MATRIX METHOD FOR ANALYSIS OF NETWORK ACCURACY BASED ON THE BEAM DYNAMIC THEORY

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1. INTRODUCTION

Based upon tolerances to a closed orbit distortion (orbit distortion), the requirements to the alignment accuracy of the magnetic system of accelerators are specified. The orbit distortion and aperture losses of a vacuum chamber are the result of many factors: manufacturing errors, fringe magnetic fields, size and motions of the beam at the moment of injection, etc. Thus, the orbit distortion caused by misalignment is usually assumed as 0.1-0.2 of the vacuum chamber size, i.e a few millimeters, though an accelerator circumference may range from 100 meters up to 10 kilometers. This corresponds to an alignment accuracy of 0.l- 0.2 mm. These very tight tolerances require notable expenses for the alignment system. It is important to be sure that the tolerances are realistic and not overstated. Starting the development of the alignment system, a surveyor faces the following questions:

1. What is a necessary absolute and relative accuracy?
2. What is a length of the region of relative accuracy?
3. What is an optimal smoothing curve which would not result in the orbit distortion?
4 What should be the scheme of measurements and appropriate instruments?

A minimum number of components to be realigned and an appropriate amplitude of the orbit distortion should be determined in the realignment process. The discussed method is aimed to help in answering these questions. Its practical application for VEPP-4 alignment system is presented.

2. MATRIX OF THE ORBIT DISTORTION

Errors in positioning centers of gradient magnets (quads) are a major source of the orbit distortion. The following consideration concerns these elements only. The orbit distortion is calculated with the use of the theory of betatron motion [1,2]. The betatron motion of particles in the Y direction transverse to the beam trajectory (in radial or vertical direction) is calculated by the following equation:

$$Y(s) = a\sqrt{\beta(s)}\cos(\varphi(s) + \vartheta_0), \tag{1}$$

where: s is the beam coordinate along the trajectory, a and ϑ are defined by initial conditions, $\beta_{(s)}$ is the betatron function, specified by the magnetic lattice.

The betatron phase is determined as $\varphi(s)=\int_0^s \frac{ds}{\beta(s)}$. The frequency of the betatron oscillations is specified as: $\nu_{r,z}=\frac{1}{2\pi}\int_0^{s_0} \frac{ds}{\beta_{r,z}(s)}$ where: s_0 is the perimeter of the closed orbit. Numerically calculated β- function and betatron phases can be obtained from accelerator physicists.

The distorted orbit is calculated by the method of variation of parameters *a* and ϑ from equation (1). The solution is found on an assumption of closed orbit after revolution, and an angle *Y'(s)* which the particle will have because the displacement of the quad center at value *x* should be $Y' = \frac{Gl}{E}\times x$, where G is the gradient of the magnetic field in the quad, *l* is the length of the quad, *E* is a particle energy. The quantity *E/Gl* = *F* is a focus distance of quad, and *F* >>*l*. Most of accelerators comply with this requirement. β -functions achieve the minimum and maximum in the quad center, therefore, it is sufficient to calculate the orbit distortion in these points. The amplitude of the orbit distortion in quad #k is:

$$Y_k = \frac{\sqrt{\beta_k\beta_i}}{2F_i \sin \pi\nu} \cos(\varphi_k - \varphi_i \mp \pi\nu) \times x_i \tag{2}$$

$$-\pi\nu \ \ for\ k \geq i \ \ , \ +\pi\nu \ \ for\ k < i \ ,$$

where F_i is the focus distance of quad *#i* , x_i is a displacement of quad *#i* from its ideal position, *k*, *i* = 1.. . *N*, *N* is a quantity of quads.

Since equation (2) is linear in *x* and *Y*, the orbit distortion caused by the displacements of all quads is a sum of Y_k over *i*

Suppose, $\vec{Y}$ is a N-dimensional vector of the orbit distortion and $\vec{x}$ is a N-dimensional vector of the quad displacements. Then, a matrix form of equation (2) may be written as

$$\vec{Y} = A \times \vec{x}, \text{ where } A_{i,k} = \frac{\sqrt{\beta_k\beta_i}}{2F_i \sin \pi\nu} \cos(\varphi_k - \varphi_i \mp \pi\nu) \equiv a_{i,k} \cos(\varphi_k - \varphi_i \mp \pi\nu)$$

The matrix A is a response-matrix or a matrix of the orbit distortion. For different categories of accelerators matrix elements $a_{i,k}$ range form 5 to 30. For a successful operation of an accelerator, the orbit. distortion with respect to the ideal orbit is less important than a displacement of a real orbit from the centers of quads, i.e. $Y_i \to Y_i - x_i$. Matrix *A'* = *A* - *I* is used instead of the matrix *A* in the following analysis.

The analysis of elements of the matrix *A* which a surveyor can easily obtain would indicate the following: elements to which positioning the magnet system is most sensitive and places along the beam orbit where the orbit distortion is maximal. Thus, one can determine particular elements which should be aligned with higher accuracy, as well as places where special attention should be paid to the positioning of the vacuum chamber and other equipment.

3. COVARIATION MATRIX AND COEFFICIENT OF THE ORBIT DISTORTION

For estimation of accuracy we use a standard deviation. We assume that (Y) is a r.m.s. value of the orbit distortion, and $\langle x \rangle$ is a similar value for the quad displacements. If x_i stands for noncorrelated errors of the quad alignment and $\langle x \rangle = \sigma_x$, we obtain $\langle Y \rangle = \gamma\sigma_x$ with

$\gamma = \sqrt{\frac{Sp\left(A \times A^{tr}\right)}{N}}$. We define the coefficient of the orbit distortion as γ . If the alignment

errors are noncorrelated, $y \approx \frac{\langle G \rangle \langle l \rangle \sqrt{N}}{2\overline{B}\nu\left|\sin \pi\nu\right|}$, where $\overline{B}$ is an average magnetic field along the beam orbit. For most accelerators $\gamma \sim 20$ - 50.

The alignment errors are always correlated to each other, depending on the scheme of measurements and the alignment procedure. A corresponding covariation matrix is $B_x = \langle x \rangle^2 * B_x'$, where $(x) = \sqrt{\frac{Sp(B_x)}{N}}$. The equation for y has the form:

$$\gamma = \sqrt{\frac{Sp\left(A \times B_x' \times A^{tr}\right)}{N}} \tag{3}$$

Simulations show that the coefficient can be reduced by a factor 3-15 taking into account the correlation in the quad positioning. In order to calculate the maximum value of the orbit distortion the coefficient should be multiplied by a factor $\sqrt{\beta_{max}/\overline{\beta}}$.

The coefficient g calculated for VEPP-4 is 53 for radial and 39 for vertical directions in the assumption that the alignment errors are noncorrelated. It is reduced down to 13 and 23, respectively, taking into account the correlation.

4. FOURIER ANALYSIS AND SMOOTHING CURVE.

4.1. Spectral sensitivity of the magnet system

The matrix of the orbit distortion *A* makes it possible to perform an analysis of the sensitivity of the magnet system to certain Fourier frequencies in a distribution of the quad displacements.

Suppose that the distribution of the displacements along the orbit is $\delta x_i = a_n \cos(2\pi \frac{s_i}{s_0} n + \varphi_0)$, where s_0 is the orbit perimeter, n is the number of harmonic, φ_0 is the initial phase, a_n is the amplitude of the harmonic. In this case $\sigma_x^2 = 0.5a_n^2$

Since δx is a discrete function of the azimuth s_i, the number of Fourier harmonics is limited by N/2, where N is the number of quads. Elements of the covariation matrix Bx are given by :

$$B_{x_{i,j}} = 0.5a_n^2 \cos 2\pi \frac{s_i - s_j}{s_0} n \; . \tag{4}$$

The spectral coefficient of the orbit distortion γ_n caused by harmonic #n is given in (3) and (4).

Finally, $\langle Y \rangle^2 = 0.5 \sum_1^{N/2} \gamma_n^2 a_n^2$. Figure 1 shows γ_n vs. n for horizontal and vertical directions calculated for the VEPP-4 collider.

One can see that the sensitivity grows up with n and has its first maximum at $n \approx \nu_r$ and $n \approx \nu_z$ respectively. An especially high sensitivity is for $n \approx k \pm \nu_{r,z}$, where k is the number of cells. Long straight sections provide local maximums within a range $v < n < k - v$ due to the distortion of homogeneity of the magnetic lattice. Other colliders usually have a similar spectral sensitivity.

Let us analyze the spectral sensitivity of VEPP-4 for $n \le \nu_z$, where the amplitudes of alignment have maximum values. The coefftcients are determined by equations (3,4) using the matrix A or $A - I$.

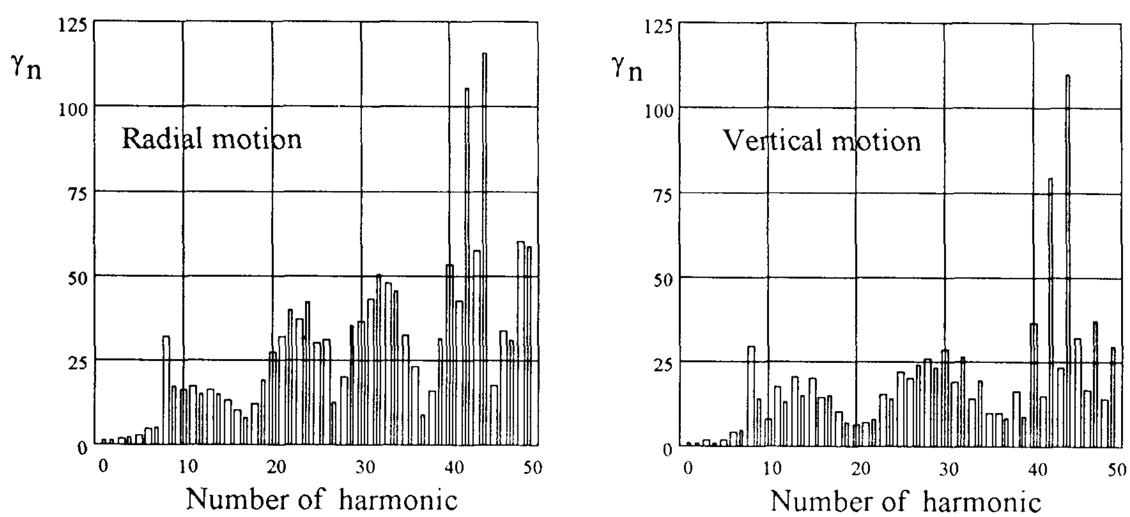

Fig .1 Spectral sensitivity of VEPP-4 magnet system

The results are summarized in Table 1. When the matrix A is replaced by $A - I$, the spectral sensitivity of magnet system for harmonics with numbers $n < \nu/2$ is considerably reduced. This fact means that for these harmonics the orbit distortion has about the same value as the quad displacement. This effect does not appear for other harmonics.

Table 1

Spectral sensitivity of VEPP-4 magnet system

$\gamma \backslash n$	1	2	3	4	5	6	7	8	9
Matr. A	0.72	0.94	1.1	2.3	1.5	4.9	4.4	29.6	13.8
Matr. $A - I$	0.017	0.23	0.37	1.7	1.0	5.0	4.0	28.4	14.0

4.2 Spectral analysis of errors

The B_x matrix allows one to carry out a spectral analysis of the alignment errors for schemes of different kind and different accuracy of measurements.

The results of the spectral analysis of the alignment errors for the VEPP-4 collider are given in Figure 2. Amplitudes of harmonics tend to diminish as n grows up. For the horizontal direction this tendency is sharper than that for the vertical one because of a stronger correlation in the quad positioning.

4.3. Local and global accuracy. Smoothing curve.

The surveyor can easily calculate which harmonics are important and which are not from the point of view of the orbit distortion with the help of the spectral sensitivity of the magnetic system and the results of the spectral analysis of alignment errors. If the quantity $a_n \times \gamma_n$ results in a significant orbit distortion, the harmonic with number n shall be considered as a critical. The wave length of such a harmonic with a minimal number n determines the area wherein a precise

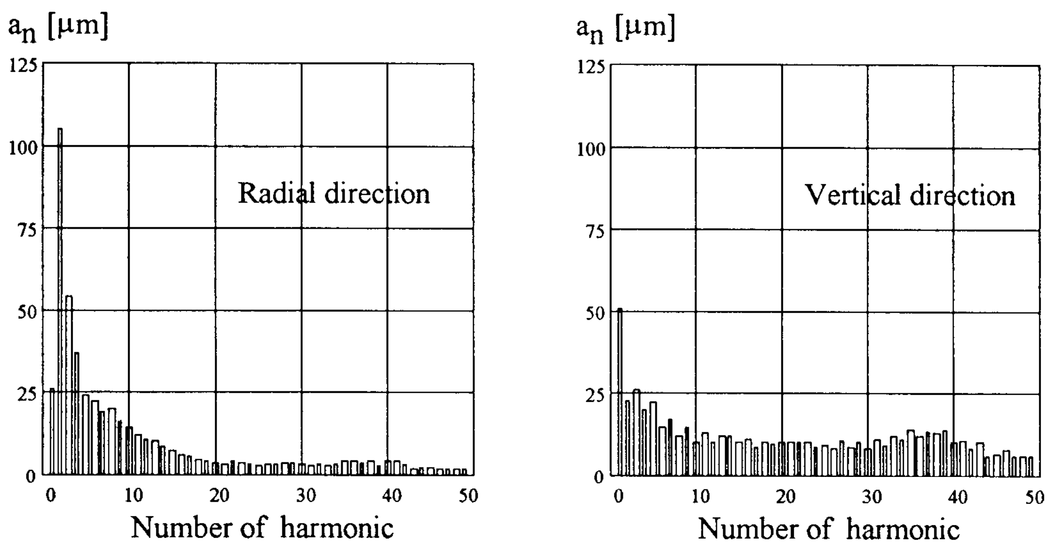

Fig .2 Spectral content of alignment errors for VEPP-4 relative alignment has to be performed. We call it local accuracy.

Simulations for a few samples of the magnetic lattice and for VEPP-4 (see Table 1, bottom line) show that y < 1 for $n < \nu/2$ and y > 1 for $n > \nu/2$.

Let us assume that y << 1 and g is a small orbit distortion, for example, g is equal to 10% of an allowed distortion. In this case the quantity g/γ_n is a tolerance for the global alignment accuracy and accordingly, $\lambda_n/4$ defines the region of global accuracy. For VEPP-4 $g \approx 0.2\text{mm}, \gamma_2 \approx 0.2, a_2 \approx 1\text{mm}$. It means that two perpendicular diameters of the ring can be different from their ideal values within 1mm.

The smoothing curve is defined by the global accuracy. In realignment process a control survey is carried out. The surveyor has a set of the quad displacements from the ideal positions. The Fourier analysis of the displacements should be performed to calculate amplitudes and phases of harmonics. The quantity of $\gamma_n * a_n$ is calculated and summarized over *n* until this sum is comparable with the orbit distortion limit (10-15%). The smoothing curve is computed as a sum of Fourier components up to an order *n*.

Methods of defining the smoothing curve using local polynomes sometimes can prove to be incorrect as they can create high order harmonics to which the magnet system is most sensitive. In this case an additional analysis of such curves is necessary.

5. CONCLUSION

The proposed method provides an opportunity to unite successfully the efforts of surveyors and physicists. The requirements on the alignment accuracy are determined. There is a possibility to select an optimum scheme of measurements and an alignment procedure at a project phase. In the realignment process the surveyor can determine the orbit distortion and reduce the number of elements requiring alignment.

The proposed method can be applied both to circular and linear accelerators.

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