ABSTRACT

A simple mathematical model is developed to help optimize the energy for frontier accelerator facilities. The expected new physics per unit cost shows a broad maximum as the energy of the new facility is increased, verifying the intuitive expectation that steps of a factor of 3 to 15 in energy for new facilities is reasonable, fairly independent of the details. As an example, I consider the extension of mass reach for $W'/\gamma_3$ that would be provided by a Really Large Hadron Collider (RLHC) beyond that of the Large Hadron Collider (LHC). I then comment briefly on aspects of the approval process for a RLHC.

I. INTRODUCTION

When planning for a future high energy physics facility, such as the Really Large Hadron Collider (RLHC), the energy is a critical parameter in setting both the cost and physics capabilities. Without specific theoretical guidance, one expects that the bigger the jump in energy, the more physics discoveries will be made. Such reasoning does not establish an optimal energy, however, and other factors such as cost must be taken into account.

Intuitively, based on past choices of energy, a factor of 3 to 15 seems about right. Given the great cost of frontier facilities, we can no longer afford duplication and the energy must be substantially above that of existing facilities. Above some factor, even with advances in technology, the costs increase rapidly and provide a practical limit.

In this paper, we develop a simple model of the benefit/cost ratio -- the physics per buck -- to better understand and validate our intuition.

II. SIMPLE PHYSICS/COST MODEL

Given the absence of specific theoretical guidance for physics thresholds at RLHC energies, we simply assume a priori that each factor of two in energy or mass reach has an equal discovery potential.

By the time the RLHC comes into operation, the LHC will have thoroughly explored the mass reach provided by its energy, $E_{(LHC)} = E_1 = 7$ TeV per beam, for the various potential physics discoveries awaiting us, predicted or not by the theorists. For some "typical" new physics, we make the approximation that the mass reach scales as

$$M_2/M_1 = (E_2/E_1)^p$$

("practically everything is a straight line on a log-log plot"). As discussed below, the exponent $p$ will depend on how the luminosity is scaled. In evaluating "physics per buck," we therefore take the new physics capabilities as being proportional to $\ln E_2/E_1$, where $E_1$ and $E_2$ are the energies of the old and new facilities, respectively.

For a given technology for the new facility, we assume a linear cost model

$$C = a + bE$$

where the fixed costs $a$ are made up of R&D, detectors, etc., and the linear costs $bE$ are mainly production costs for magnets, cryogenics and tunnel. We define $E_0 = a/b$, the energy for which the energy-dependent costs equal the fixed costs.

The function that we want to maximize is then

$$\text{physics per buck} = \frac{K \ln E_2/E_1}{E_2 + E_0}$$

(3)

While the constant $K$ is a most important parameter in the real world, here we will adjust it to give a normalization of unity at the peak value. This function is plotted in Figure 1 for several values of $E_0$.

FIG. 1: Normalized “physics per buck” for the RLHC as a function of its energy, as given by Eq. 3 for the values of $E_0$ indicated (in TeV), using the LHC energy of $E_1=7$ TeV.
As expected, the higher the $E_0$, i.e., the higher the fixed costs relative to the linear costs, the higher is the optimal energy -- having paid all those fixed costs, we might as well extend the energy. In the approximation of negligible fixed costs ($E_0 = 0$), the function becomes relatively simple, of the form $(\ln x) / x$, and has its maximum value at $x = E_0 / E_1 = e = 2.72$.

The "physics per buck" curves fall off slowly above the very broad maxima. Figure 2 shows the optimal energy $\hat{E}$ in this model as a function of $E_0$, together with $E_+$, the energy for which the "physics per buck" rises to 90% of its peak value and $E_-$, the energy for which it falls to 90% of the peak.

FIG. 2: Optimal RLHC energy $\hat{E}$ given by the peak position of the curves in Fig. 1. Also shown are the energies at which the "physics per buck" has fallen to 90% of its optimum value.

For the ambitious goals of the low-field RLHC group, the fixed costs would be $1$ billion and the linear costs $\$ (20 to 40) million/TeV, giving $E_0 = 25$ to 50 TeV. To the extent that the actual fixed and linear costs scale together by the same factor, $E_0$ will not change. In this range of $E_0$, RLHC energies of roughly 25 to 80 TeV yield a "physics per buck" within 10% of the peak value, consistent with the intuitive factors listed in the Introduction.

III. A SPECIFIC PHYSICS EXAMPLE

As a specific example, I consider the production of heavy charged intermediate vector bosons, $W'$, with subsequent decay to $e^\pm$. This is one of the standard "bellwether" reactions used to examine the reach of new accelerators and I have chosen it for its simple and robust topology, signature and trigger.

Very heavy particles, such as of interest here, are centrally produced. Because of the rapid fall-off of parton distributions at high energies, the most probable production is with the two partons (one in each beam) having roughly equal energies.[1] For the heaviest observable particles, this results in most being produced within a rapidity of $|y| < 0.5$. With only a two body final state, a general purpose detector will have close to full geometric acceptance. The high transverse momentum of the decay $e^\pm$ will provide a robust trigger.

At these energies, electromagnetic calorimeters provide excellent resolution and the sharp cut-off at the Jacobian peak should be highly distinctive. With good spatial resolution in the calorimeter, it should be easy to distinguish electrons and photons by looking for the high momentum track segment just in front of the calorimeter. Even with a large number (say, 100) of interactions from the same beam crossing, the number of tracks per square foot at a radius of 1 or 2 meters should be manageable.

The production cross section and decay probability were calculated by Stephane Keller [2] for several proton-proton center of mass energies, $\sqrt{s} = 2E$, using the standard $W$ couplings, but varying the mass, for cuts of $|y| < 3$ and $E_T > E/1000$. I took the mass reach to be that mass corresponding to 10 events per $10^7$ sec ("Snowmass year"). The results are shown in Figure 3. As a check, this procedure gave a $W'$ mass reach of 6.2 TeV for the LHC at $10^5$ cm$^{-2}$sec$^{-1}$, embarrassingly close to the 6 TeV quoted by Hinchliffe and Womersley.[3]

FIG. 3: The points show the mass reach as a function of proton-proton center-of-mass energy for luminosities of $10^{33}$, $10^{34}$ and $10^{35}$cm$^{-2}$sec$^{-1}$, under the assumptions described in the text. (The 14-TeV point for $10^{35}$ is unavailable.) The straight lines are given by Eq. 4 for these three luminosities.

As shown in Figure 3, the RLHC mass reach for the luminosities and energies considered is well approximated by

$$M(W) = 26TeV\left(\frac{E}{50TeV}\right)^{0.7}\left(\frac{L}{10^{34}}\right)^{0.15}.$$  (4)
For a fixed luminosity (say, $10^{34}$) independent of energy, the mass reach thus scales as $E^{0.7}$. Note that since protons are broad-band beams of gluons and quarks, with increasing numbers effective as the energy of the protons is increased, the proton-proton luminosity does not have to scale as $E^2$ as for $e^+e^-$ colliders. However, for such scaling, the above formula would indeed give a linear scaling with energy, $M(W') = E/2$ for a luminosity of $10^{34}$ times $(E/50 \text{ TeV})^2$.

IV. COMMENTS ON THE RLHC APPROVAL PROCESS

Before approval can be expected for RLHC, we will have to demonstrate both feasibility and interest with LHC successes. On the one hand, we need to get experience with, and find solutions for: high rates in detectors, including the overlap of many events; radiation hardness, for both accelerator and detector components; beam scraping and safe extraction of Gigajoule beams; synchrotron radiation and gas desorption in a cryogenic environment; operational efficiency for huge cryogenic magnet systems; etc. On the other hand, we need to get exciting physics from the LHC in order to catch the imagination of others -- scientists, Congress and the public at large -- and get their support.

Today's climate greatly favors the internationalization of large science projects, in spite of the added complexity this brings. Thus, the community needs to start soon to develop an international RLHC collaboration. We will need time, patience and luck in dealing with large numbers of Presidents, Prime Ministers, legislators, Ministries of Finance, etc.

Sometimes big projects are easier to sell than small ones. They catch the imagination. But, as we have painfully seen, if a project grows too big, it becomes an easy target for budget cutters. We need to aim at some optimum size and then carefully control costs.

V. CONCLUSION

I have described a simple, but adequate model to optimize the cost-benefit ratio, the "physics per buck," for the energy of a new frontier accelerator. The production of heavy $W'$ was taken as an example, and the mass reach of high energy proton-proton colliders for $W'$ was shown to scale as a power of the energy of the beam, $M(W') = E^{0.7}$ for fixed luminosity and $M \sim E$ for luminosity scaling as $E^2$.

The optimization model is insensitive to details and shows a broad maximum in "physics per buck" as a function of accelerator energy, with a slow dependence on the ratio of fixed to linear costs for the facility. The model confirms our intuition and past choices of energy increases -- a factor of roughly 3 to 15 is reasonable.

In the real world, a number of other considerations come into play when setting the energy of a new accelerator: economics, budgets, geography, geology, sociology, landowners, and politics -- not to mention technical and scientific considerations.

REFERENCES

