Cosmic Microwave Background: Past, Future, and Present

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1 Introduction

Anisotropies in the Cosmic Microwave Background (CMB) carry an enormous amount of information about the early universe. The anisotropy spectrum depends sensitively on close to a dozen cosmological parameters, some of which have never been measured before. Experiments over the next decade will help us extract these parameters, teaching us not only about the early universe, but also about physics at unprecedented energies. We are truly living in the Golden Age of Cosmology.

One of the dangers of the age is that we are tempted to ignore the present data and rely too much on the future. This would be a shame, for hundreds of individuals have put in countless human-years building state-of-the-art instruments, making painstaking observations at remote places on and off the globe. It seems unfair to ignore all the data that has been taken to date simply because there will be more and better data in the future.

In this spirit, I would like to make the following claims:

1. We understand the theory of CMB anisotropies.
2. Using this understanding, we will be able to extract from *future* observations extremely accurate measurements of about ten cosmological parameters.
3. Taken at face value, *present* data determines one of these parameters, the curvature of the universe.
4. The present data is good enough that we should believe this measurement.

The first three of these claims are well-known and difficult to argue with; the last claim is more controversial, but I will present evidence for it and hope to convince you that it is true. If you come away a believer, then you will have swallowed a mouthful,

for the present data strongly suggest that the universe has zero curvature. If you believe this data, then you believe that (a) a fundamental prediction of inflation has been verified and (b) because astronomers do not see enough matter to make the universe flat, roughly two-thirds of the energy density in the universe is of some unknown form.

2 Anisotropies: the past

When the universe was much younger, it was denser and hotter. When the temperature of the cosmic plasma was larger than about 1/3 eV, there were very few neutral hydrogen atoms. Any time a free electron and proton came together to form hydrogen, a high energy ($E > 13.6$ eV) photon was always close enough to immediately dissociate the neutral atom. After the temperature dropped beneath 1/3 eV, there were no longer enough ionizing photons around, so virtually all electrons and protons combined into neutral hydrogen. This transition—called recombination—is crucial for the study of the CMB. Before recombination, photons interacted on short time scales with electrons via Compton scattering, so the combined electron-proton-photon plasma was tightly coupled, moving together as a single fluid. After recombination, photons ceased interacting with anything and traveled freely through the universe. Therefore, when we observe CMB photons today, we are observing the state of the cosmic fluid when the temperature of the universe was 1/3 eV.

Because the perturbations to the temperature field are very small, of order 10^{-5}, solving for the spectrum of anisotropies is a linear problem. This means that different modes of the Fourier transformed temperature field do not couple with each other: each mode evolves independently. Roughly, the large scale modes evolve very little because causal physics cannot affect modes with wavelengths larger than the horizon.[1] When we observe anisotropies on large angular scales, we are observing the long wavelength modes as they appeared at the time of recombination. Because these modes evolved little if at all before recombination, our observations at large angular scales are actually of the primordial perturbations, presumably set up during inflation [1].

Inflation also set up perturbations on smaller scales, but these have been processed by the microphysics. The fluid before recombination was subject to two forces: gravity and pressure. These two competing forces set up oscillations in the temperature [2]. A small scale mode, begins its oscillations (in time) as soon as its wavelength becomes comparable to the horizon. Not surprisingly, each wavelength oscillates with a different period and phase. The wavelength which will exhibit the largest anisotropies is the one whose amplitude is largest at the time of recombination.

[1] Recall that the horizon is the distance over which things are causally connected.

Figure 1 illustrates four snapshots in the evolution of a particularly important mode, one whose amplitude peaks at recombination. Early on (top panel) at redshifts larger than 10^5, the wavelength of this mode was larger than the horizon size. Therefore, little evolution took place: the perturbations look exactly as they did when they were first set down during inflation. At $z \sim 10^4$, evolution begins, and the amplitudes of both the hot and cold spots decrease, so that, as shown in the second panel, there is a time at which the perturbations vanish (for this mode). A bit later (third panel) they show up again; this time, the previous hot spots are now cold spots and vice versa (compare the first and third panels). The amplitude continues to grow until it peaks at recombination (bottom panel).

Figure 1 shows but one mode in the universe. A mode with a slightly smaller wavelength will "peak too soon": its amplitude will reach a maximum before recombination and will be much smaller at the crucial recombination time. Therefore, relative to the maximal mode shown in Fig. 1, anisotropies on smaller scales will be suppressed. Moving to even smaller scales, we will find a series of peaks and troughs corresponding to modes whose amplitudes are either large or small at recombination.

An important question to be resolved is at what angular scale will these inhomogeneities show up? Consider Fig. 2 which again depicts the temperature field at decoupling from the mode corresponding to the first peak. All photons a given distance from us will reach us today. This distance defines a surface of last scattering (which is just a circle in the two dimensions depicted here, but a sphere in the real universe). This immediately sets the angular scale θ corresponding to the wavelength shown, $\theta \simeq$ (wavelength/distance to last scattering surface). If the universe is flat, then photons travel in straight lines as depicted by the bottom paths in Fig. 2. In an open universe, photon trajectories diverge as illustrated by the top paths. Therefore, the distance to the last scattering surface is much larger than in a flat universe. The angular scale corresponding to this first peak is therefore smaller in an open universe than in a flat one.

The spectrum of anisotropies will therefore have a series of peaks and troughs, with the first peak showing up at larger angular scales in a flat universe than in an open universe. Figure 3 shows the anisotropy spectrum expected in a universe in which perturbations are set down during inflation. The RMS anisotropy is plotted as a function of multipole moment, which is a more convenient representation than angle θ. For example, the quadrupole moment corresponds to $L = 2$, the octopole to $L = 3$, and in general low L corresponds to large scales. The COBE [3] satellite therefore probed the largest scales, roughly from $L = 2$ to $L = 30$. The first peak shows up at $L \simeq 200$ in a flat universe, and we do indeed see a trough at smaller scales and then a later peak at $L \simeq 550$. This sequence continues to arbitrarily small scales (although past $L \simeq 1000$ the amplitudes are modulated by damping). We also observe the feature of geodesics depicted in Fig. 2: the first peak in an open universe is shifted to much smaller scales.

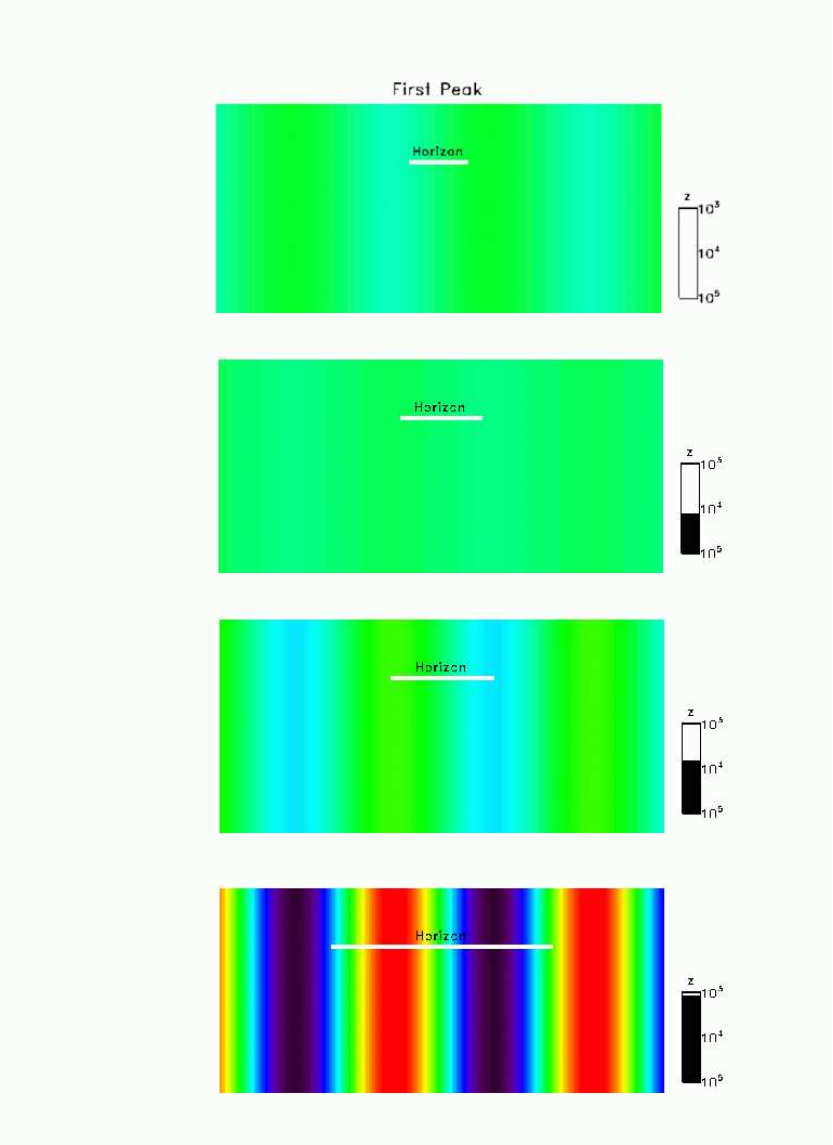

Figure 1: Four snapshots in the evolution of a Fourier mode. The top panel shows the anisotropy field to this one mode very early on, when its wavelength is still much larger than the horizon (shown as white bar throughout). The second panel, at redshift $z > 10^4$ shows a time at which the amplitude of the oscillations is very small. The third panel shows the amplitude getting larger; note that the hot and cold spots in the third panel are out of phase with those in the top panel. Finally, the bottom panel shows that at recombination, the amplitude has reached its peak. The side bar shows the redshift ranging from 10^5 at top to 10^3 in bottom panel.

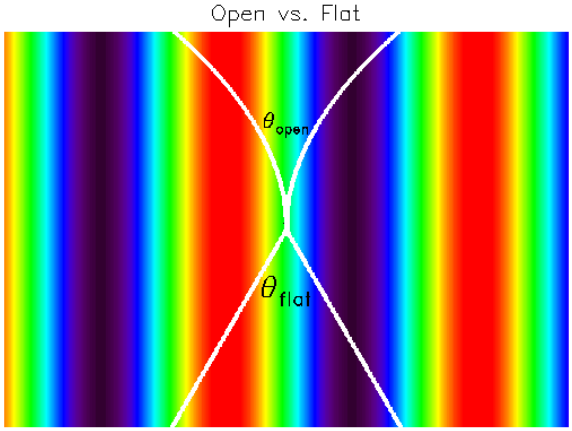

Figure 2: Photon trajectories in an open and flat universe. The same physical scale—in this case the one associated with maximal anisotropy—projects onto smaller angular scales in an open universe because geodesics in an open universe diverge.

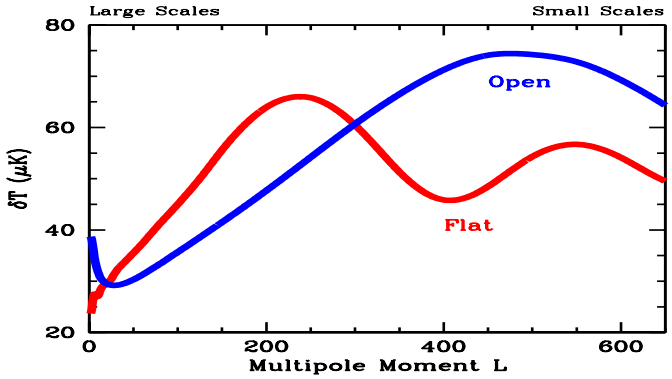

Figure 3: The spectrum of anisotropies in an open and flat universe. Plotted is the expected RMS anisotropy in micro Kelvin as a function of multipole moment. The series of peaks and troughs—the first several of which are apparent in the flat case—continues to small scales not shown in the plot. These are shifted to the right in the open case, so only the first peak shows up here. These curves are for a particular choice of cosmological parameters, corresponding to standard Cold Dark Matter.

An important aspect of Fig. 3 is the accuracy of the predictions. Although I have given a qualitative description of the evolution of anisotropies, I and many other cosmologists spent years developing quantitative codes to compute the anisotropies accurately [4]. This activity anticipated the accuracy with which CMB anisotropies will be measured and therefore we strove for (i) accuracy and (ii) speed. The former was obtained through a series of informal discussions and workshops, until half a dozen independent codes converged to answers accurate to within a percent. Speed is important because ultimately we will want to churn out zillions of predictions to compare with observations in an effort to extract best fit parameters. Fortunately, Seljak and Zaldarriaga [5] developed CMBFAST, a code which runs in about a minute on a workstation. None of these developments are particularly surprising: perturbations to the CMB are small, and therefore the problem is to solve a set of coupled linear evolution equations. The fact that there are many coupled equations makes the problem challenging, but the fact that these are linear more than compensates.

3 Anisotropies: the future

Figure 4 shows why cosmologists are so excited about the future possibilities of the CMB. First, the top panel shows that people are voting with their feet. There are literally hundreds of experimentalists who have chosen to devote their energies to measuring anisotropies in the CMB. Over the coming decade, this will lead to observations by more than a dozen experiments, culminating in the efforts of the two satellites, MAP and Planck. Some of these results are beginning to trickle in. In particular, Viper [6], MAT [7], MSAM [8], Boomerang NA [9], and Python [10] have all reported results within the last year.

The middle panel in Fig. 4 shows the expected errors after all this information has been gathered and analyzed. Take one multipole moment, at $L = 600$ say. We see that the expected error is of order 5μK, while the expected signal is about 50μK. At $L = 600$, therefore, we expect a signal to noise of roughly ten to one. Notice though that this estimate holds for all the multipoles shown in the figure. In fact, it holds for many not shown in the figure as well: it is quite possible that Planck will go out to $L \simeq 2000$. So, we will have thousands of data points, each of which will have signal to noise of order ten to one, to compare with a theory in which it is possible to make linear predictions! No wonder everyone is so excited.

The final panel in Fig. 4 shows the ramifications of getting this much information about a theory in which it is easy to make predictions. The exact spectrum of anisotropies depends on about ten cosmological parameters: the baryon density, curvature, vacuum density, Hubble constant, neutrino mass, epoch of reionization, and several parameters which specify the primordial spectrum emerging from inflation. Figure 4 shows the expected errors in four of these parameters [11]. In each case, all

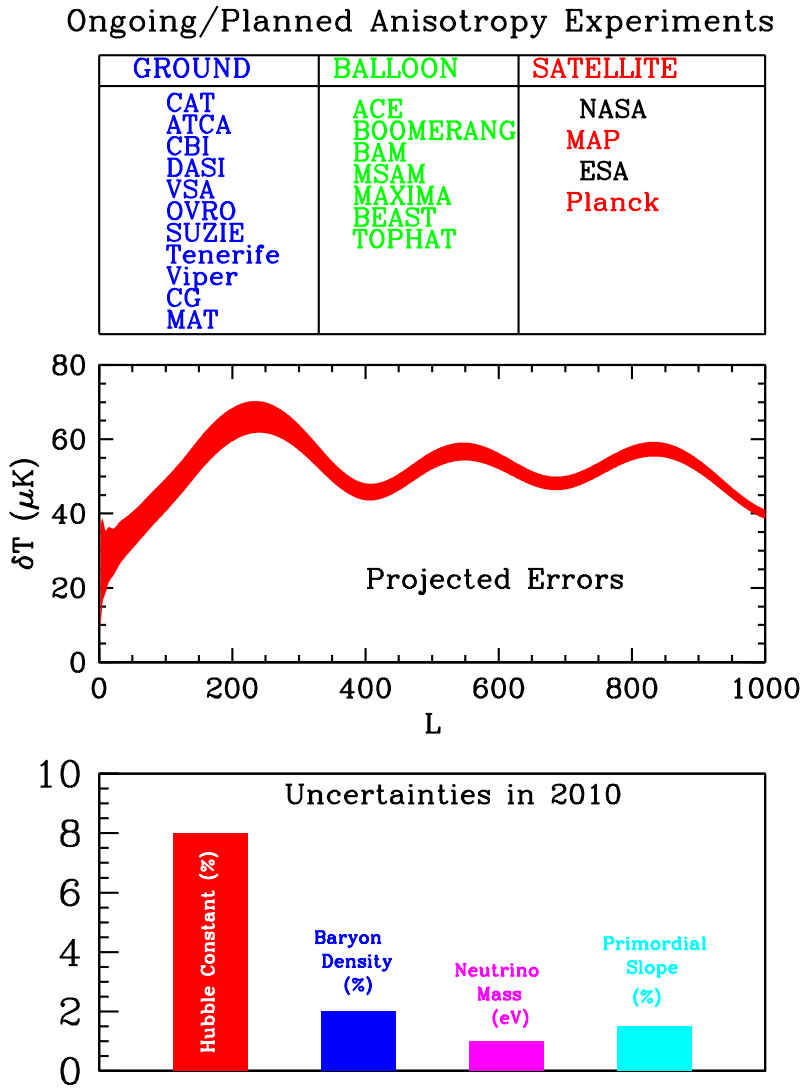

Figure 4: The future of CMB anisotropies. *Top panel:* Experiments expected to report anisotropy results within the next decade. *Middle panel:* Expected uncertainty on the anisotropy after these results come in. *Bottom panel:* Anticipated uncertainties in several cosmological parameters as a result of all this information.

(roughly ten) other parameters have been marginalized over. That is, the uncertainty in the Hubble constant stated allows for all possible values of the other parameters.

The uncertainty in the Hubble constant, of five to ten percent, comes down significantly if one assumes the universe is flat. In any event, this uncertainty is still smaller than the current estimates from distance ladder measurements [12]. The very small uncertainty on the baryon density is smaller than the five percent number obtained by looking at deuterium lines in QSO absorption systems [13]. More importantly, the systematics involved in the two sets of determinations are completely different. If the two determinations agree, we can be very confident that systematics are under control. The upper limit on the neutrino mass is particularly interesting given recent evidence for non-zero neutrino masses. The CMB alone will not go down to 0.07 eV, the most likely number from atmospheric neutrino experiments [14], but it will certainly probe the LSND region ($m_\nu \simeq 2-3$ eV) [15]. Further, it is possible that, in conjunction with large scale structure [16] and weak lensing measurements [17], we will get to the range probed by atmospheric neutrinos.

The final bar in the bottom panel shows the predicted uncertainty in the slope of the primordial spectrum. While one might reasonably ask, "What difference does it matter if we know the baryon density or the Hubble constant to five percent or two percent accuracy?" the slope of the primordial spectrum and other inflationary parameters are different. For every inflationary model makes predictions about the primordial perturbation spectrum. The more accurately we determine the parameters governing the spectrum, the more models we can rule out. So it is extremely important to get the primordial slope and other inflationary parameters as accurately as possible. These may well be our only probe of physics at energies on the order of the GUT scale.

Along these lines, I should mention several recent developments in the field of parameter determination. The first is an argument made by several groups for measuring polarization [18]. They show that accurate measurement of polarization will decrease the uncertainty in the primordial slope by quite a bit. Even though currently planned experiments may well do a nice job measuring polarization, there will still be work to do even after Planck. So we can look forward to proposals for a next generation experiment which measures polarization, and I believe we should strongly support such efforts.

Another development in the field of parameter determination is the realization that a large part of the uncertainty in some parameters (especially some of the inflationary ones) is contributed by treating the reionization epoch as a free parameter. In fact, it is a function [19] of the cosmological parameters and some astrophysical parameters. Recently, Venkatesan [20] has argued that we can use our very rough knowledge of the astrophysical parameters together with the reionization models to reduce the errors on the cosmological parameters.

4 Anisotropies: the present

It is time to confront the data. Figure 5 shows all data as of November 1999. There are two features of this compilation worthy of note. First, note that data reported within the last year are distinguished from earlier results, illustrating in a very graphic way the progress of the field. Second, Fig. 5 *understates* this progress because it was produced before the late November release of the the Boomerang North America "test" flight [9]. Indeed, the results which follow do not include this test flight. The papers describing the Boomerang release are fascinating if only because one can compare the results of all data pre-Boomerang with the test flight data. Both subsets of the data have enough power to constrain the curvature by themselves. They produce remarkably consistent results.

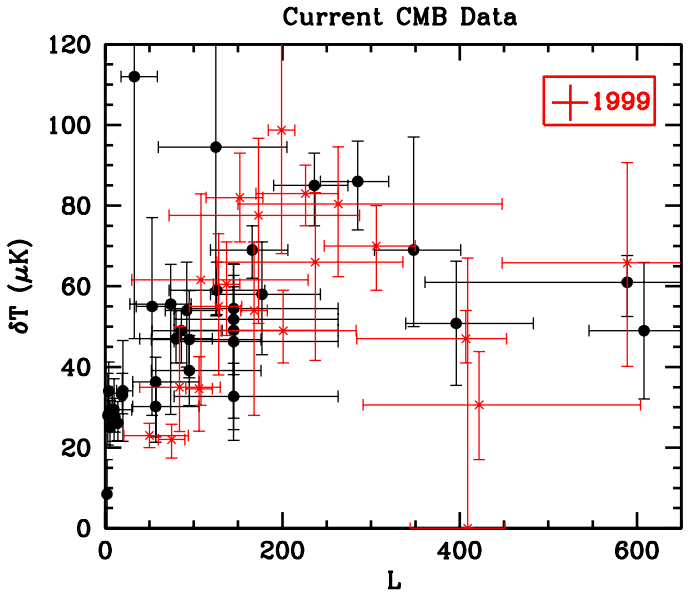

Figure 5: Current measures of CMB anisotropy. Points marked as crosses refer to measurements reported within the past year. Included are all data as of November 1999.

The data in Fig. 5 show a clear peak at around the position expected in flat models. Indeed, a number of groups [21] have analyzed subsets of this data and found it to be consistent with a flat universe and inconsistent with an open one. I

will briefly describe my efforts with L. Knox [22]. We accounted for a number of facts which make it difficult to do a simple "chi-by-eye" on the data. First, every experiment has associated with it a calibration uncertainty: all the points from a given experiment can move up or down together a given amount. We account for this by including a calibration factor for each experiment and including a Gaussian prior on this factor with a width determined by the stated uncertainties. Second, the error bars in the plot are slightly misleading because the errors do not have a Gaussian distribution. In particular, the cosmic variance part of the error is proportional to the signal itself, so the error gets much larger than one would expect at high δT. In other words, the distribution is highly skewed, with very high values of δT not impossible. The true distribution is close to a log-normal distribution [23], and we have accounted for this in our analysis. Finally, as alluded to above, there are many cosmological parameters in addition to the curvature. We do a best fit to a total of seven cosmological parameters (in addition to eighteen calibration factors).

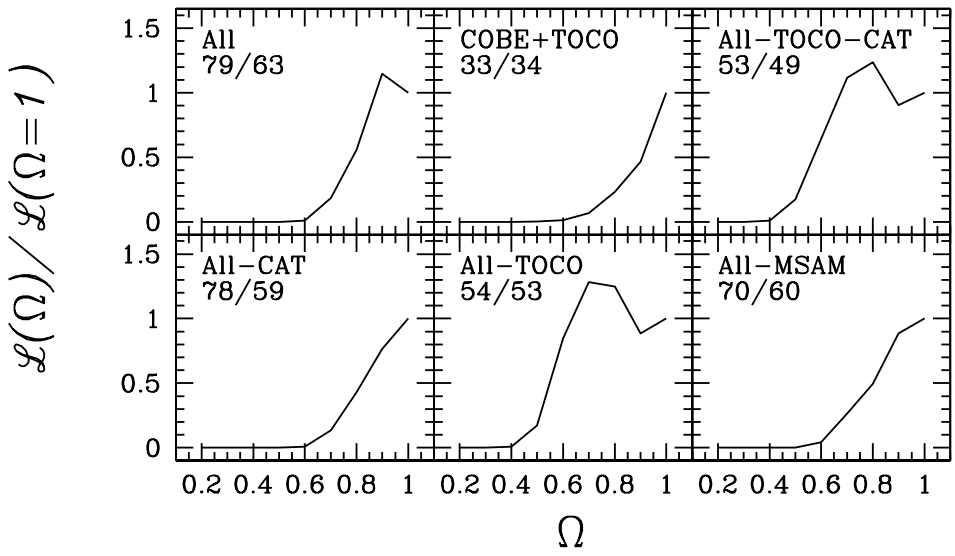

Figure 6: Ratio of likelihood of Ω to $\Omega = 1$ (flat) for different sets of experiments. Top left panel shows results using all data; other panels show the same ratio using only subsets of the data.

The top left panel of Fig. 6 shows our results. The likelihood peaks at total density Ω very close to one (no curvature) and falls off sharply at low Ω. A universe with total density equal to 40% of the critical density is less likely than the flat model by a factor of order 10^7. This ratio is key because observations [24] of the matter

density in the universe have converged to a value in the range $0.3 - 0.4$ of the critical density. We can combine these two results to conclude that there must be something else besides the matter in the universe. This conclusion probably sounds familiar to you, as the recent discoveries of high redshift supernovae [25] also strongly suggest that there is more to the universe than just the observed matter: there is dark energy in the universe. The exciting news is that we now have independent justification of these results using CMB + Ω_{matter} determinations.

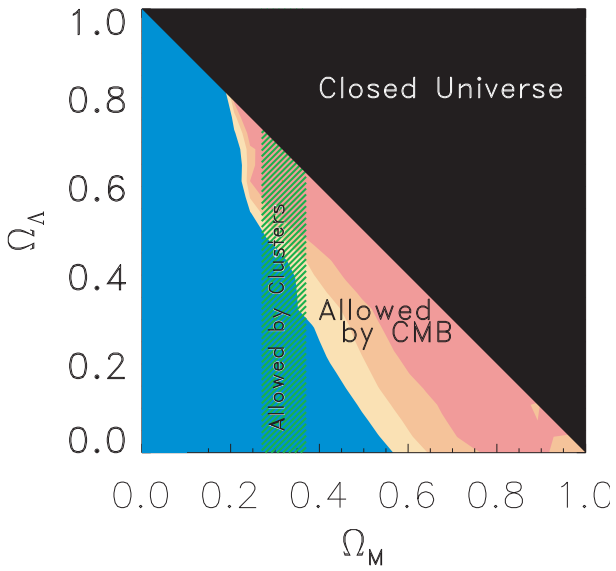

Figure 7: Constraints on the vacuum and matter densities in the universe. Shown are one-, two-, and three-sigma regions allowed by the CMB and best-fit region of the matter density from clusters.

One way to depict this information that has been popularized by the supernovae teams is to plot the constraints in a space with vacuum energy and matter density as the two parameters. As shown in Fig. 7, the strongest constraints on the matter density come from observations of baryons and dark matter in clusters of galaxies. We obtain contours in this plane from the CMB shown in Fig. 7. Note that the flat line runs diagonally from top left to bottom right and is strongly favored by the CMB. The data are so powerful that some discrimination is appearing along this line. Very large values of Ω_Λ are disfavored, and, at a much smaller statistical level, so is $(\Omega_\Lambda = 0, \Omega_{matter} = 1)$. The main result, though, is that the intersection of the regions allowed by clusters and the CMB is at $\Omega_\Lambda \sim 0.6$, in remarkable agreement with the high redshift supernovae results.

This concludes my arguments for the first three claims advanced in the introduction. Undoubtedly, many of you have heard them in various forms over the past few years. Now let us turn to the hardest claim to justify, the claim that we should indeed believe the powerful conclusions of the CMB results. I will focus on two arguments. First, one might be worried about the possibility that the weight of these conclusions rests on one experiment, and one experiment might be wrong. The remaining panels of Fig. 6 show that this is not a problem. We have tried removing any one data set to see how our conclusions about Ω are affected; in all cases, the conclusion stands. We even tried removing pairs of data sets and again saw no change. One has to argue for a bewildering set of coincidences if one were to disbelieve the statistical conclusions.

The second class of arguments hinges on something that was not possible until very recently. Ultimately, skeptics will be convinced if different experiments get the same signal when measuring the same piece of sky. Until now, this test has been difficult to carry out for two reasons. First, at least at small scales, only a very small fraction of the sky has been covered, so there has been little overlap. This has changed a bit over the last year and obviously will change dramatically in the coming years. Second, different experiments observe the sky differently: they smooth with different beam sizes and use different chopping strategies to subtract off the atmosphere. Recently, we have developed techniques which "undo" the experimental processing, thereby allowing for easy comparisons between different experiments [8].

To illustrate the map-making technique, let us model the data D in a given experiment as

$$D = BT + N \tag{1}$$

where T is the underlying temperature field; B is the processing matrix which includes all smoothing and chopping; and N is noise which is assumed to be Gaussian with mean zero and covariance matrix C_N. To obtain the underlying temperature field T, we need to invert the matrix B. This inversion is carried out by constructing the estimator $\hat{T}$ which minimizes the χ^2:

$$\chi^2 \equiv (D - B\hat{T})C_N^{-1}(D - B\hat{T}). \tag{2}$$

We find

$$\hat{T} = \tilde{C}_N B C_N^{-1} D. \tag{3}$$

This estimator will be distributed around the true temperature due to noise, where the noise covariance matrix is

$$\tilde{C}_N \equiv \left\langle (\hat{T} - T)(\hat{T} - T) \right\rangle = \left(B^T C_N^{-1} B \right)^{-1}. \tag{4}$$

Not surprisingly, maps made from modulated data are extremely noisy. By definition, modulations throw out information about particular modes. For example, a modulation which takes the difference between the temperature at two different

points clearly cannot hope to say anything useful about the sum of the temperatures. So looking at a raw, demodulated map is a very unenlightening experience. There are two ways of getting around this noisiness and producing a reasonable-looking map. Before I discuss them, though, it is important to point out that even without any cleaning up, the maps in their raw noisy states are very useful. They can be analyzed in the same manner as the modulated data, with the huge advantage that the signal covariance matrix is very simple to compute. Previously, calculating the signal covariance matrix required doing a multi-dimensional integral for every covariance element. In the new "map basis," the signal covariance matrix simplifies to

$$\langle T_i T_j \rangle = \sum_L \frac{2L+1}{4\pi} P_L(\cos(\theta_{ij})) C_L. \tag{5}$$

Indeed, one way to think of a map is that it is the linear combination of the data for which the signal (and therefore its covariance) is independent of the experiment. The noise covariance Eq. (4) accounts for all the experimental processing.

Nonetheless, we would like to produce nice looking maps, if only to use to compare different experiments. One way to do this is to Wiener filter the raw map, multiplying the estimator in Eq. (3) by $C_T(C_T + \tilde{C}_N)^{-1}$, which is roughly the ratio of signal to (signal plus noise). Noisy modes are thereby eliminated from the map.[2]

An example of the Weiner filter is shown in Fig. 8. The two panels are two different years of data taken by the MSAM experiment [8]. It is well established that the two data sets are consistent [26, 27]. I show these because it is important to get a sense of what constitutes good agreement. Most of the features are present in both experiments, but there are several—for example the hot spot at RA $\simeq -135$ and the cold spot at RA $\simeq -120$ in the 1992 data—which do not have matches. This is not surprising: the same regions in the 1994 experiment may have been noisy so that, in the process of throwing out the noise, the Wiener filter also eliminated the signal. Another feature of these maps which is readily apparent is that they only have information in one direction. There is very little information about declination. As a corollary, the exact shapes of the hot and cold spots in the two data sets do not agree, nor should they. Another way of saying this is to point out that there are some modes remaining in the maps which are noisier than others (*e.g.* the shapes of the spots are noisy modes). Is there a more systematic way to eliminate noise than the Wiener filter?

A different technique is illustrated in Fig. 9 in a setting which is more challenging. Whereas the two years of MSAM data both had very high signal to noise and both were taken with the same instrument at the same frequencies, the two years of Python data [10, 28] shown were taken with completely different instruments (bolometers in

[2] A simple way to derive this factor is to put in a Gaussian prior in for the signal T, effectively adding to the χ^2 in Eq. (2) the term $TC_T^{-1}T$. Minimizing this new χ^2 leads to the Wiener factor.

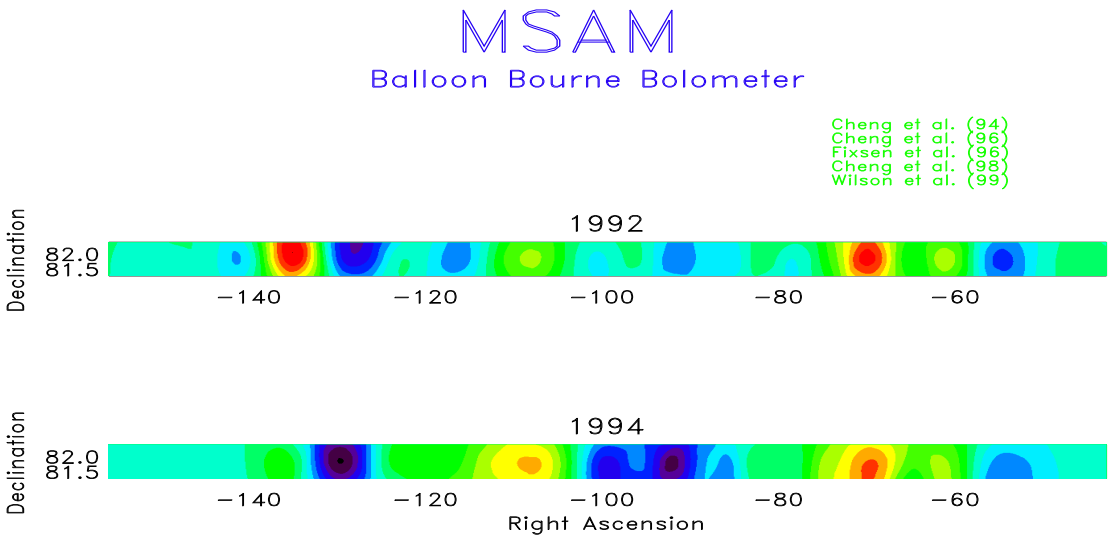

Figure 8: Maps of two years of data from the MSAM experiment. Note that, due to the horizontal scanning strategy, there is very little information in the vertical direction.

the 1995 data and HEMTs in the 1997 data) at completely different frequencies (90 vs. 40 GHz). They are therefore subject to a completely different set of systematics and foregrounds. Further, the 1997 data is part of a much larger region of sky covered; to get very large sky coverage, the team sacrificed on signal to noise per pixel. Therefore, the signal to noise ratios of the two years are very different.

To make the maps in Fig. 9, I started with the raw maps and then decomposed the data into signal to noise eigenmodes [29]. By ordering the data in terms of signal to noise, we can gradually and systematically eliminate the noisiest modes. This has already been done on the 1995 data in the bottom panel. The top panel contains all modes with S/N greater than about 1.5. As indicated by the bars, there are very few such modes, on the order of ten. Nonetheless, many features are found in both maps. There is the triplet of cold spots extending diagonally from -15° to -10° azimuth. There is the cold spot at -4° azimuth, and the hot spot at 0°, and then finally the cold spot at the far right. It appears to me that these two maps agree—after far too many hours staring at them—as well as the MSAM maps. In fact, the β test advocated by Bond, Jaffe, and Knox [27] confirms this agreement.

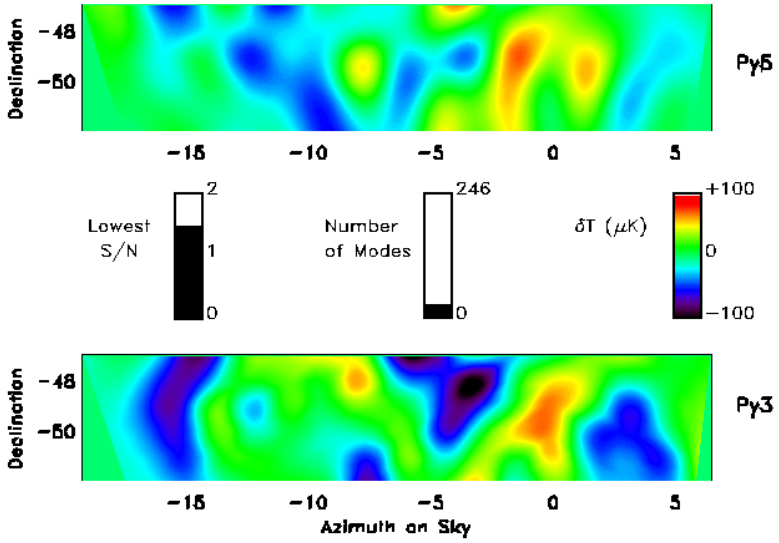

Figure 9: Two years of Python data. The bottom panel shows data from 1995; the top panel contains much noisier 1997 data. In both cases, noisy modes have been eliminated so that only modes with S/N greater than 1.5 are retained. The middle bar shows that (for Python 97) there are of order 15 such modes out of 246 pixels in the region.

5 Conclusion

The first acoustic peak in the CMB has been detected at an angular position corresponding to that expected in a flat universe. This confirms the fundamental prediction of inflation that the universe is flat. It also offers independent evidence for the existence of dark energy with negative pressure. This is but the first of many grand results we expect to come out of the CMB over the coming decade.

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Discussion

Sherwood Parker (University of Hawaii): Inflation is motivated, in part, by the uniformity of the black body radiation coming from places that did not have time to communicate since the origin of the expanding universe. Is there any data that would exclude the following possibility: (1) the universe is much, and possibly infinitely, larger than the part we can see; (2) the universe is much, possibly infinitely,

older than 15 billion years; and (3) there was a gravitationally driven infall of part of it that was reversed at a high energy by phenomena beyond the reach of present experiments?

Dodelson: It would be interesting to work out the predictions of theories other than inflation. At present, the best alternative is topological defects, which fare very poorly when confronted with the data. If you can work out some prediction of your model, it would be wonderful: we need alternatives to inflation if only to serve as strawmen. Regarding your specific model, I don't know what you mean by larger than we can see: the standard cosmology has this built in. If the age was much older than 15 billion years, one would wonder why the oldest objects are roughly 10-15 billion years old.

Jon Thaler (University of Illinois): If Ω_Λ is 70% and Ω_M is 30%, do we still need non-baryonic dark matter?

Dodelson: Yes, due to limits from nucleosynthesis and structure formation.