THE APPLICATION OF THE PRINCIPAL CURVE ANALYSIS TECHNIQUE TO SMOOTH BEAM LINES

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Introduction

The smoothness of a beam line refers to the quality of the relative positioning of a number of adjacent beam guiding components. The fact that smoothness is of highest priority when positioning magnets can be seen in the local tolerances imposed by the beam optics. In the past, smoothing has been done by separating horizontal and vertical misalignments and then applying some sort of analytical or manual "feathering" technique. The Stanford Linear Collider (SLC) did not easily lend itself to this sort of smoothing because of the highly coupled nature of its pitched and rolled beam line. This paper will discuss an attempt to develop a. repeatable method which is independent of the inconsistencies of human judgment and can simultaneously smooth in two or more dimensions.

Goals

Four major goals were defined for the smoothing algorithm used on the SLC alignment. The first, was to simultaneously model errors for both horizontal and vertical directions. Secondly, a smooth curve whose shape was suggested by the data and not by a predetermined model was implied by the fact that unknown systematic errors were being eliminated. Thirdly, this curve must be a reproducibly fit, independent of the inconsistent nature of human judgment. Fourth. the result of the procedure was to minimize the number and size of magnet movements to reach the final alignment criteria.

Smoothing The SLC Beam Lines

The alignment tolerances set out for the SLC show how smoothness is more important than absolute positioning for beam transport. For this machine a global positioning envelope is set to ±5 mm for every arc magnet, while the relative alignment of three adjacent magnets must be within ±0.100 mm (Figure 1). Probably the biggest enemies in achieving this last tolerance are unmodelled systematic measurement errors.

The pitched and rolled sausage-link beam line formed by the arc magnets makes this modeling particularly difficult. The absolute design shape of the path is a series of curves and straight sections in pitched and rolled planes. This form does not readily lend itself to fitting with polynomials or splines. The large coupling of X and Y motions also prevents the separation of smoothing operations into two components.

Solution

The complication of an irregularly shaped beam line was eliminated by subtracting out, the actual size and shape of the beam lines. leaving a series of residual misalignments for a string of magnets. Figure 2 shows an example plot of X and Y residuals which now must be modeled. This plot indicates a highly systematic trend which can be explained by unaccounted systematic errors inherent in the measurement procedure. These errors cause a strain or bowing when the observed network is adjusted using least squares. holding the endpoints fixed at their ideal coordinates.

Principal curve analysis was chosen to simultaneously pass a one dimensional curve through the X and Y residual misalignments mapped out along the Z-axis.* This curve will pass through the middle of the data set such that the sum of the squared errors in all variables are minimized. If the procedure were allowed to iterate many times the curve would approach almost all the data points thus creating a form which may or may not be considered smooth. This presents a problem of determining what is smooth.

The 0.1 mm local alignment. tolerance suggests a calculable smoothness criteria. Figure 3 shows how a 0.1 mrad angular offset of one magnet direction to its neighbors will result in a 0.1 mm transverse offset at beam line. If this threshold is exceeded, the curve is defined as no longer being smooth.

One of the goals of smoothing was to minimize movements of the magnets to a smooth curve. However, if an outlier from the trend curve exists, it may artificially bias the fitting routine and draw the curve away from the general neighborhood trend. For this reason a robustness estimator is included in the modeling program to weight out these points.

The end result has been a series of transformation programs which standardize the measured magnet misalignments to a common reference line where they can be modeled. A one dimensional principal curve which passes through the middle of the multidimensional data set is then fitted. This curve is non-parametric with its shape suggested by the data. Through robustness estimators and physics defined smoothness criteria, the curve is reproducibly fit to the trend of the alignment

* "Principal Curves and Surfaces" by Tremor Hastie; SLAC Report - 276, 11/84.

data. What results is a table of movements to be applied to the magnets to align them to the smooth trend curve.

Observation Plan and Adjustment

The horizontal network consists of a series of direction sets measured from stations directly on the magnet reference points. The observation plan is designed so that each point including the positions occupied by the instrument is sighted at least three times. What results is an extremely long and narrow network made up of directions. To strengthen the network in the beam direction, distances between reference points are measured with the Distinvar. These observations are then adjusted in a least squares routine where the endpoints of the network are held as knowns. This constraint often causes the bowing seen in the residual misalignments plotted in Figure 2.

The vertical network is built up through overlapping leveling runs which span the same reference points observed in the horizontal measurements. This network is also reduced by holding fixed the same end points of the total level line. The residual misalignments in Y as shown in Figure 2 tend to be more random due to the lack of unmodeled systematic errors encountered in leveling.

Example

At first this technique was applied only to arc magnets ignoring junctions between the arcs and the special sections of the beam line such as the final focus. This was done because evenly spaced magnets with a homogenous observation plan were easy to describe and model. However, this homogeneity was broken when one proceeded into the final focus where magnets were irregularly spaced and each had multiple reference points for the alignment. The following example was the first attempt at smoothing the transition between the south arc and the final focus.

Figure 4 shows the misalignments in X and Y which must, be modeled. Figures 5 shows the step by step fitting of the data in the Y direction. The X direction is also being smoothed during the same operation but is not shown here. Note the application of the robustness measure on the data points part way down the beam line. Figure 6 shows the final fitted curve in the X direction. The “rough” look to the plot results from the uneven scale on the horizontal axis verses the vertical axis (60,000 mm verses 1.2 mm). Table 1 summarizes the final movements to be applied to the magnets. All changes of 60 microns or greater are flagged for adjustment. Finally, Table 2 shows the results of a check measurement of the same area. The only remaining adjustments are ones caused by mechanical modifications to the magnets after the initial alignment.

Further Improvements to the Technique

Two immediate improvements are suggested through our experience. The first involves the weighting of points in the smoothing routine so that a small area of magnets can be "patched in" to existing elements. This would enable one to force the direction of the beam line if movements of one section were not desirable. The second improvement is not quite as straight forward. It involves the definition of what is smooth. How does one stop the iteration process on sections of a beam line where the alignment tolerances are not as well understood as they are for the arcs. If this problem can be solved a general smoothing algorithm for any beam transport line could be developed.

Conclusion

Through the application of principal component analysis a successful smoothing procedure was developed for the highly coupled SLC beam line. The routine allows for a flexible yet repeatable fitting of data which may be superimposed with with unmodeled systematic measurement errors. A smoothness criteria was developed which reflected beam alignment tolerances and still allowed the number of magnet adjustments to be minimized. The technique still needs improvement to make it a general smoothing program but its application to the SLC has already been highly successful.

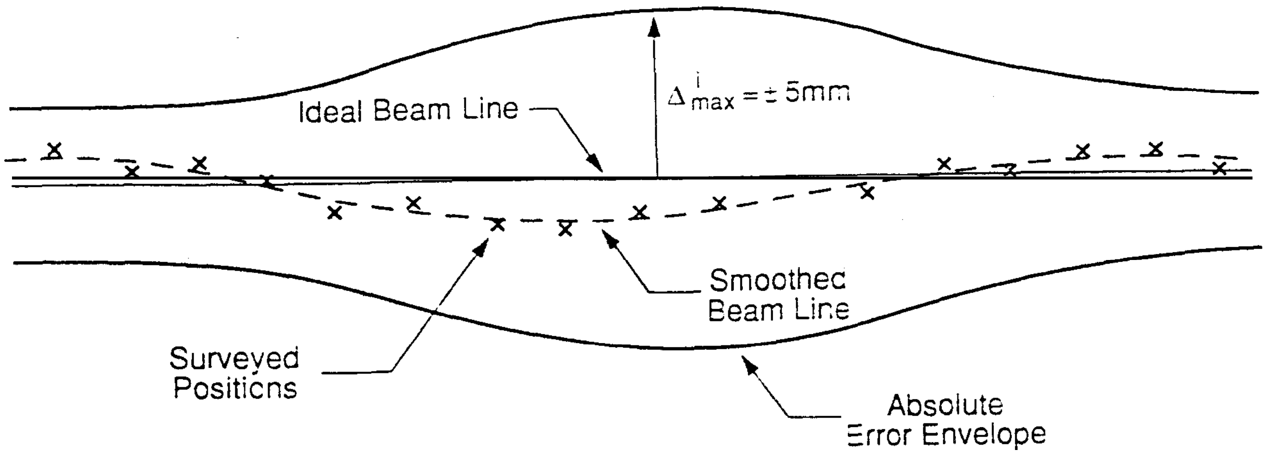

Figure 1. Global Positioning Envelope

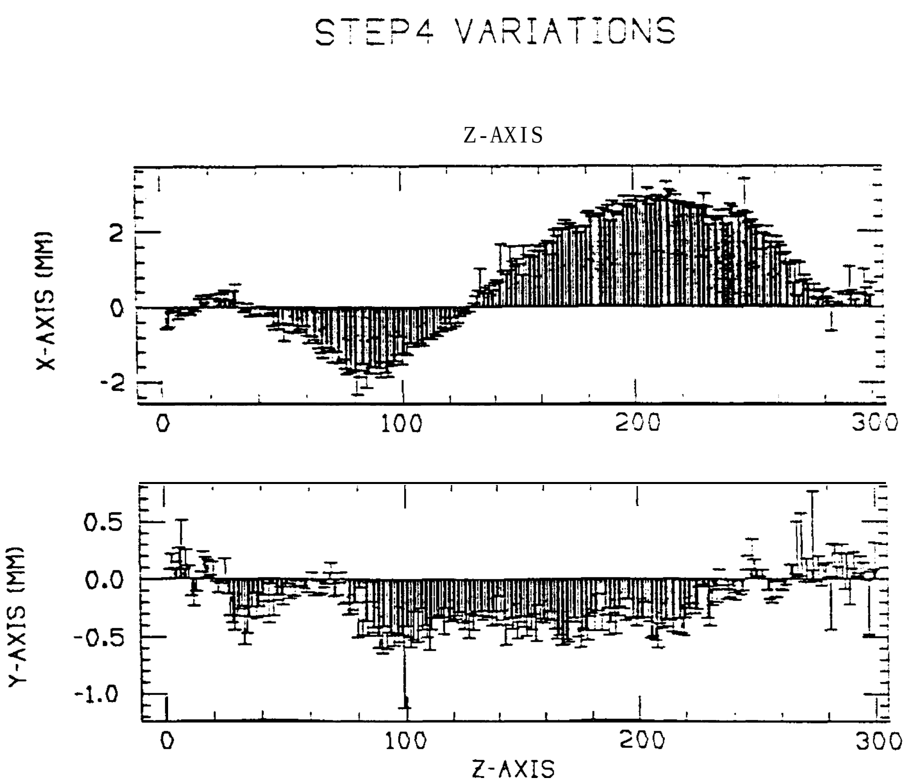

Figure 2. X and Y Residuals

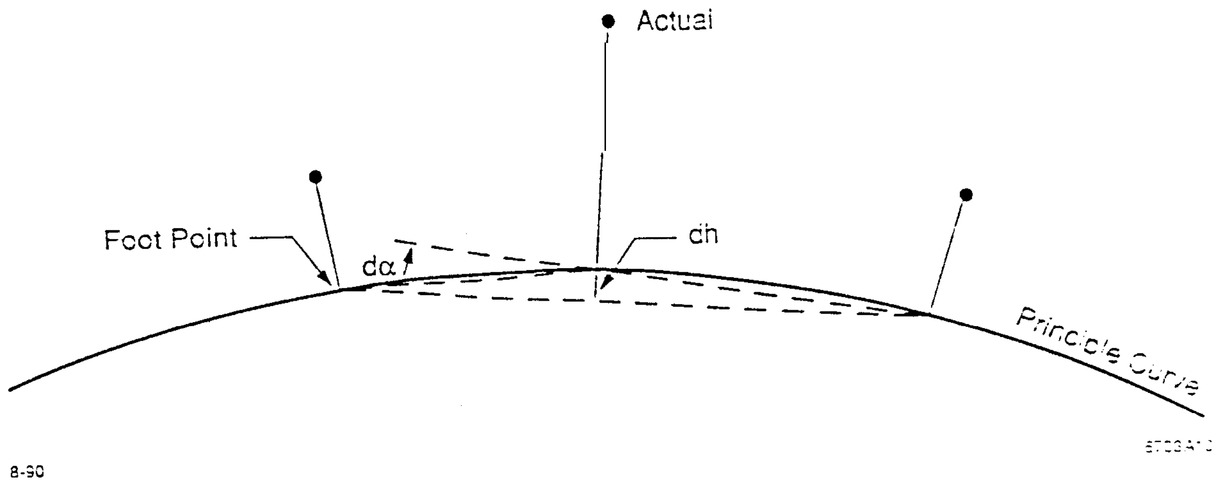

Figure 3. Smoothness Criteria

$d\alpha \leq 0.1$ mrad
dh 0.1 mm

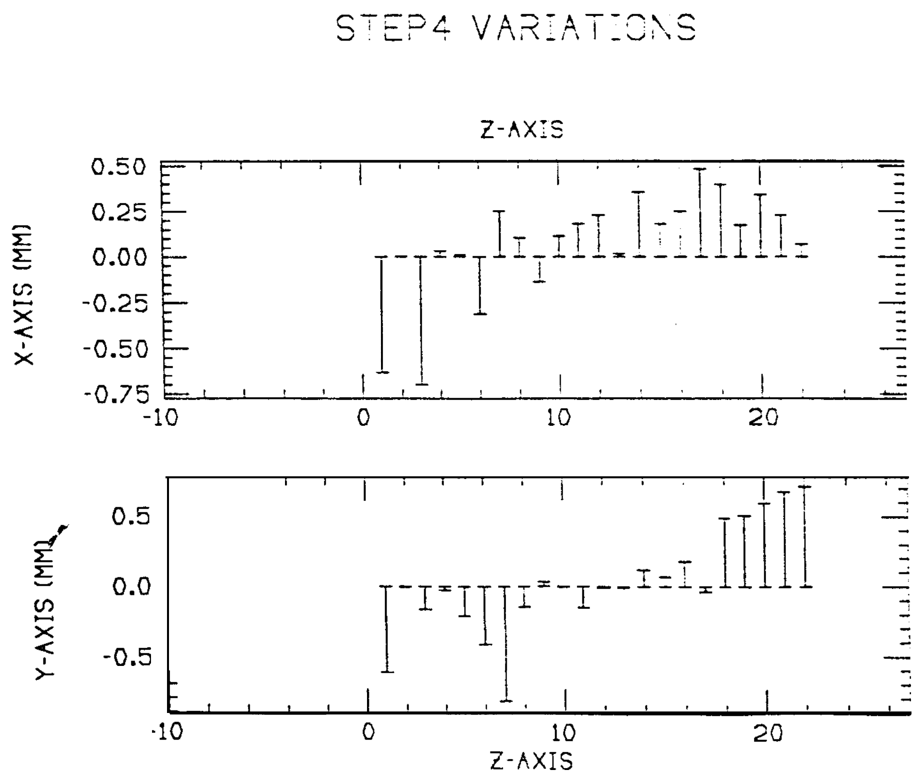

Figure 4. X and Y Residuals

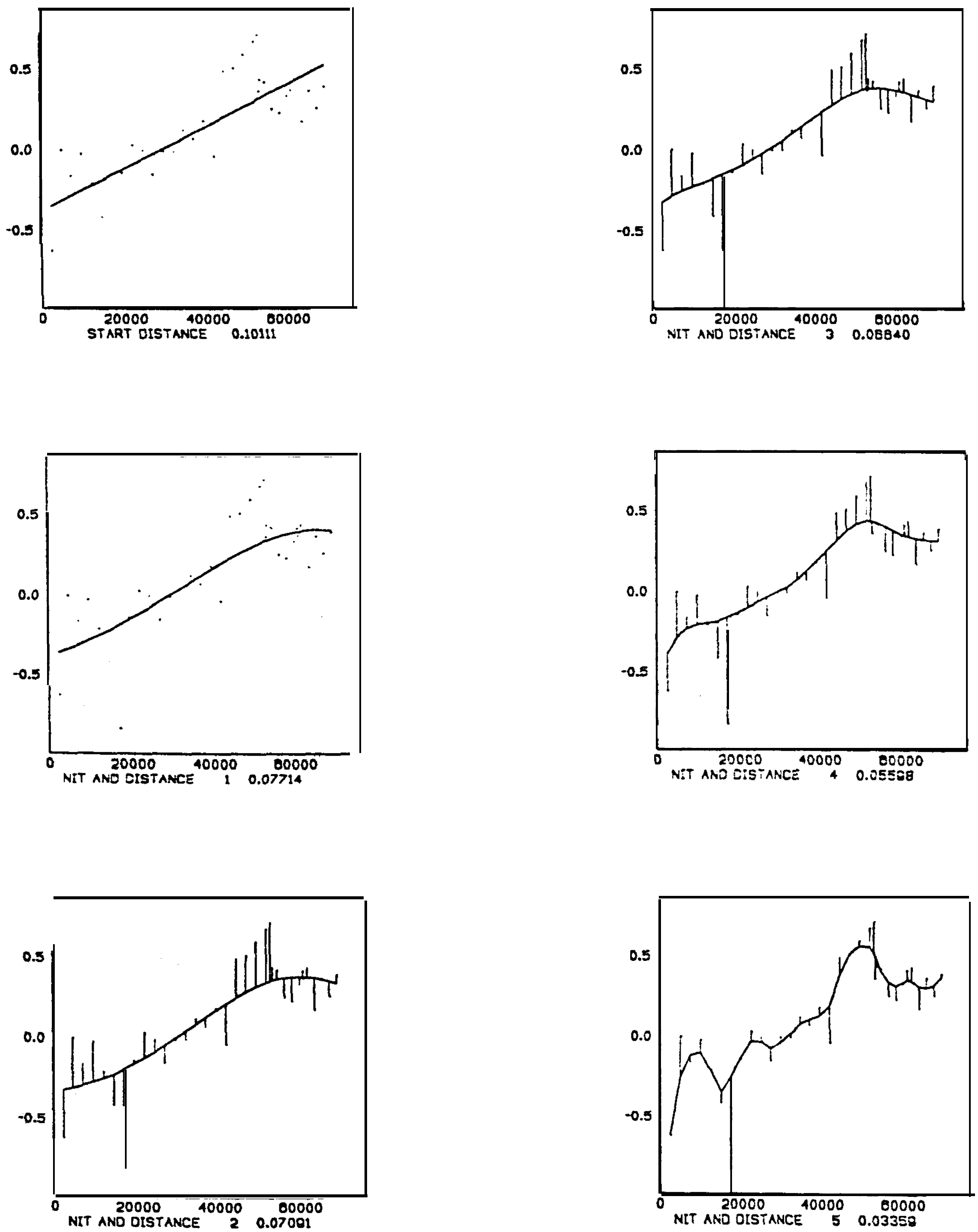

Fig. 5. Step-by-Step Fitting of a Smooth Curve in Y Direction

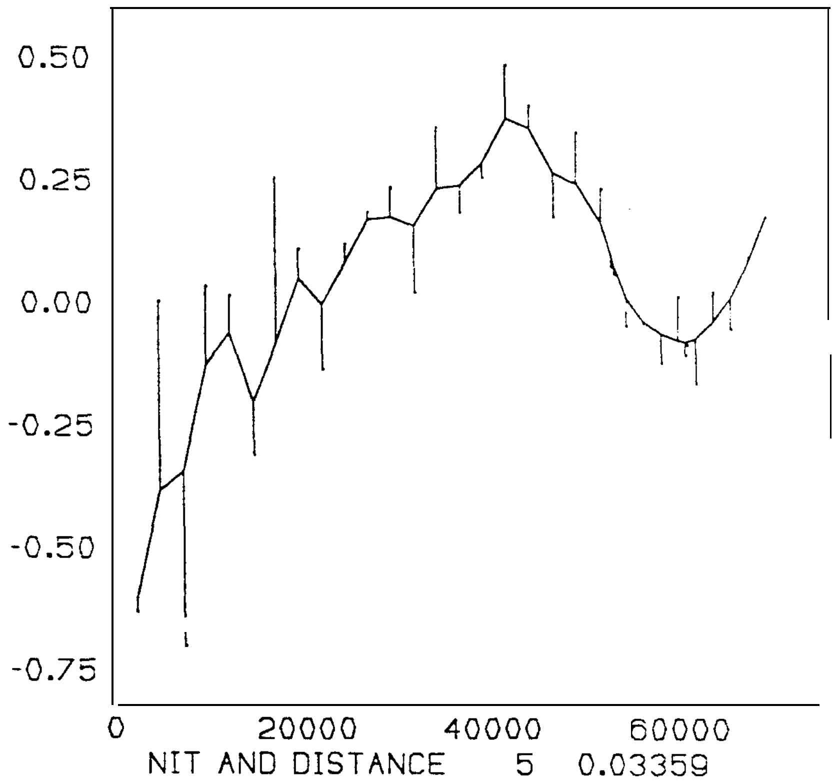

Fig. 6. Final Fitted Curve in X-Direction

STEP4 DIAL GAGE ADJUSTMENTS - 02-04-89 - 10:36

NAME	Z[mm]	X[mm]	Y[mm	
XS2301FF	0.01081	0.13784	0.23016	* *
XS2302FF	-0.01273	-0.37920	-0.28359	* *
XS2303FF	-0.00280	0.42136	-0.06511	* *
XS2304FF	-0.00719	-0.23363	-0.17675	* *
XS2305FF	0.00058	-0.15426	0.01493	* *
XS2306FF	0.00852	0.21948	0.23377	* *
XS2307FF	0.02288	-0.30323	0.66675	* *
XS2308FF	0.00039	-0.11449	0.01219	*
XS2309FF	-0.00389	0.17062	-0.12944	* *
XS2310FF	-0.00170	-0.03913	-0.06088	*
XS2311FF	0.00300	-0.06695	0.11618	* *
XS2312FF	0.00005	-0.07959	0.00205	*
XS2313FF	0.00085	0.16814	0.03980	*
XS2314FF	-0.00088	-0.14156	-0.04550	*
XS2315FF	0.00099	0.05692	0.05766	
XS2316FF	0.00012	-0.00191	0.00801	
XS2317FF	0.00372	-0.22948	0.29115	* *
XS2318FF	-0.00177	-0.15982	-0.16799	* *
XS2319FF	-0.00106	0.04017	-0.12900	*
XS2320FF	-0.00102	-0.17158	-0.16960	* *
XSFF01FF	-0.00103	-0.11324	-0.23440	* *
XSFF01RF	-0.00120	0.02223	-0.27376	*

MEAN AND VARIANCE FOR Z X AND Y

ZMEAN:	0.00075663	ZVAR:	0.00680846
XMEAN:	-0.04324045	XVAR:	0.18852785
YMEAN:	-0.00287997	YVAR:	0.21795110

Table 1.

Magnet Movements After the First Iteration

STEP4 DIAL GAGE ADJUSTMENTS - 02-10-89 - 16:58

NAME	Z[mm]	X[mm]	Y[mm]	
XS2301FF	0.00003	0.00039	0.00062	
XS2302FF	-0.00103	-0.01428	-0.02283	
XS2303FF	0.00802	-0.30181	0.18715	* *
XS2304FF	-0.00001	-0.00246	-0.00024	
XS2305FF	-0.00038	0.00092	-0.00983	
XS2306FF	0.00047	0.00876	0.01287	
XS2307FF	0.00052	-0.00364	0.01501	
XS2308FF	-0.00038	0.01081	-0.01174	
XS2309FF	-0.00009	-0.02333	-0.00305	
XS2310FF	-0.00023	0.00609	-0.00808	
XS2311FF	0.00030	0.00893	0.01178	
XS2312FF	-0.00009	-0.02992	-0.00382	
XS2313FF	0.00069	0.02967	0.03198	
XS2314FF	-0.00048	-0.03095	-0.02498	
XS2315FF	0.00020	-0.00269	0.01184	
XS2316FF	-0.00004	0.04543	-0.00285	
XS2317FF	-0.00004	-0.03912	-0.00333	
XS2318FF	0.00001	-0.04687	0.00112	
XS2319FF	0.00005	-0.00008	0.00615	
XS2320FF	-0.00053	0.08747	-0.08847	* *
XSFF01FF	-0.00024	-0.10377	-0.05333	*
XSFF01RF	-0.00003	-0.11914	-0.00660	*

MEAN AND VARIANCE FOR Z X AND Y

ZMEAN:	0.00030547	ZVAR:	0.00176313
XMEAN:	-0.02361905	XVAR:	0.07592055
YMEAN:	0.00178909	YVAR:	0.04824993

Table 2.

Magnet Movements After a Second Iteration