different from that of E949, using K⁺ decay in flight from a separated 12 GeV beam.

3.4 Search for $K_L^0 \to \pi^0 \nu \bar{\nu}$

Despite the importance of a measurement of the branching fraction for the decay $K_L^0 \to \pi^0 \nu \bar{\nu}$, progress to date has come only as a by-product of other experiments, primarily from the KTeV collaboration. Recently, this group has published results based on two techniques. They are distinguished by the detection of $\pi^0$ decays to either $\gamma \gamma$ or $\gamma e^+ e^-$. The experimental difficulties in the experiment result from the lack of knowledge of the initial state (because the beam is neither monoenergetic nor very small) and the paucity of information about the final state. For the $2\gamma$ decay mode, only the energy and position of the two photons is measured, and the $K_L^0$ decay point is inferred by constraining the $\gamma \gamma$ invariant mass to the $\pi^0$ mass. For Dalitz decays, the $K_L^0$ decay point is determined by the vertex position of the $e^+ e^-$ pair. In both cases, copious backgrounds exist from $K_L^0 \to \pi^0 \pi^0$ and $K_L^0 \to \pi^0 \pi^0 \pi^0$ decays and from hyperon decays.

The KTeV apparatus is shown in Fig. 7. It consists of a magnetic spectrometer and a CsI electromagnetic calorimeter for the detection of charged decay products and photons, respectively. Particle identification is provided by the CsI calorimeter and transition radiation detectors in the case of electrons and by a hadron filter and muon scintillator hodoscope in the case of muons. A set of lead-scintillator sandwich $\gamma$ veto counters is situated around the decay region. Figure 7 shows the distribution in the $p_T$ of the $\pi^0$s detected using Dalitz decays. Based on no detected events in the interval $160\ MeV/c < p_T < 240\ MeV/c$, a limit was set [31] $B(K_L^0 \to \pi^0 \nu \bar{\nu}) < 5.9 \times 10^{-7}$ [90% CL]. A somewhat worse limit, $B(K_L^0 \to \pi^0 \nu \bar{\nu}) < 1.8 \times 10^{-6}$ [90% CL] was set [32] by the KTeV collaboration using the $2\gamma$ decays of the $\pi^0$, in which one event consistent with background was found. The limits are well above the SM prediction of $\sim 3 \times 10^{-11}$.

Further significant improvement in sensitivity will come only from dedicated experiments, and two such experiments are contemplated. E926 [15, 33] at BNL proposes to use a low energy pulsed beam and a detector capable of measuring $\gamma$ positions and directions so as to reconstruct the $K_L^0$ decay position and time, and hence determine the $K_L^0$ momentum by time of flight. This allows, in principle, complete determination of the decay kinematics. Coupled with a hermetic photon veto, the proponents believe they can detect $\sim 65$ events with a signal/background ratio greater than two. Significant experimental challenges
Figure 7: The KTeV apparatus, configured for the rare decay experiment E799, is shown at the top. The histogram on the bottom is the distribution in the transverse momentum of the $\pi^0$ reconstructed from Dalitz decays, with Monte Carlo simulated backgrounds superimposed.
arise in achieving the $\gamma$ veto efficiency needed and in measuring the $\gamma$ positions, angles, and times to the required precision. In addition, the very intense neutral beam (a mix of kaons, neutrons, and photons) results in the potential for large dead-times due to the requirement that events with as little as 1-2 MeV energy deposited in an extensive veto system be rejected. This experiment is being considered for funding by the U.S. DOE and NSF.

The KAMI collaboration is studying the possibility of mounting an experiment [34] at Fermilab with approximately the same sensitivity goal. It would use a higher energy beam. The proponents believe the superior $\gamma$ veto efficiency that in principle can be achieved at higher energy will allow a large signal to background ratio to be achieved without the $K^0_L$ momentum determination using time of flight—which is not possible at high energy—and without the $\gamma$ direction determination, which could be done. Events would be isolated by very hermetic $\gamma$ vetos of high efficiency and by inferring the $K^0_L$ decay position by containing the invariant mass of two detected $\gamma$’s to the $\pi^0$ mass, and then selecting events with a $\pi^0$ transverse momentum above that from $K^0_L \rightarrow \pi^0\pi^0$ decays. This collaboration has an approved R&D program and anticipates producing a proposal early in 2001 if it is successful.

### 3.5 Search for $K^0_L \rightarrow \pi^0 e^+ e^-$ and $K^0_L \rightarrow \pi^0 \mu^+ \mu^-$

In principle, a measurement of $B(K^0_L \rightarrow \pi^0 l^+ l^-)$ would allow the determination of the CKM parameter $\eta$ because there is a significant direct CP violating contribution. The rate is about one third that for $K^0_L \rightarrow \pi^0 \nu \bar{\nu}$, because there are three final states (three neutrino flavors) in the latter. There are also contributions from indirect CP violation and from CP conserving amplitudes with $2 \gamma$ intermediate states. The experimental situation is even more difficult due to backgrounds from less interesting processes, which are discussed below.

The KTeV collaboration has recently reported results [35] of a search for both modes. The apparatus was identical to that used for the $K^0_L \rightarrow \pi^0 \nu \bar{\nu}$ search discussed in the previous section. Figure 8 shows data for these search modes after final selection criteria were applied. In both searches, two events are within the signal region and consistent with the expected background level. The resulting limits on the branching fractions are $B(K^0_L \rightarrow \pi^0 e^+ e^-) < 5.64 \times 10^{-10}$ and $B(K^0_L \rightarrow \pi^0 \mu^+ \mu^-) < 3.4 \times 10^{-10}$, both at 90% CL. Both searches are limited by backgrounds. In the case of $K^0_L \rightarrow \pi^0 e^+ e^-$, it is dominated by $\gamma e^+ e^-$ decay with final state radiation, and is minimized by kinematic cuts to eliminate events in which one of the photons is either along the direction of the $e^+$ or $e^-$ or in which the a photon momentum is opposite that of the $e^+ e^-$ pair in the $K^0_L$ center of mass. The superb resolution of the KTeV calorimeter helps in minimizing this background by allowing a relatively small signal box; nonetheless, the background is about
100 times the expected signal. In the case of $K^0_L \rightarrow \pi^0 \mu^+ \mu^-$, the background is from $K^0_L \rightarrow \pi^0 \pi^+ \pi^-$ decays in which both the $\pi^+$ and $\pi^-$ decay. In this respect, the KTeV apparatus is not well optimized, in that $\pi$ decay in the center of the analyzing magnet can go undetected and result in an apparently large muon momentum and hence large value for the reconstructed $K^0_L$ mass. Eliminating this type of $\pi$ decay background was the motivation for two successive magnets in the BNL E871 search for $K^0_L \rightarrow \mu^\pm e^\mp$.

### 4 Summary

There has been significant recent progress in experimental tests of lepton flavor conservation using kaons and in measurements of rates for rare decays of kaons from which quark mixing matrix elements can be deduced. LFV in the charged sector has been searched for in $K^0_L \rightarrow \mu^\pm e^\mp$ and $K^+ \rightarrow \pi^+ \mu^+ e^-$ decays and improved upper limits have been set. The expected rate from loop diagrams with neutrino mixing, would contribute to this class of LFV processes at a rate well below conceivably observable levels. Nearly all models for new physics allow the possibility of LFV, and the current upper limits further restrict such models at a mass scale of order 100 TeV/c$^2$ for full mixing.

The progress in CKM matrix element measurements using rare decays has been in modes involving K decays to either lepton pairs or a pion and lepton pairs, including the first observation of $K^0_L \rightarrow e^+ e^-$, a precise measurement of
B(K^0_L \rightarrow \mu^+\mu^-), and the first observation of K^+ \rightarrow \pi^+\nu\bar{\nu}. Improved upper limits on K^0_L \rightarrow \pi^0\ell\bar{\ell} have also been set. Although the current sensitivities for the K \rightarrow \pi\ell\bar{\ell} modes are insufficient to provide an incisive test of standard model predictions, important progress is being made in improving experimental techniques and new experiments are proposed that could provide such a test.

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