# PELLISSIER H5 HYDROSTATIC LEVEL*

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## 1. INTRODUCTION

Conventional spirit leveling using double scale invar rods has been in use at SLAC for some time as the standard method of obtaining very precise height difference information. Typical accuracy of +/- 100 µm and better can routinely be achieved. Procedures and software have evolved to the point where the method is relatively fast and reliable. However, recent projects such as the Final Focus Test Beam have pushed the requested vertical positioning tolerances for alignment of quadrupoles to the 30 µm level. It is apparent that conventional spirit leveling cannot achieve this level of accuracy. To meet the challenge, the alignment group contracted with Pellissier, Inc. to develop a portable hydrostatic leveling system. The H5 grew out of this development effort and is expected to provide the needed accuracy and ease of use required for such vertical positioning projects.

Fig. 1 H5

The H5 hydrostatic level is a portable instrument that under ideal operating conditions will provide elevation differences with an accuracy of +/- 5 µm over double leg closed loop surveys. The H5 incorporates several features that eliminate problems common with hydrostatic leveling, primarily errors due to thermal gradients along the fluid tube. It utilizes self-checking software and automatic water level detection to reduce observational errors. Design features also have made the instrument reasonably quick and easy to operate when used on a flat surface. The instrument can be adapted for use in a wide variety of environments by using support fixtures and brackets. The H5 is robust and operators require little training to become proficient in its use. It has been successfully employed on several projects including the FFTB project at SLAC, as well as the Green Bank Telescope project for the NRAO and the SSC project in Texas.

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The co-authors wish to dedicate this paper to the memory of Pierre Pellissier. Throughout his life, Pierre strove to improve hydrostatic leveling techniques. His work culminated in the development of the H5. Unfortunately, it wasn't meant for him to experience the fruits of his work.

## 2. HYDROSTATIC LEVELING

Principles of hydrostatic leveling have been known for a long time. The Egyptians are known to have filled ditches with water to transfer elevations. Also, they knew about the principle of communicating pipes. (The Romans quite obviously did not know about this principle, otherwise they would not have built the wonderful aqueducts such as the Pont du Gard.)

Today, hydrostatic leveling is used in a wide variety of applications, but typically only as static installations on a very large scale. Recent advances in electronics and sensing technology have allowed for the design of relatively compact static systems that are capable of very high accuracy. These systems are used for monitoring position change and long term tilt effects in accelerator alignment and crustal deformation studies. Some attempts have been made to design and implement a portable system; but these systems have typically been cumbersome to move and difficult to use. The H5 is the only portable system that provides accuracy much greater than conventional precise spirit leveling while maintaining ease of operation and a relatively high degree of efficiency.

### 2.1 Methods

Hydrostatic levels are usually implemented using one of three approaches: the height-transfer method, which is the one commonly associated with hydrostatic leveling and employed in the H5, the pressure-transfer method, and the weight-comparison method. All three methods employ two reservoirs or "wells" connected by a flexible tube. The instruments are filled with some fluid, most often water due to the ease of its handling, environmental friendliness, well known properties, and low cost.

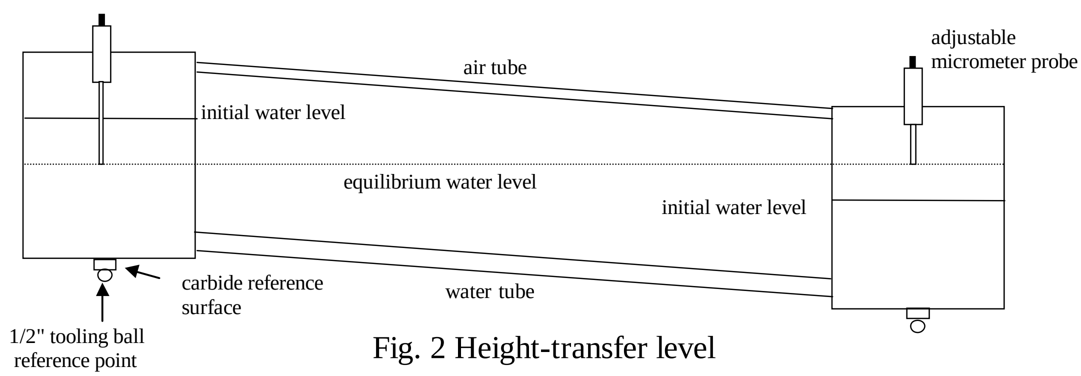


Fig. 2 Height-transfer level

A tube that connects the air volumes over the well water surfaces is necessary in the height-transfer and weight-comparison method. This ensure the atmospheric pressures over both wells are equal and the water is then free to flow between the two wells. Any change in the elevation between the wells causes water to flow from one into the other, creating a new equilibrium surface. The surface of the water at each well will conform to the same gravitational equipotential surface or "level" surface.

With the height-transfer method, changes in the water level height relative to the well are measured. This is most commonly accomplished by optical micrometers, mechanical micrometers, or by inductive or capacitive proximity gauges. By taking the difference of the micrometer readings, a value for the height difference is obtained. This is only a relative measurement and has no meaning in an absolute sense since the two micrometers are not calibrated to each other. In order to get useful information, a series of height difference measurements are made between reference points forming an elevation network. The elevation differences are observed between points such that a closed loop of elevation differences has been measured in a manner identical to conventional leveling operations.

The pressure-transfer method implements the principle that equipotential surfaces within the fluid are at constant hydrostatic pressure to derive height difference information. "Pressure transducers within the fluid will measure a height difference h related to fluid density and pressure and the acceleration due to gravity. It is made up of a flexible tube containing a fluid of uniform and known density, terminated at each end by a pressure gauge. The height difference is determined from the difference in pressure between the ends."[1]

The weight-comparison method employs weight or load sensor cells which accurately determines the weight of each well. Since the height of the liquid column in each well is a linear function of the measured weight, elevation changes can be measured very accurately.

## 2.2 Sources of error

Hydrostatic leveling by the height-transfer method is subject to many error sources that must be dealt with in order to achieve micron level results. The most significant errors are: bubbles in the fluid line, temperature variations of the system, accelerations of the fluid, differing gas pressures above the wells, and observational.

### 2.2.1 Bubbles in the fluid line

Bubbles cause two problems. If small bubbles are present in the fluid line, the instrument will behave similarly as if the tube was not insulated. With a change in temperature, the small air bubbles expand at a greater rate than the water. This causes rapid variations in the surface level of each well. A large bubble, one that blocks the entire tube, poses a much greater problem. Surface tension forces cause the bubble to resist water pressure differences of up to a millimeter or two. The H5 uses a pump to nullify this error. By circulating water throughout the entire system, any bubbles will be pumped out of the fluid line.

### 2.2.2 Temperature variations of the system

The largest source of error is due to thermal gradients in the fluid tube that cause the density of the fluid to be non-uniform. (e.g., If two wells are positioned accurately at the same elevation but have different water temperatures, the water levels would be at different heights. This difference stems from the fact that the density of water changes with temperature. Hence, water of different temperature will form water columns of different heights.) To alleviate this problem,

Fig. 3 Internal view of master unit, pump at lower right

the H5's pump (see Fig. 3) circulates the water throughout the entire system. It also utilizes an insulated fluid tube so that not only can a uniform temperature of the fluid be forced, but is also maintained long enough to obtain accurate data (3-5 minutes).

Differential thermal expansion of the wells can also be safely ignored since the pump not only forces a uniform temperature of the water, but also other strategic internal parts including the wells. The H5 has a well coefficient of less than 1 µm per degree Celsius and the difference in well temperatures is never greater than 0.5 degree Celsius.

### 2.2.3 Accelerations of the fluid

When the valves at each well are opened, water travels back and forth from one well to the other through the fluid tube gradually settling to equilibrium. This action can be described as a damped pendulum. Before any accurate readings can be made, a calculated time is allowed to pass. This calculation is based on the kinematic viscosity of the fluid, gravity, length and diameter of the tube, and the diameter of the wells. The key to success is to have the system settle before the water in the system gets a chance to change temperature. The H5's well diameter and its tube length and diameter were constructed to optimize the settling time. It takes approximately one minute for the water to reach equilibrium.

A hydrostatic height-transfer level must be stationary to function properly. If a substantial force is applied to any part of the instrument while the fluid is settling, pressure fluctuations occur and result in erroneous data. The operators must make certain that the integrity of the setup will not be jeopardized during an observation. The H5 is therefore limited, as all hydrostatic levels, to differential leveling of stationary monuments only.

### 2.2.4 Differing gas pressures above the wells

The water in the wells of a hydrostatic height-transfer level will not lie in the same equipotential surface if gas pressures above them are not the same. All hydrostatic height-transfer levels, including the H5, have two tubes, a fluid tube and a gas tube. The gas tube equalizes the gas pressure in both wells by linking the two head spaces together, allowing the water in the fluid tube to move freely from one well to the other.

### 2.2.5 Observational errors

Observational errors chiefly involve the accurate detection of the water surface. Water will climb the inside walls of the wells due to capillary rise and an errant measurement will result. The two variables in determining this rise are the diameter of the wells and the surface tension of the fluid. A surfactant is added to the water in the H5 reducing surface tension and the wells have been designed with a diameter so that the calculated rise is negligible.

Wavelets will form on the surface of the fluid and can cause several micron-sized-errors. This can be caused by vibrations from a non-stable setup or from driving the probe (see Fig. 4) toward the fluid surface. As the probe approaches the fluid surface, a wavelet will strike the tip. This subsequent contact will not be observed at the surface, but at the crest of the wave. Reduction of the surface tension minimizes the effect. The probe system of the H5 is designed to (see Fig. 5) cause nearly no vibrations resulting in a minimal formation of wavelets. In addition, since the process is automated, the resulting effect is constant and falls out by the differential nature of the measurement.

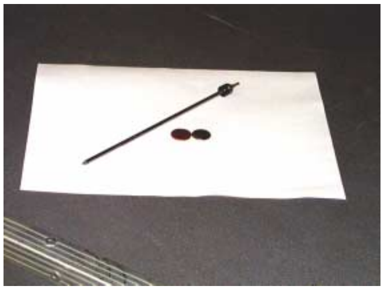
Fig. 4 Platinum tipped probe

Manual detection of the fluid surface and recording of the data introduce operator biases. The H5 automates this process. As the probe comes into contact with the water, an electronic connection is made between the probe tip and the water. A switch is triggered and the motor is turned off. A data collector interfaces each micrometer and records the data digitally.

Even though the circulating pump achieves a uniform temperature of the water, thermal expansion resumes during the 3-5 minutes of observation. Because the readings were initiated with equal well temperatures, it is reasonable to believe that the rate of temperature change will be equal in both wells with respect to time. Readings of both wells must be made at approximately the same instant (+/- 5 seconds) or thermal expansion in one well may be greater or less than the other. The H5 has a clutch on each motor that allows the operators to time their measurement within the tolerance specified.

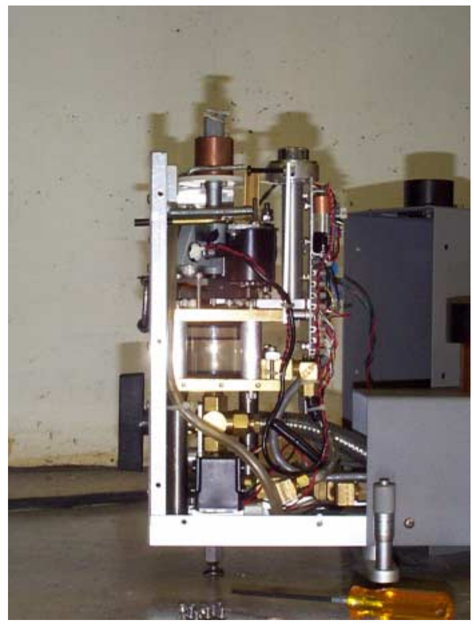
Fig. 5 Internal view of slave unit, motor is above and to the right of water well

## 3. MEET THE H5

The H5 is a compact and very accurate hydrostatic level, easily up to ten times better than current optical methods. Less than two hours of training time is required for two operators to be proficient in its use.

### 3.1 Description of the hardware

The H5 hydrostatic level is composed of a master unit, a remote or “slave” unit, and a flexible hose assembly that connects the two reservoirs and air volumes. Through experience, the optimal length of hose has been determined to be about 15 meters. Using a longer hose will begin to cause larger thermal errors than can be compensated by the instruments design features, and the instrument becomes less and less portable.

The master unit is larger than the slave unit. It houses not only its own well, but also a water reservoir for the system and a pump that circulates water throughout both units. As the water circulates, it comes into contact with all components of the system that are susceptible to thermal effects. This equalizes the temperature of the system while the pump is running and circulating water. When the pump is turned off to take a set of observations, a time period of 3-5 minutes is available during which time the temperature of the system remains stable. This period of thermal stability allows the operator to take observations free from all thermally induced errors.

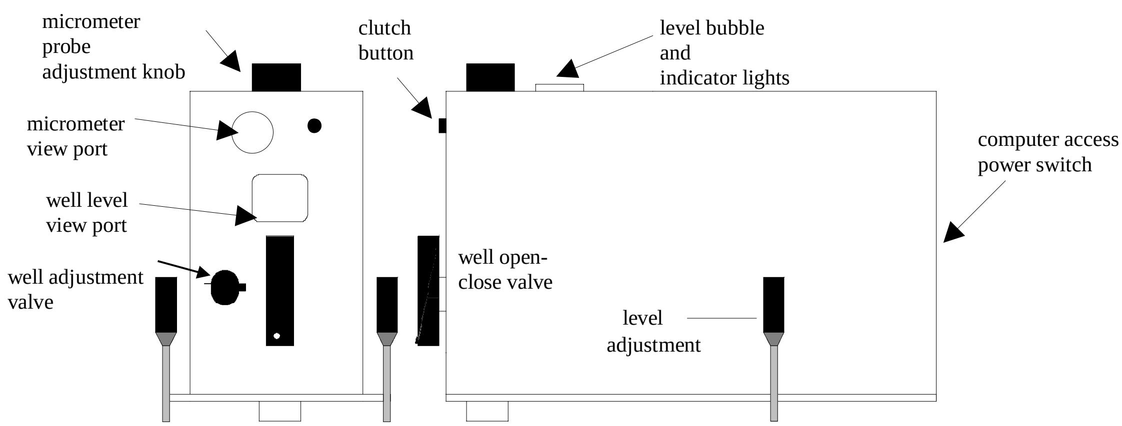


Fig. 6 H5 hydrostatic level master unit

Table 1 H5 specific

| | |
|---|---|
| Accuracy (each setup) | +/- 5 µm |
| Repeatability (each setup) | +/- 2 µm |
| Vertical range (each setup) | +/- 25 mm |
| Master end | 9" w x 22" l x 14" h, 40 lb. |
| Slave end | 5" w x 16" l x 14" h, 15 lb. |
| Tube length | 15 m |
| Tube assembly o.d. | 60 mm |
| Total weight | 125 lb. |

### 3.2 Using the H5

The observations are taken in a manner comparable to performing differential leveling with a conventional sight level. The master unit could be considered the "level" and the slave unit the “rod”. The master unit is set up at a point within range of both the backsight and foresight points. Since the vertical range of the instrument is only 25 mm for any single observation, some care is required to ensure the setup is within this range of measurement.

Experience has shown that the best way to ensure the master and slave units are within range of each other is to perform a coarse conventional vertical leveling run over the points of interest prior to using the H5. Once these elevations are known within a few millimeters, the positions for the master unit are defined and the ideal elevation of the master unit at each of these points can be calculated. With this information, it is a simple matter to set the elevation of the master contact point either by using shim blocks or in the case of the FFTB project, an adjustable invar rod that acts as a hard contact point for the master. Typically a 0.5” diameter tooling ball is used as the contact point, but since the actual surface of the carbide contact is flat, the instrument can be modified to sit on a spherical surface with a larger diameter if necessary.

The slave unit is set up on the backsight point and a data set is observed. This will establish the HI (instrument height) of the master unit. The slave unit is then moved to the foresight station without moving the master unit and another data set is observed. From this second data set the height difference between the backsight and foresight stations is computed. By repeating this procedure in reverse direction, a loop closure is achieved and the height differences obtained should be the same within +/- 5 µm.

## 4. LABORATORY TESTS

To provide an indication of the reliability of the H5, a test network was set up at SLAC in a controlled and stable environment. A network of points consisting of 0.5” tooling balls was established on the floor of the alignment calibration laboratory. It is built into the foundation of the Linear Accelerator, which is approximately 10 meters below ground level. In this location the lab has a very stable thermal environment and is well protected from ground motion because it sits on the base rock layer. Precision, repeatability, and accuracy of the H5 were tested.

### 4.1 Precision

The lab itself is a long room, approximately 6 meters wide and 40 meters long. Ten points were established in a 10 meter by 6 meter rectangle located in the center of the lab. Using the H5, each and every delta elevation was measured. This created an observation set of 45 observations and 10 unknowns, the unknowns consisting of the elevations of each point rather than the height differences between points.

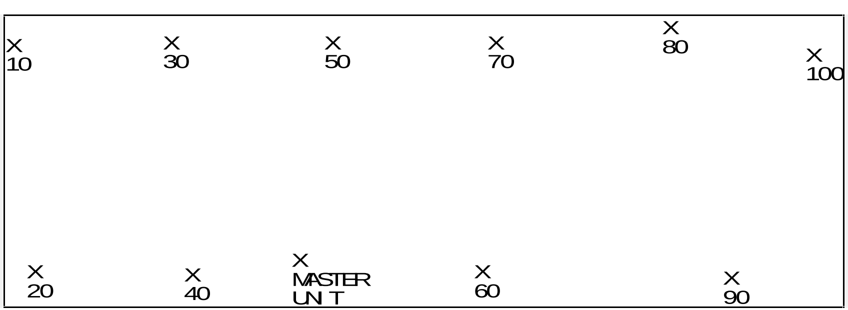


Fig. 7 Network layout

Table 2  Precision test output

| | |
|---|---|
| Degree of freedom | 36 |
| Variance component for height difference | 0.353556 |
| Sigma a priori | 1.000000 |
| Sigma a posteriori for height difference | 0.594605 |

See Appendix A for raw data

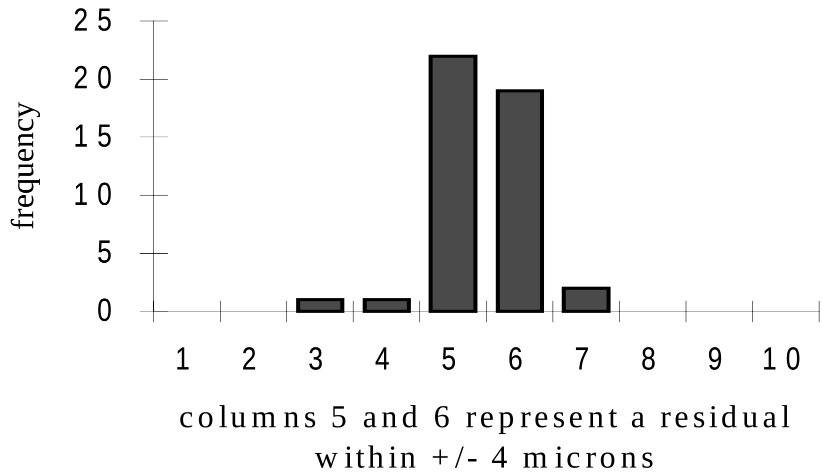


Fig. 8  Histogram of height differences residuals

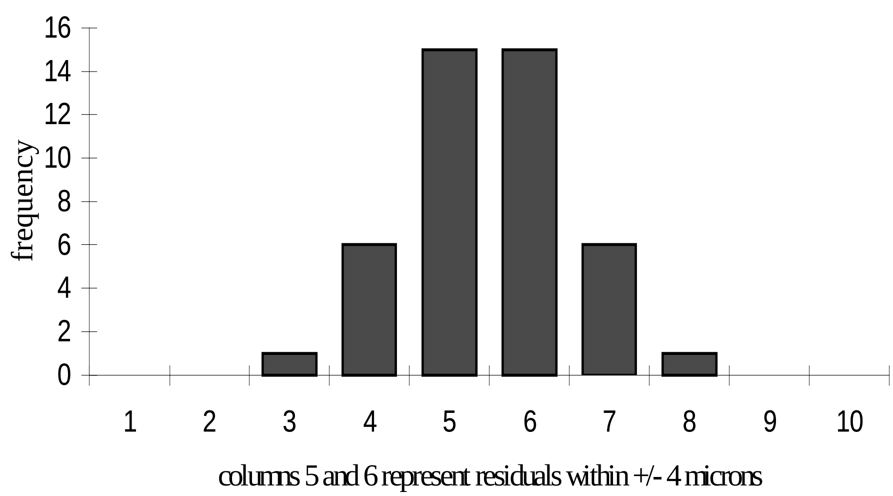


Fig. 9  Histogram of normal distribution

## 4.2 Repeatability

To test the instrument's repeatability, two more points were established in the lab. They were placed approximately 30 meters apart and at each end of the room. The points again consisted of 0.5" tooling balls and a third point was set in the middle of the two to accommodate the master unit. Fifteen level loops were measured, i.e. 30 differences in elevation.

Table 3 Repeatability test output

| | |
|---|---|
| Average dh | 1.476 mm |
| Standard deviation | 3 µm |
| Maximum observation | 1.483 mm |
| Minimum observation | 1.470 mm |
| Normal distribution from 1.471 to 1.481 mm | 90% |

See Appendix B for raw data

## 4.3 Accuracy

To test for accuracy, it was necessary to have a reference of higher accuracy. In the micron realm, this becomes very difficult. The only instrument available to the authors that is capable of micron measurements is SLAC's Coordinate Measurement Machine (CMM), a Leitz PMM 12106. A fixture measuring 1" x 12" x 24" was constructed of aluminum. Three holes were drilled and tapped to accommodate bolts used as feet and five 1.5" tooling ball cups were mounted on the surface. Some care was definitely required to clamp the plate without warping it. First, the H5 measured the delta elevations of the five tooling balls and then the fixture was sent to the CMM room. The adjusted H5 data was then transformed into the CMM coordinate system.

The procedure for the H5 was to backsight one of the five balls, foresight the remaining four, and then foresight the original backsight to close the loop. This procedure was repeated for each ball and resulted in 25 observations.

Table 4 H5 fixture output

| | |
|---|---|
| Degree of freedom | 21 |
| Variance component for height difference | 0.761905 |
| Sigma a priori | 1.000000 |
| Sigma a posteriori for height difference | 0.872872 |

See Appendix C for raw data

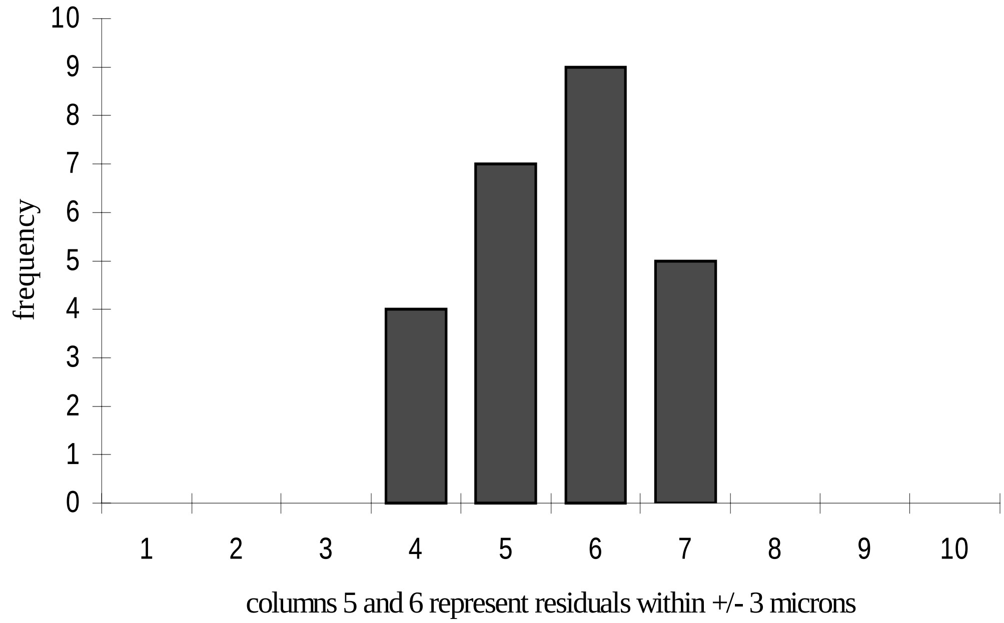


Fig. 10 Histogram of height differences residuals

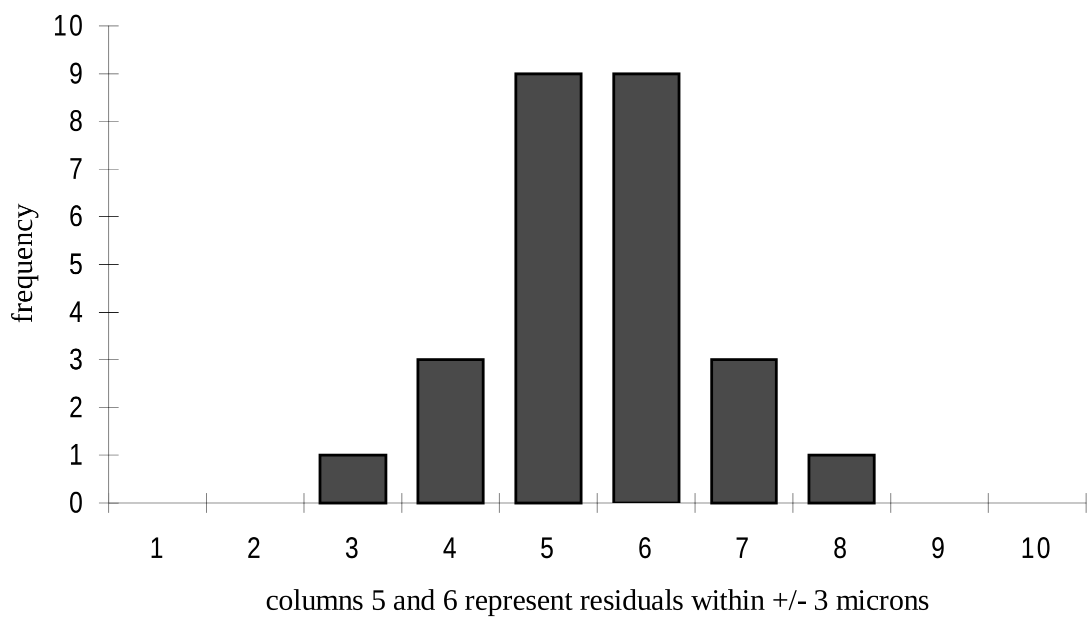


Fig. 11 Histogram of normal distribution

The fixture was then measured by the CMM and the H5's output was transformed into this system. Of the CMM data, all fiducials were held fixed in elevation, one fiducial was held in two dimensions, and one fiducial was held in all three dimensions. This allowed the plate to translate in the horizontal plane and rotate about the axes.

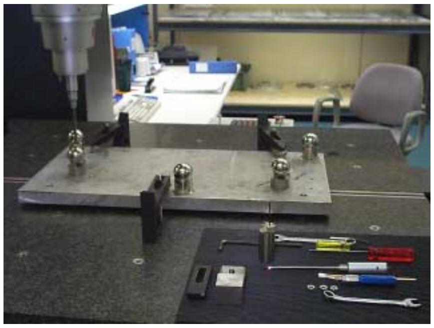

Fig. 12 CMM and fixture

Table 5 Transformation output

| Transformed H5 fiducial | residual |
|---|---|
| 1 | .002 |
| 2 | -.002 |
| 3 | -.002 |
| 4 | .004 |
| 5 | -.002 |
| A posteriori standard deviation | .002 |

See Appendix C for transformation data and parameters

### 4.4 Results

The manufacturer's claim of accuracy of +/- 5 µm appears to be a reliable number, maybe even a little conservative. But the claim of a precision good to +/- 2 µm could not be confirmed, although it came close to the measured +/- 3µm. However, if the maximum and minimum measurements are removed from the sample, 1.470 and 1.483, the standard deviation is reduced to 2 µm. Perhaps a larger sample would prove these observations to be extreme (95% uncertainty) and hence rejected.

## 5. FIELD EXPERIENCE

For the Final Focus Test Beam project at SLAC, the alignment objective was to position quadrupole centers on a straight line within 30 microns. This required a more precise method for obtaining height differences than conventional spirit levels. So, the H5 was selected to provide the needed precision. The major challenge of using the H5 on a beam line is to position the instrument at the correct elevation and maintain its stability. The FFTB magnets are positioned approximately 2 meters above the floor. In order to position

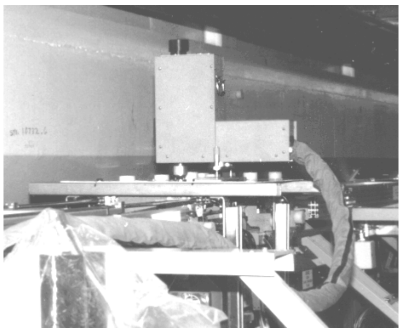

Fig. 13 Slave unit on FFTB quadrupole support frame

the H5 slave and master units at this height, specially fabricated support frames and wall brackets were used. A uni-strut frame was erected around each quadrupole. This provided the slave unit with a resting place. The top plate, where the slave unit sits, had cutouts to match the pattern of the fiducial spheres on which the slave would take its observations.

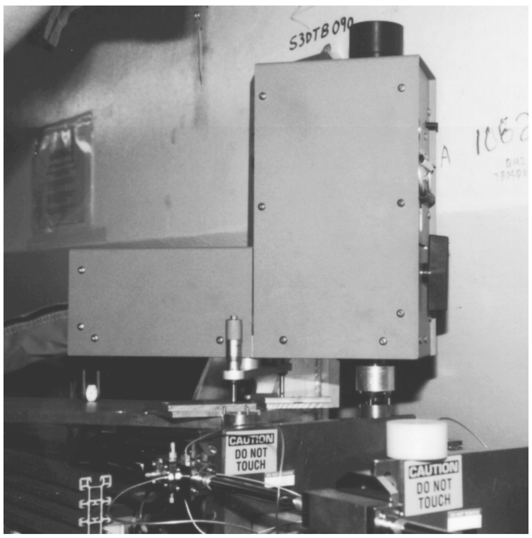


Fig. 14 Slave unit on FFTB support frame

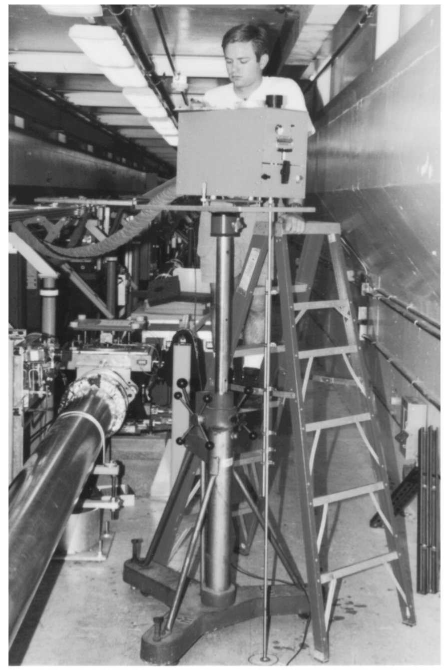

Fig. 15 Master unit on FFTB support

The data collected turned out to be better than expected. The least count of the adjustment was 10 µm and the computed residuals were all 0 or 10 µm. The FFTB is situated with the upstream end located in the beam switchyard housing and the downstream end covered by cement blocks on a parking lot. Needless to say, temperature gradients are quite severe from one end to the other. The tests in the lab lacked this variable so the thermal error inhibiting features of the H5 were not truly tested. However, the output from the FFTB run leads one to believe that in a test setting, not practical, +/- 5 µm would be obtainable.

## 6. CONCLUSION

The PELLISSIER H5 has definitely filled the role of a precise vertical level at SLAC. Even though a little planning is needed when performing level loops on beam lines (e.g., construction of support stands), the accuracy obtained from the H5 far out weighs these minor nuisances. Once a friendly data collection environment has been established, accurate data can be collected at a fairly fast pace. The H5 is currently not being manufactured. This is a shame because it would certainly be a valuable tool in any laboratory's survey and alignment department.

## 7. APPENDIX A

Data from the precision test:

| Differences | | dh(m) | Residuals | Standard Deviation of the residual |
|---|---|---|---|---|
| 10 | 20 | 0.011361 | 0.000005 | 0.000004 |
| 10 | 30 | 0.003859 | -0.000000 | 0.000004 |
| 10 | 40 | 0.024305 | 0.000002 | 0.000004 |
| 10 | 50 | 0.001620 | -0.000009 | 0.000004* |
| 10 | 60 | 0.016175 | 0.000001 | 0.000004 |
| 10 | 70 | 0.015974 | -0.000002 | 0.000004 |
| 10 | 80 | -0.005172 | -0.000001 | 0.000004 |
| 10 | 90 | 0.023546 | 0.000004 | 0.000004 |
| 10 | 100 | 0.004382 | 0.000000 | 0.000004 |
| 20 | 30 | -0.007498 | -0.000001 | 0.000004 |
| 20 | 40 | 0.012948 | 0.000002 | 0.000004 |
| 20 | 50 | -0.009729 | -0.000002 | 0.000004 |
| 20 | 60 | 0.004816 | -0.000001 | 0.000004 |
| 20 | 70 | 0.004622 | 0.000002 | 0.000004 |
| 20 | 80 | -0.016525 | 0.000002 | 0.000004 |
| 20 | 90 | 0.012187 | 0.000001 | 0.000004 |
| 20 | 100 | -0.006973 | 0.000001 | 0.000004 |
| 30 | 40 | 0.020447 | 0.000004 | 0.000004 |
| 30 | 50 | -0.002231 | -0.000001 | 0.000004 |
| 30 | 60 | 0.012314 | -0.000000 | 0.000004 |
| 30 | 70 | 0.012116 | -0.000001 | 0.000004 |
| 30 | 80 | -0.009030 | 0.000001 | 0.000004 |
| 30 | 90 | 0.019680 | -0.000002 | 0.000004 |
| 30 | 100 | 0.000522 | -0.000001 | 0.000004 |
| 40 | 50 | -0.022671 | 0.000002 | 0.000004 |
| 40 | 60 | -0.008129 | 0.000000 | 0.000004 |
| 40 | 70 | -0.008330 | -0.000003 | 0.000004 |
| 40 | 80 | -0.029468 | 0.000006 | 0.000004* |
| 40 | 90 | -0.000760 | 0.000001 | 0.000004 |
| 40 | 100 | -0.019919 | 0.000002 | 0.000004 |
| 50 | 60 | 0.014545 | 0.000001 | 0.000004 |
| 50 | 70 | 0.014344 | -0.000003 | 0.000004 |
| 50 | 80 | -0.006804 | -0.000003 | 0.000004 |
| 50 | 90 | 0.021912 | -0.000001 | 0.000004 |
| 50 | 100 | 0.002749 | -0.000004 | 0.000004 |
| 60 | 70 | -0.000198 | -0.000000 | 0.000004 |
| 60 | 80 | -0.021347 | -0.000002 | 0.000004 |
| 60 | 90 | 0.007369 | 0.000001 | 0.000004 |
| 60 | 100 | -0.011790 | 0.000001 | 0.000004 |
| 70 | 80 | -0.021152 | -0.000005 | 0.000004 |
| 70 | 90 | 0.007566 | 0.000000 | 0.000004 |
| 70 | 100 | -0.011595 | -0.000001 | 0.000004 |
| 80 | 90 | 0.028710 | -0.000003 | 0.000004 |
| 80 | 100 | 0.009554 | 0.000001 | 0.000004 |

| | | | | |
|---|---|---|---|---|
| 90 | 100 | -0.019158 | 0.000002 | 0.000004 |

# 8. APPENDIX B

Data from the repeatability test:

| Leg | dh(mm) |
|---|---|
| 1 2 | -1.474 |
| 2 1 | 1.475 |
| 1 2 | -1.475 |
| 2 1 | 1.476 |
| 1 2 | -1.475 |
| 2 1 | 1.480 |
| 1 2 | -1.475 |
| 2 1 | 1.472 |
| 1 2 | -1.476 |
| 2 1 | 1.472 |

| Leg | dh(mm) |
|---|---|
| 1 2 | -1.477 |
| 2 1 | 1.483 |
| 1 2 | -1.474 |
| 2 1 | 1.470 |
| 1 2 | -1.479 |
| 2 1 | 1.471 |
| 1 2 | -1.477 |
| 2 1 | 1.478 |
| 1 2 | -1.481 |
| 2 1 | 1.475 |

| Leg | dh(mm) |
|---|---|
| 1 2 | -1.475 |
| 2 1 | 1.481 |
| 1 2 | -1.472 |
| 2 1 | 1.477 |
| 1 2 | -1.477 |
| 2 1 | 1.474 |
| 1 2 | -1.477 |
| 2 1 | 1.478 |
| 1 2 | -1.475 |
| 2 1 | 1.479 |

# 9. APPENDIX C

Data from the accuracy test

| Differences | | dh(m) | Residual | Standard Deviation of the residual |
|---|---|---|---|---|
| 5 | 4 | 0.003030 | 0.000000 | 0.000003 |
| 5 | 3 | 0.001967 | 0.000001 | 0.000003 |
| 5 | 2 | 0.003130 | 0.000004 | 0.000003 |
| 5 | 1 | 0.000654 | 0.000000 | 0.000003 |
| 5 | 5 | 0.000000 | 0.000000 | 0.000003 |
| 4 | 5 | -0.003025 | 0.000005 | 0.000003 |
| 4 | 3 | -0.001064 | -0.000000 | 0.000003 |
| 4 | 2 | 0.000096 | -0.000000 | 0.000003 |
| 4 | 1 | -0.002379 | -0.000003 | 0.000003 |
| 4 | 4 | 0.000000 | 0.000000 | 0.000003 |
| 3 | 2 | 0.001160 | 0.000000 | 0.000003 |
| 3 | 1 | -0.001312 | 0.000000 | 0.000003 |
| 3 | 5 | -0.001966 | 0.000000 | 0.000003 |
| 3 | 4 | 0.001065 | 0.000001 | 0.000003 |
| 3 | 3 | 0.000000 | 0.000000 | 0.000003 |
| 2 | 1 | -0.002474 | -0.000002 | 0.000003 |
| 2 | 5 | -0.003123 | 0.000003 | 0.000003 |
| 2 | 4 | -0.000094 | 0.000002 | 0.000003 |
| 2 | 3 | -0.001155 | 0.000005 | 0.000003 |
| 2 | 2 | 0.000002 | 0.000002 | 0.000003 |
| 1 | 2 | 0.002476 | 0.000004 | 0.000003 |
| 1 | 3 | 0.001309 | -0.000003 | 0.000003 |
| 1 | 4 | 0.002373 | -0.000003 | 0.000003 |
| 1 | 5 | -0.000656 | -0.000002 | 0.000003 |
| 1 | 1 | -0.000003 | -0.000003 | 0.000003 |

Transformation:

Point 1 was held fixed in X and Y. Point 5 was held fixed in Y. All the points were held fixed in Z. The bare minimum was held fixed in the horizontal plane and the remaining points were allowed to shift with the transformation. Column W represents the adjusted H5 output.

Common points (mm)

| | H5 | | | CMM | | |
|---|---|---|---|---|---|---|
| name | U | V | W | X | Y | Z |
| 1 | .000 | .000 | -1.101 | .000 | .000 | .000 |
| 2 | 139.331 | 47.719 | 1.371 | 139.331 | 47.719 | 1.504 |
| 3 | 218.120 | 287.383 | .211 | 218.120 | 287.383 | .000 |
| 4 | 173.381 | 473.430 | 1.275 | 173.381 | 473.430 | 1.570 |
| 5 | -20.584 | 538.820 | -1.755 | -20.584 | 538.820 | .000 |

Statistics

number of iterations : 2

Residuals on:

| name (mm) | U | V | **W** | X | Y | Z |
|---|---|---|---|---|---|---|
| 1 | .00002 | .00000 | **.00231** | .00000 | .00000 | .00000 |
| 2 | -.00001 | .00000 | **-.00196** | -.01462 | .00249 | .00000 |
| 3 | -.00002 | .00000 | **-.00216** | -.00602 | .00157 | .00000 |
| 4 | .00003 | .00000 | **.00403** | -.01634 | .00262 | .00000 |
| 5 | -.00002 | .00000 | **-.00222** | .00000 | -.00040 | .00000 |

a posteriori standard deviation : .001973335

Transformed coordinates

| name (mm) | YTR | ZTR | DX | DY | DZ | XTR |
|---|---|---|---|---|---|---|
| 1 | .00000 | .00000 | -.00231 | .00000 | .00000 | -.00231 |
| 2 | 139.34562 | 47.71651 | 1.50596 | .01462 | -.00249 | .00196 |
| 3 | 218.12602 | 287.38143 | .00216 | .00602 | -.00157 | .00216 |
| 4 | 173.39734 | 473.42738 | 1.56597 | .01634 | -.00262 | -.00403 |
| 5 | -20.58400 | 538.82040 | .00222 | .00000 | .00040 | .00222 |

Transformation parameters

| | | | | |
|---|---|---|---|---|
| shift in X direction | : | .007971 | +/- | 0.001973 |
| shift in Y direction | : | -.001041 | +/- | 0.001973 |
| shift in Z direction | : | 1.098662 | +/- | 0.001677 |
| | | | | |
| rotation around X axis | : | -.05417703038 | +/- | 0.00023270394 |
| rotation around Y axis | : | -.41483532348 | +/- | 0.00053078368 |
| rotation around Z axis | : | .00005388618 | +/- | 0.00029675729 |
| | | | | |
| scale factor | : | 1.000000000 | +/- | 0.000000000 |

Rotation matrix

| | | |
|---|---|---|
| .9999737895601 | .0000077865682 | .0072401748655 |
| -.0000009404666 | .9999995529441 | -.0009455742275 |
| -.0072401789915 | .0009455426345 | .9999733425233 |

## 10. REFERENCES


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