THE SPECTRAL METHOD FOR PRECISION ESTIMATE OF THE CIRCLE ACCELERATOR ALIGNMENT

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INTRODUCTION

The rigour of the requirements to the geometry of the ring accelerator position is well-known. They tier in the necessity to take account of the error harmonic set. The point is, that the separate harmonic components of the magnet position errors have different effect on the machine operation. It follows from the solution of the equation for the disturbed particle motion in the closed orbit. This solution for the strong focusing accelerator with an error not more than 30% can be written as follows [1]:

$$\delta Y_i = -\frac{R^2}{R_0}\sum_{k=0}^{N/2}\frac{G_{(\Delta H/H)_k}}{\nu^2 - k^2}\cos[k\Theta_i - \Omega_{(\Delta H/H)_k}] \qquad (1)$$

where δY_i is the particles orbit distortion in the centre of the quadrupole magnet under number i;

R and R_0 are the mean orbit radius and orbit radius in the magnets;

$G_{(\Delta H/H)_k}$ and $\Omega_{(\Delta H/H)_k}$ are the amplitude and phase of the harmonic of k order of the expansion into the Fourier series of the relative disturbances of the magnet field;

ν is beam betatron frequency;

N is the quadrupole capacity;

Θ is the azimuth coordinate.

The relative disturbances of the magnet field are related to magnet position errors in the following equations:

$$\frac{\Delta H_R}{H} = \pm\frac{\Delta R \cdot Gr}{H} \quad ; \quad \frac{\Delta H_Z}{H} = \mp\frac{\Delta Z \cdot Gr}{H} \quad , \qquad (2)$$

where Gr - the magnet field gradient;

ΔR - the radial position errors;

ΔZ - the vertical position errors.

The upper sign in the formulae (2) concerns F quadrupoles, the lower sign concerns D quadrupoles.

As it follows from expression (1), the disturbance harmonics, the nearest to betatron frequency, give rise to the biggest orbit distortion. Therefore, the corresponding harmonics of the magnet alignment errors should be considered specially. But the conventional methods do not allow to extract the dangerous harmonics from the alignment error population.

1. The basis for the spectral method.

As a rule, for big accelerator alignment the precision ring geodesic networks are used. They have a polygon form. The measuring elements of network can be distances between the neighboring points and the offsets or angles. The point capacity usually coincides with quadrupole capacity, and the distance values correspond to value of the distances between the quadrupoles. Since our problem is of an estimation character, let us take this polygon as regular. We take account too of the following characteristic properties of the accelerator alignment:

1. The requirements to the magnet precise position are given in polar coordinate system.
2. The alignment errors in lateral to beam axis directions are the most critical.
3. The regular position of geodetic points in circumference, the design and use similarity without separating some of them starting requires to consider the ring network as free geodetic system.

The basis for the spectral method is the set of n equations (n is the points capacity), which connect the errors of the measuring values Δh (or $\Delta\beta$) and As with the errors of the radial coordinates of the network points ΔR (see fig.1):

$$\Delta R_{i-1} - 2\Delta R_i \cos\varphi + \Delta R_{i+1} = L_{R_i} \quad ; \qquad (3)$$

where for measured h and s

$$L_{R_i} = -2[\Delta h_i - \sin\frac{\varphi}{2}(\Delta s_i + \Delta s_{i+1})] \quad ; \qquad (4)$$

for measured β and s

$$L_{R_i} = \frac{1}{\rho} s \cos\frac{\varphi}{2} \cdot \Delta\beta_i + \sin\frac{\varphi}{2}(\Delta s_i + \Delta s_{i+1}) \quad ; \qquad (5)$$

where ρ - radian with units of $\Delta\beta$.

The coefficient matrix of the equation set (3) has the dependent columns. Therefore the unknowns ΔR can be determined by the certain conditions, namely, by absence of the displacement of the system weight centre, that is condition for free geodetic network. The solution of the set (3) is founded on the expansion of the ΔR and L_R values into closed Fourier series. It can be realized by the known

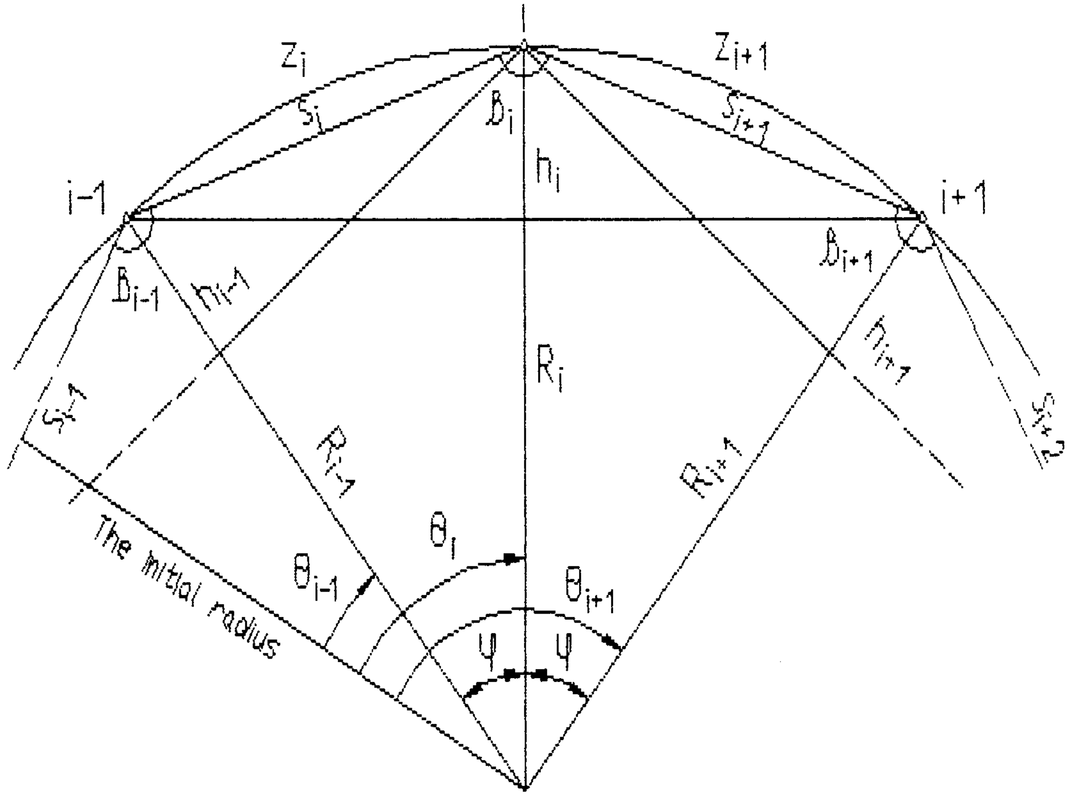

Fig. 1 The scheme of the ring geodetic network

formulae for expansion into trigonometrical series of the n values X, which are regularly arranged in the period [2]:

$$X_i = \sum_{k=0}^{r} G_k \cdot \cos(k\Theta_i - \Omega_k) \quad , \tag{6}$$

where $r = \frac{n}{2}$ at even n, $r = \frac{n-1}{2}$ at not even n;

G_k is an amplitude of harmonic under order k;

$$G_k = \sqrt{A_k^2 + B_k^2} \quad ; \tag{7}$$

Ω_k is a phase of harmonic under order k;

$$\Omega_k = arctg\left(\frac{B_k}{A_k}\right) \quad ; \tag{8}$$

A_k , B_k axe trigonometrical coefficients,

$$A_0 = \frac{1}{n}\sum_{i=1}^{n} X_i \ ; \quad B_0 = 0 \ ; A_k = \frac{2}{n}\sum_{i=1}^{n} X_i \cos k\Theta_i \ ; \quad B_k = \frac{2}{n}\sum_{i=1}^{n} X_i \sin k\Theta_i \ ; \tag{9}$$

$k = 1, 2, 3, \ldots . \frac{n-1}{2}$ at not even n,

$k = 1, 2, 3, \ldots, \frac{n}{2} - 1$ at even n.

With the even n the coefficient values of last harmonic under order $n/2$ are calculated by formulae:

$$A_{n/2} = \frac{1}{n}\sum_{i=1}^{n}(-1)^i X_i \ ; \qquad B_{n/2} = 0 \ . \tag{10}$$

We will further consider the ring network with the even capacity of points. Suppose that the ΔR and L_R values are periodic functions of the azimuthal angle Θ with period of 2π and expand these functions into closed trigonometrical series:

$$\Delta R_i = \sum_{k=0}^{r} G_{R_k} \cos(k\Theta_i - \Omega_{R_k}) \quad , \tag{11}$$

$$L_{R_i} = \sum_{k=0}^{r} g_{R_k} \cos(k\Theta_i - \omega_{R_k}) \ . \tag{12}$$

Using the expressions (11), (12), we can write the equation (3) as

$$\sum_{k=0}^{r} G_{R_k} \cdot [\cos(k\Theta_{i-1} - \Omega_{R_k}) - 2\cos\varphi \cdot \cos(k\Theta_i - \Omega_{R_k}) + \cos(k\Theta_{i+1} - \Omega_{R_k})] =$$

$$\sum_{k=0}^{r} g_{R_k} \cos(k\Theta_i - \omega_{R_k})$$

The both sides of the derived equality contain the series. It is known, that the equality is possible if the series are term-by-term equal. That is, the expression is correct:

$$G_{R_k} \cdot \cos(k\Theta_{i-1} - \Omega_{R_k}) - 2\cos\varphi \cdot \cos(k\Theta_i - \Omega_{R_k}) + \cos(k\Theta_{i+1} - \Omega_{R_k}) = \\ = g_{R_k} \cos(k\Theta_i - \omega_{R_k}) \tag{13}$$

Taking into account that

$$k\Theta_{i-1} = k\Theta_i - k\varphi \quad , k\Theta_{i+1} = k\Theta_i + k\varphi \quad , \tag{14}$$

from (13) it follows that:

$$2 \cdot G_{R_k} \cdot \cos(k\varphi - \cos\varphi) \cdot \cos(k\Theta_i - \Omega_{R_k}) = g_{R_k} \cos(k\Theta_i - \omega_{R_k}) \tag{15}$$

We have equality of two harmonic waves, which have the similar frequencies and directions. In this case their amplitudes and phases are equal too, that is

$$2 \cdot G_{R_k} \cdot \cos(k\varphi - \cos\varphi) = g_{R_k} \quad , \tag{16}$$

$$\Omega_{R_k} = \omega_{R_k} \quad , \tag{17}$$

If to label

$$C_{R_k} = \frac{1}{2\cos(k\varphi - \cos\varphi)} \quad , \tag{18}$$

so

$$G_{R_k} = C_{R_k} g_{R_k} \; . \tag{19}$$

Expression (19) allows to correspond the harmonic amplitudes of the measurement error functions to the errors of the radial position of the network points. Note, that because of indeterminate form of the initial equation set the coefficient C_{R_k}, which can be called as "coefficient of the harmonic amplification", at $k = 1$ is indeterminate. It can be explained by the conditions for the free geodetic network: the first harmonic of the radial errors causes the parallel displacement of the network points relative to its weight centre.

2. Some distinctions of the estimate of the measurement error harmonic set.

Let us investigate the correlation matrix of the vector of the trigonometrical coefficients

$$I = \begin{pmatrix} A_0 \\ A_1 \\ \dots \\ A_r \\ B_0 \\ B_1 \\ \dots \\ B_r \end{pmatrix} \quad ,$$

which is linear function of the some measurement vector

$$J = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ \dots \\ X_n \end{pmatrix} \quad :$$

$$I = M \cdot J \tag{20}$$

The measurement errors are random values, which are characterized by correlation matrix

$$J = \begin{pmatrix} m_x^2 & & & \\ & m_x^2 & & \\ & & \dots & \\ & & & m_x^2 \end{pmatrix}$$

where m is r.m.s. of the measurement.

As it follows from expressions (9-10), the matrix of linear transformation in the equation (20) has following form:

$$M = \frac{2}{n} \cdot \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \dots & \frac{1}{2} \\ \cos\ \Theta_1 & \cos\ \Theta_2 & \dots & \cos\ \Theta_n \\ \cos 2\Theta_1 & \cos 2\Theta_2 & \dots & \cos 2\Theta_n \\ \dots & \dots & \dots & \dots \\ \cos r\Theta_1 & \cos r\Theta_2 & \dots & \cos r\Theta_n \\ 0 & 0 & & 0 \\ \sin\ \Theta_1 & \sin\ \Theta_2 & \dots & \sin\ \Theta_n \\ \sin 2\Theta_1 & \sin 2\Theta_2 & \dots & \sin 2\Theta_n \\ \dots & \dots & \dots & \dots \\ \sin r\Theta_1 & \sin r\Theta_2 & \dots & \sin r\Theta_n \end{pmatrix} \tag{21}$$

Let us define the form of the correlation matrix K_I for vector of trigonometrical coefficients:

$$K_I = M \cdot K_J \cdot M^T \quad , \tag{22}$$

where M^T is transposed matrix M.

The matrix K_I has dimension $(2r + 1) * (2r + 1)$.

The following equations hold for points, regular placed in the circumference:

$$\sum_{i=1}^{n} \cos k\Theta_i = \sum_{i=1}^{n} \sin k\Theta_i = 0 \quad ,$$

$$\sum_{i=1}^{n} \cos k\Theta_i \cdot \sin j\Theta_i = \sum_{i=1}^{n} \cos k\Theta_i \cdot \cos j\Theta_i = \sum_{i=1}^{n} \sin k\Theta_i \cdot \sin j\Theta_i = 0 \quad ,$$

if $k \neq j$ and $k + j < n$,

$$\sum_{k=1}^{n} \cos^2 k\Theta_i = \sum_{k=1} n \sin^2 k\Theta_i = \frac{n}{2} \ .$$

Subject to these equations the matrix K_I will have form:

$$K_I = \begin{pmatrix} \frac{m_x^2}{n} & 0 & 0 & \dots & 0 \\ 0 & \frac{m_x^2}{n} & 0 & \dots & 0 \\ 0 & 0 & \frac{m_x^2}{n} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 \end{pmatrix} \tag{23}$$

From the derived matrix the following conclusions can be done:

1. The diagonal terms, which characterize error variances of the trigonometrical coefficients at k = 1, 2, 3, ... $\frac{n}{2}$ - 1 are equal each other:

$$m^2_{A_0} = m^2_{A_{n/2}} = \frac{m_x^2}{n} \ ; \quad m^2_{B_0} = m^2_{B_{n/2}} = 0 \ ;$$

$$m^2_{A_k} = m^2_{B_k} = \frac{2m_x^2}{n} \ ; \tag{24}$$

2. expressions (24) define error variances for trigonometrical coefficients of the measurement value errors: offsets, angles, distances, elevations.

3. all not diagonal terms are equal to zero. Consequently, the errors of the trigonometrical coefficients are not correlated to each other.

Differentiating (7), we get:

$$dG_k = dA_k \cos \Omega_k + dB_k \sin \Omega_k \ .$$

Using absence of correlation, let us go to r.m.s. values:

$$m^2_{G_k} = m^2_{A_k} \cos^2 \Omega_k + m^2_{B_k} \sin^2 \Omega_k \ .$$

Or, subject to (24),

$$m_{G_k} = m_{A_k} = m_{B_k} = \gamma_k \tag{25}$$

Correspondingly $\gamma_0 = m_{A_0}$; $\gamma_{n/2} = m_{A_{n/2}}$.

Let us substitute in the formula, similar (7), for A_k and B_k terms their r.m.s. values. We will get:

$$\Gamma_k = \sqrt{m^2_{A_k} + m^2_{B_k}} \ .$$

Subject to (25) we will have:

$$\Gamma_k = m_{G_k} \cdot \sqrt{2} = \gamma_k \cdot \sqrt{2} \ . \tag{26}$$

Considering the sense of formula (7), one may say, that we have got the r.m.s. amplitude of the k-th harmonic.

If γ_k characterizes the error part of each value X_i, due to error harmonic of k, that Γ_k characterizes the r.m.s. amplitude of this harmonic. For harmonics of $k = 0$ and $k = n^2$ we will have:

$$\Gamma_0 = m_{G_0} = \gamma_0$$

$$\Gamma_{n/2} = m_{G_{n/2}} = \gamma_{n/2} \; . \tag{27}$$

3. The harmonic set estimation of the ring network geodetic points radial position errors.

Let us consider the geometrical conditions in the ring network. By analogy with known level method we have here three conditions too:

$$\begin{cases} g_{h_0} - g_{s_0} \cdot \sin\frac{\varphi}{2} = 0 \quad or \quad g_{\beta_0} = 0 \; , \\ a_{R_1} = 0 \; , \\ b_{R_1} = 0 \; , \end{cases} \tag{28}$$

where g_{h_0}, g_{s_0} , g_{β_0} are amplitudes of the zero harmonics of the $\Delta h, \Delta s, \Delta\beta$; a_{R_1}, b_{R_1} are trigonometrical coefficients of the first harmonic of LR; values.

The methodology of level and its final results will not be considered here. But from condition equation set (28) one can see, that by level only zero and first harmonics of the measured values will have got corrections. The rest error harmonics of measured values at the final of level are not corrected and all go into the error harmonics of the point position.

To estimate the r.m.s. amplitudes of harmonics of $k = 0, 2, 3, 4, \ldots, n/2$ orders let us use the outcome from expression (19):

$$A_{R_k} = C_{R_k} \cdot a_{R_k} \quad ; \qquad B_{R_k} = C_{R_k} \cdot b_{R_k} \; . \tag{29}$$

using (7), (12), let us write in more detailed form one of the equations (29), for example, the first one:

$$A_{R_k} = \frac{2}{n} C_{R_k} \left[\frac{1}{\rho} s \cdot \cos\frac{\varphi}{2} \sum_{i=1}^{n} \Delta\beta_i \cos k\Theta_i + \sin\frac{\varphi}{2} \sum_{i=1}^{n} (\Delta s_i + \Delta s_{i+1}) \cos k\Theta_i \right] \; .$$

The formula set (14) allows to get easily the expression of following form:

$$A_{R_k} = \frac{2}{n} C_{R_k} \left[\frac{1}{\rho} s \cdot \cos\frac{\varphi}{2} \sum_{i=1}^{n} \Delta\beta_i \cos k\Theta_i + \sin\frac{\varphi}{2} (1 + \cos k\varphi) \sum_{i=1}^{n} \Delta s_i \cos k\Theta_i + \right.$$

$$\left. + \sin\frac{\varphi}{2} \sin k\varphi \sum_{i=1}^{n} \Delta s_i \sin k\Theta_i \right] \; .$$

Or, that is equivalently,

$$A_{R_k} = C_{R_k}\left[\frac{1}{\rho}s\cdot\cos\frac{\varphi}{2}a_{\beta_k} + \sin\frac{\varphi}{2}(1+\cos k\varphi)\cdot a_{s_k} + \sin\frac{\varphi}{2}\sin k\varphi\cdot b_{s_k}\right] .$$

Here coefficient A_{R_k} is the linear function of the trigonometrical coefficients a_{β_k}, a_{s_k} and b_{s_k} of the expansion into the series of the measured value errors, namely of angles and distances. Using the absence of the correlation between the coefficient errors one can easily enough come to r.m.s. values:

$$\gamma^2_{R_k} = C^2_{R_k}\left[\frac{1}{\rho^2}s^2\cdot\cos^2\frac{\varphi}{2}\gamma^2_{\beta_k} + \sin^2\frac{\varphi}{2}(1+\cos k\varphi)^2\cdot\gamma^2_{s_k} + \sin^2\frac{\varphi}{2}\sin^2 k\varphi\cdot\gamma^2_{s_k}\right] .$$

Substituting for γ_{β_k} and γ_{s_k} the equivalent values, defined by (24, 25), taking into account (26), we will have for the case of angles and distances measurement:

$$\Gamma_{R_k} = \frac{2}{\sqrt{n}}\cdot C_{R_k}\sqrt{\frac{1}{\rho^2}\cdot s^2\cdot\cos^2\frac{\varphi}{2}\cdot m^2_\beta + 2\sin^2\frac{\varphi}{2}(1+cosk\varphi)\cdot m^2_s} \tag{30}$$

Doing the similar operation, we will have for the case of offset and distances measurement:

$$\Gamma_{R_k} = \frac{4}{\sqrt{n}}\cdot C_{R_k}\sqrt{m^2_h + 2\sin^2\frac{\varphi}{2}(1+cosk\varphi)\cdot m^2_s} \tag{31}$$

For harmonics of $k = 0$ and $k = n/2$ orders the Γ_{R_k} values are $\sqrt{2}$ as low.

4. The harmonic set estimation of the vertical position errors,

To estimate the harmonic set of the vertical position errors we will use the same supposition.

Assume that: n is the number of geodetic points in the network;

i is the current point number.

The equations connecting the measured elevations z with altitudes Z have the following form:

$$Z_i - Z_{i-1} = z_i .$$

Differentiating, we will have the n equations of the form:

$$\Delta Z_i - \Delta Z_{i-1} = \Delta z_i . \tag{32}$$

Suppose that ΔZ and Δz values are periodic functions of the azimuthal angle Θ with period of 2π and expand these functions into closed trigonometrical series:

$$\Delta Z_i = \sum_{k=0}^{r} G_{Z_k}\cos(k\Theta_i - \Omega_{Z_k}) , \tag{33}$$

$$\Delta z_i = \sum_{k=0}^{r} g_{z_k} \cos(k\Theta_i - \omega_{Z_k}) \ , \tag{34}$$

where $G_{Z_k}, g_{z_k}, \Omega_{Z_k}, \omega_{z_k}$ are defined by formulae (7-10), where ΔZ and Δz values substituted for X.
Using (33), (34), let write the initial equations (32) as

$$\sum_{k=0} G_{Z_k} \left[\cos(k\Theta_i - \Omega_{Z_k}) \cos(k\Theta_{i-1} - \Omega_{Z_k})\right] = \sum_{k=0} g_{z_k} \cdot \cos(k\Theta_i - \omega_{z_k})$$

As the series equality is possible only by the each-by-each equality of their terms, that, taking account of (14), after easy trigonometrical transformations we will have:

$$2 \cdot G_{Z_k} \sin\frac{k\varphi}{2} \cos(k\Theta_i - \Omega_{Z_k} - \frac{k\varphi}{2} - \frac{\pi}{2}) = g_{z_k} \cdot \cos(k\Theta_i - \omega_{z_k}) \ .$$

As earlier we have equality of the two harmonic waves with the same frequencies and directions. Consequently their amplitude and phase are equal, that is:

$$2 \cdot G_{Z_k} \sin\frac{k\varphi}{2} = g_{z_k} \ .$$

From here

$$G_{Z_k} = C_{Z_k} \cdot g_{z_k} \ , \tag{35}$$

where

$$C_{Z_k} = \frac{1}{2 \cdot \sin\frac{k\varphi}{2}} \tag{36}$$

Note, that coefficient of the harmonic amplification for zero harmonic is indeterminate. This is the result of the indeterminate form of the initial equation set (32). But in this case failure to take into account the zero harmonic **AZ** reduces network without any deformations to the mean horizontal plane.

The r.m.s. value of the elevation measurement error m characterizes the random values, which are not correlated. Using it with help of expressions (24-26) one can go to r .m.s amplitudes of expansions of these values into closed trigonometrical series. As result we will have:

$$\Gamma_{z_k} = \frac{2m_z}{\sqrt{n}} \ ,$$

$$\Gamma_{z_{n/2}} = \frac{2m_z}{\sqrt{n}} \ .$$

Using now the equation (35), after the transfer to r.m.s. values we will get r.m.s. amplitudes of the vertical point position errors:

$$\begin{aligned} \Gamma_{Z_k} &= C_{Z_k} \cdot \Gamma_{z_k} = \tfrac{2m_z}{\sqrt{n}} C_{Z_k} \ , \\ \Gamma_{Z_{n/2}} &= \tfrac{2m_z}{\sqrt{n}} C_{Z_k} \ . \end{aligned} \tag{37}$$

5. The estimation of the results.

With the final formulae (31, 37) the r.m.s. amplitudes for radial position errors of the network points of the operating Serpukhov accelerator with network radius of 236 m were calculated. The network includes 60 points. The measurement precision is characterized by following values in r.m.s: offset m_h = 0.04mm, distances m_s = 0.2mm, elevations m_z = 0.05mm. From resulting Table 1 we can see, that the measurement errors will have the main effect on the amplitude amplification of error harmonics of lower orders. It is explained by high values of coefficients of the harmonic amplification C_{R_k}. With the rise of harmonic order the error harmonic amplitudes quickly reduce.

Table 1.
The r.m.s. amplitudes for the radial position errors *of the ring network points at Serpukhov accelerator.*

Harmonic orders	C_{R_k}	Γ_{R_k} μcm	Harmonic orders	C_{R_k}	Γ_{R_k} μcm	Harmonic orders	C_{R_k}	Γ_{R_k} μcm
0	91.3	1064	11	0.8	20	21	0.3	7
2	30.5	711	12	0.7	17	22	0.3	6
3	11.5	267	13	0.6	14	23	0.3	6
4	6.2	143	14	0.6	13	24	0.3	6
5	3.9	91	15	0.5	11	25	0.3	6
6	2.7	62	16	0.4	10	26	0.3	6
7	2.0	45	17	0.4	8	27	0.3	6
8	1.5	35	18	0.4	8	28	0.2	6
9	1.2	28	19	0.4	7	29	0.2	6
10	1.0	23	20	0.3	7	30	0.2	4

Table 2.
The r.m.s. amplitudes for the vertical position errors of the ring network points at Serpukhov accelerator.

Harmonic orders	C_{R_k}	Γ_{R_k} μcm	Harmonic orders	C_{R_k}	Γ_{R_k} μcm	Harmonic orders	C_{R_k}	Γ_{R_k} μcm
0	9.6	123	11	0.9	12	21	0.6	7
2	4.8	62	12	0.8	11	22	0.6	7
3	3.2	41	13	0.8	10	23	0.5	7
4	2.4	31	14	0.8	10	24	0.5	7
5	1.9	25	15	0.7	9	25	0.5	7
6	1.6	21	16	0.7	9	26	0.5	7
7	1.4	18	17	0.6	8	27	0.5	7
8	1.3	16	18	0.6	8	28	0.5	6
9	1.1	14	19	0.6	8	29	0.5	6
10	1.0	13	20	0.6	7	30	0.5	3

For example the comparatively high amplitude of second harmonic points to the fact that as result of measurement errors the network points will displace into the smooth elliptic curve with minor deviation from ones due to the harmonics of the higher orders.

The similar effect is observed for vertical position of points (Table 2). But in this case the amplitude reduction is more smoothed. Due to correlation the network points take up the error position in the sufficiently smoothed curves, defined by error harmonics of the low orders. For example, the first harmonic sets off the comparatively small inclination of orbit by angle:

$$\alpha'' = \frac{\Gamma z_1}{R}\rho''.$$

In the showed example at the radius of $R = 230$ m, a = 0.1".

CONCLUSION

The derived results confirm the very important property of ring networks: the measurement errors cause the smooth deformation of the initial form. From this deformation one can sufficiently easily extract and estimate the dangerous harmonics. The absence of correlation among the error harmonic components allows to use ones for the further method development: the definition of the other precise characteristic of network, the estimation of the orbit distortion due to alignment errors and so on.

REFERENCES

[1] Коломенский А.А., Лебедев А.Н., "Теория циклических ускорителей", Москва, Физматгиз, 1962.

[2] Корн Г., Корн Т., "Справочник по математике для научных работников и инженеров", Москва, Наука, 1968.