SMOOTHING THE ERRORS IN THE EQUIPMENT ALIGNMENT ON THE JOINTS OF THE UNK SECTIONS

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As it is known, the 3000 GeV Accelerating; and Storage Complex (UNK) is being constructed on the basis of the 70 GeV IHEP accelerator.

The existing 70 GeV accelerator will be used for the initial particle beam acceleration, which will be injected into the UNK to be accelerated further. In this connection, one of the major problems is to transport the 70 GeV particle beam from the present accelerator to the UNK electromagnet. This problem involves others, including the high-precision geodetic adjustment of the equipment for the UNK and for Injection Beam Line (IBL).

In order to solve the complicated problem of the IBL components installation the following technique will be used, taking into account the IBL geometric and physical features.

For security of the general beam trace configuration and of the mutual design position of the components, the IBL was broken into separate sections to be installed independently of each other. These sections are specified by the equipment base elements, installed in the project position in advance. The quadrupole magnets positioned near the correction magnets, adjusting the beam trace, are used as the base elements. These base elements serve as the foundation for the equipment installation inside the section. Thus, each section is installed separately from each other, allowing one to make the adjustments irrespective of whether the construction work is over or the assembling work is done in neighbouring sections. Due to different accumulation of the adjustment errors in every section, the angles causing trace kinks will inevitably occur. The effect of these kinks on the orbit distortion of moving particles is compensated by the correction magnets, placed on the section joints.

Such a technique is supposed to be in use for equipment adjustment in the UNK with its orbit broken into 19 sections, each to be installed independently.

The proposed method enables one to solve the complicated problem of the precise adjustment of the electromagnetic equipment for the UNK. However, to decrease the load on the correction system it is necessary to smooth the beam orbit in the places of joints. To solve this problem the algorithm smoothing the errors in the mutual position of the equipment in the places of section joints has been developed at IHEP.

If the adjusted interval adjoins one or two sections, where the equipment has been installed (Fig.1), then during its alignment the radial offsets r_1 and r_2 of its marking base elements, denoted in Fig.1 by 1 and 2, are measured. The offset of each base element is measured relatively to the neighbouring quadrupoles with one of them on the adjusted section and the other - on the adjoining one. The differences Δr_1 and Δr_2 between the obtained values of the offsets and the theoretical ones are then used in the smoothing operations.

The idea of the smoothing process is offered to find the mathematical function Y(x), which should satisfy the following requirements:

1). it must have continuous derivatives in all points;

2). its curvature must be minimal.

The polynomial interpolation does not ensure the continuous derivatives in all points and can introduce considerable errors into the intervals between separate section joints. Often when the number of joints increases, the interpolation error does not only decrease, but begins to rise.

The interpolation using aspline function is free from these demerits. Mathematically, a spline is a special polynomial, taking in the joints the values $Y(x)=Y_i=Y(x_i)$ and providing the continuity of the derivatives at these points.

Let us take n+1 nodal points. The interpolation function f(x) in the [a,b] interval will be searched for among k-th power spline functions $S_k(x)$. Let K_n be the group of nodal points (nodes) and $a=x_0<x_1<\ldots<x_n=b$. Function $S_k(x)$ is named the $k\geq 0$ power spline function in the k_n interval if [1]

a) $S_k(x) \in C^{k-1}[a,b]$;

b) $s_k(x)$ is a polynomial of the power not greater than k for $x \in [x_{i-1}, x_i]$.

The set of the spline functions in the K_n interval is denoted via $S_k(K_n)$. The spline function $\hat{S}_k(x) \in S_k(K_n)$ is termed the interpolating spline function if

$$\hat{S}_k(x_i)=y_i=f(x_i) \qquad (i=0,1,\ldots,n). \tag{1}$$

To make the first and second order derivatives continuous, it is sufficient to use a third-order spline-polynomial (cubic spline).

It is important that in addition to the known properties, the interpolating spline function has another two extremum properties:

1. Among all the interpolating functions $\varphi(x)$, twice differentiated in [a,b], only the third-order interpolating spline function brings the minimum to the functional

$$J(\varphi)=\int_{x_0}^{x_n} (\varphi''(x))\,dx, \quad \text{i.e.}$$

$$\int_a^b (s''(x))^2\,dx \leqslant \int_a^b (\varphi''(x))^2\,dx \tag{2}$$

for all functions $\varphi(x)$ such that $\varphi(x_i)=y_i$.

2. For all $S(x) \in S_3(K_n)$ and the twice differentiated continuous functions f_n such that $f(x_i)=y_i$, the equation

$$\int_a^b (f(x)-s(x))^2 dx \leqslant \int_a^b (f(x)-S(x))^2 dx \tag{3}$$

holds true.

It should be noted that equality (3) is valid only for the interpolating spline function

$$S(x)=s(x)\ . \tag{4}$$

Property (1) actually means the curvature of the interpolation line becomes minimal.

Finally, let us define the cubic spline function for each interval $[x_i, x_{i+1}]$ as[2]

$$Y_i(x)=\frac{1}{6h_i}[m_i(x_{i+1}-x)^3+m_{i+1}(x-x_i)^3]+ \\ +\frac{1}{h_i}[(y_i-(m_ih_i^2/6))(x_{i+1}-x)+ \\ +(y_{i+1}-(m_{i+1}h_{i+1}^2/6))(x-x_i)], \tag{5}$$

where $h_i=x_{i+1}-x_i$, $y_i(x)=y(x)$, $m_i=y''(x_i)$ and i=1.2,...,n (n is the number of nodes). If x_i, y_i and m_i are known. this equation defines the spline approximation.

If one requires that the condition $Y_i(x)=y_i$ be fulfilled, then the above equation (5) for cubic splines leads to the linear system of equations. One finds m_i from it :

$$h_im_i+2(h_i+h_{i+1})m_{i+1}+h_{i+1}m_{i+2}= \\ =6[(y_{i+2}-y_{i+1})/h_{i+1}-(y_{i+1}-y_i/h_i)]. \tag{6}$$

Since this system does not determine m_i completely, it is necessary to specify additional boundary conditions. If they are $m_1=0$ and $m_n=0$, we have normal spline functions; and if they are $m_n=m_1$ and $m_{n+1}=m_2$ we have periodic ones , etc.

In our case, if either end of the section to be installed adjoin those already installed, the number of nodes n is 4. There are base elements 1 and 2 (see Fig. 2) and the installed elements 0 and 3, joining them. Therefore the values of y in the nodes are as follows:

$$y_0 = \frac{\Delta r_1 \cdot L_1}{l_1}$$

$$y_1 = y_2 = 0 \qquad (7)$$

$$y_3 = \frac{\Delta r_2 \cdot L_2}{l_2}\,.$$

Having estimated the values of y(x) from equations (5) and (6) for the places of the equipment elements in the interval to be installed, it is necessary to take them into account during the radial displacements of the elements into their design positions. Similar actions are to be taken during the height positioning of the equipment.

Using the above algorithm and making the numeric simulation for adjusting the IBL equipment, we have obtained the interpolating curves smoothing the mutual position of the IBL equipment elements and have determined the required displacement values to place the elements on these curves.

The analysis of the curves obtained allows one to conclude that the errors, introduced by spline interpolation into the mutual position of the elements, are negligible and smooth. In addition, the smoothing procedure permits to decrease the load on the correction system.

Using the simulation results one may arrive at the conclusion on the possibility of redistributing the accumulation of stochastic errors when joining a number of sections. In this case, the error line reaches its maximum curvature near the joining points.

The results of the beam dynamics computations for the IBL indicate that smoothing decreases the maximum corrector loading from 89% (before smoothing) to 42% with the 0.3% probability of their rise. The double reduction of the correction system loading will make it possible to use it to eliminate other, nongeodetic defects.

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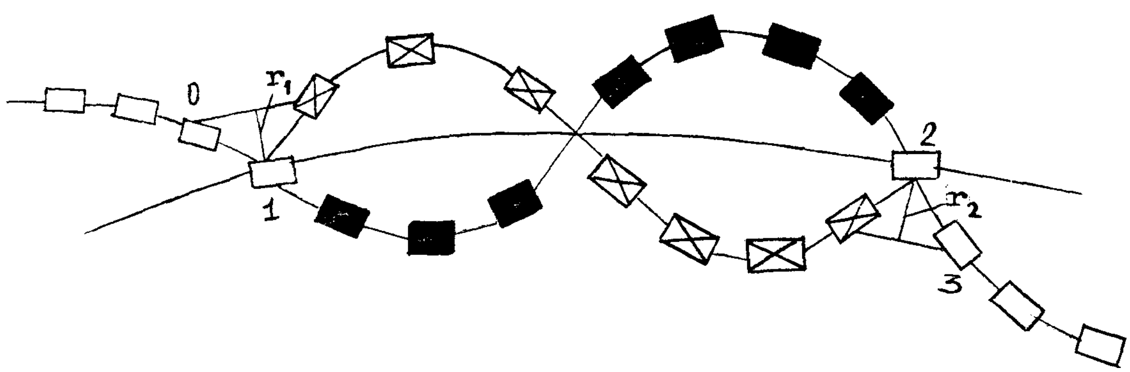

Fig.1. The diagram of the smoothing operation.

– the design position of the equipment

– the equipment position after the smoothing procedure

– installed sections

– section to be installed

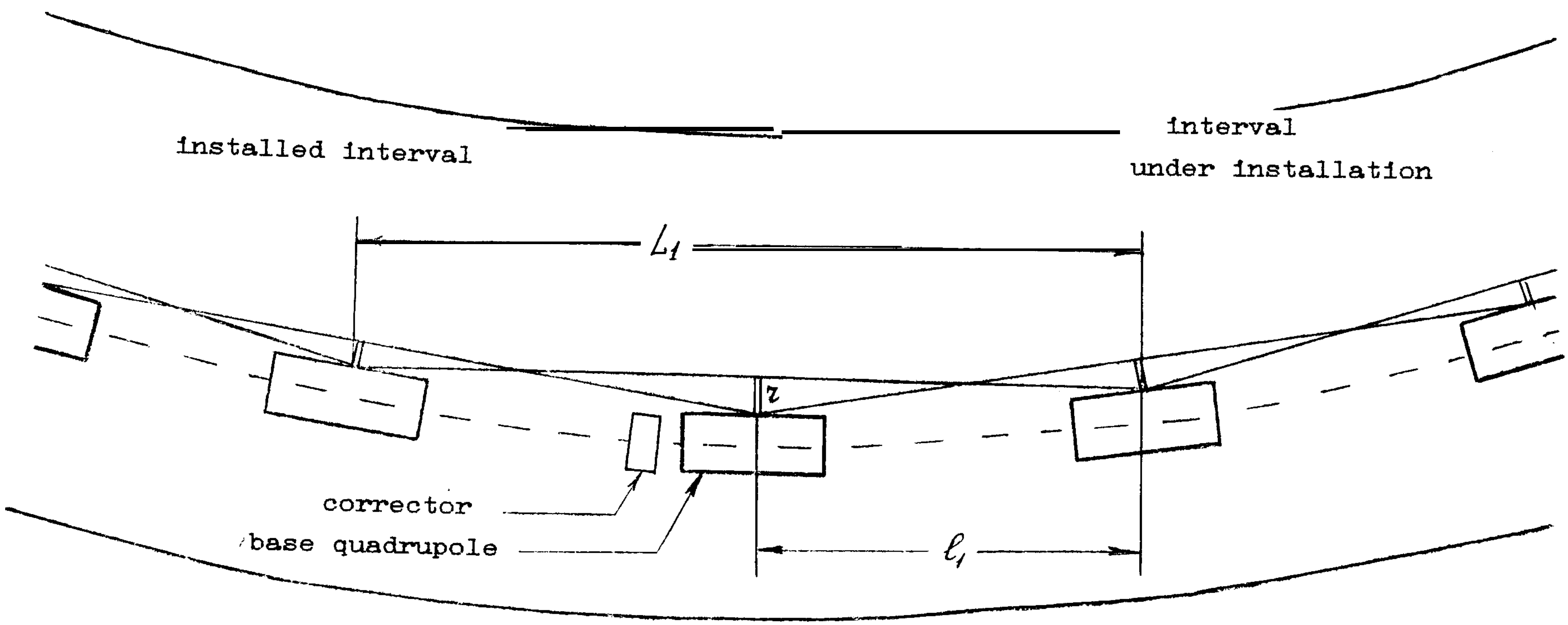

Fig. 2. The joint of the separate UNK sections