# HIGH ENERGY THEORY

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## I. Introduction

We all know that the basis of physics is experiment. Without it, theoretical physics would be reduced to mere philosophical speculation. However, it may not be as widely appreciated that the heart of experimental physics is instrumentation: Without appropriate instruments, experiment could hardly flourish. That this has always been the case can be illustrated by the figures on the next two pages. If Archimedes did not have the wonderful circular instrument in Figure 1, it might have been difficult for him to do his famous experiment.

As fully realized, especially at SLAC, linear instruments play an equally important part. In Figure 2 we give one such example. Without this marvelous linear tool, which is now in Florence, it would have been impossible for Galileo to do some of his celebrated experiments. And if we did not have those we might not even have the beginning of classical physics.

The development of linear and circular facilities played an even more basic role in high energy physics. Without these accelerators it would simply not be possible to have today's particle physics. This talk is dedicated to one of the true giants in this field, Wolfgang K. H. Panofsky.

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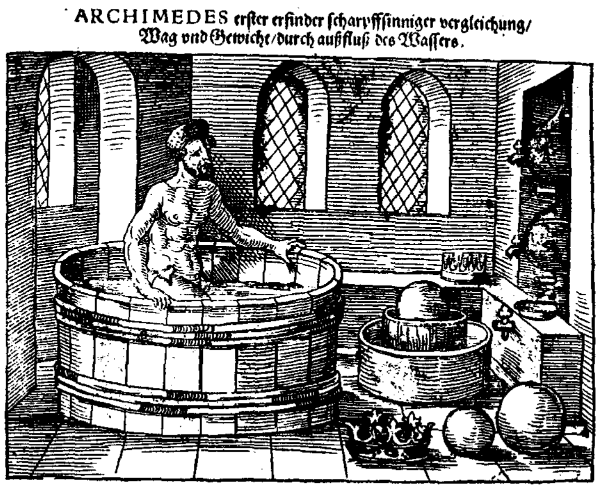



Figure 1.

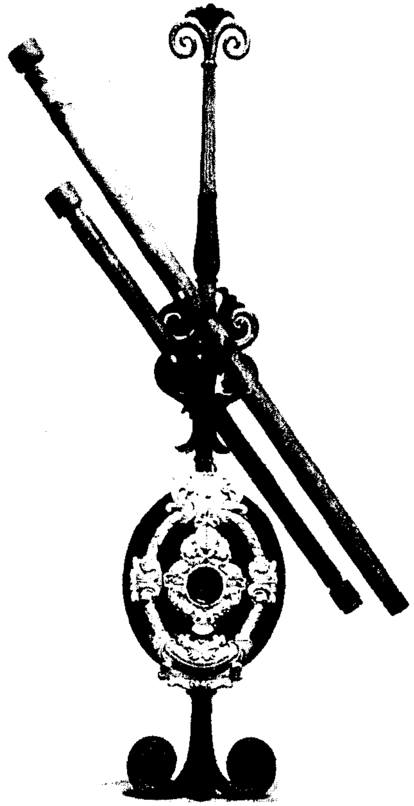


Figure 2.

## II. Present Status

High energy physics can perhaps be dated to the construction of the cyclotron by Lawrence in 1928. In theory we may regard Fermi's formulation of the weak interaction in 1932 and Yukawa's meson hypothesis of the strong interaction in 1934 as the beginning of the present era. During the fifty years since then we have made tremendous progress in our understanding of the fundamental structure of matter. The table on the next page comprises almost all the highlights in our field for more than three decades. It is of interest to note some consistent patterns:

1) With the exception of the anti-nucleon and the intermediate boson, none of these landmark discoveries was the reason for the construction of the relevant accelerator.

When Lawrence built his 184" cyclotron, the energy was thought to be below pion production. Therefore, after the cyclotron was turned on, even though pions were produced abundantly, for a long time nobody noticed them.

The progress of particle physics is closely tied to the discovery of resonances, which started with the (3,3) level found at the Chicago cyclotron. Yet even the great Fermi, when he proposed the machine, did not envisage this at all. When the Cosmotron was constructed, some high priests of theory thought that the most important high energy problem was to understand the angular distribution of pp collisions, which remains mysteriously flat even at a few hundred MeV, although at that energy s, p, d, f, g · · · waves are all involved. But as we all know, it was the dynamics of strange particles that put the Cosmotron on the map.

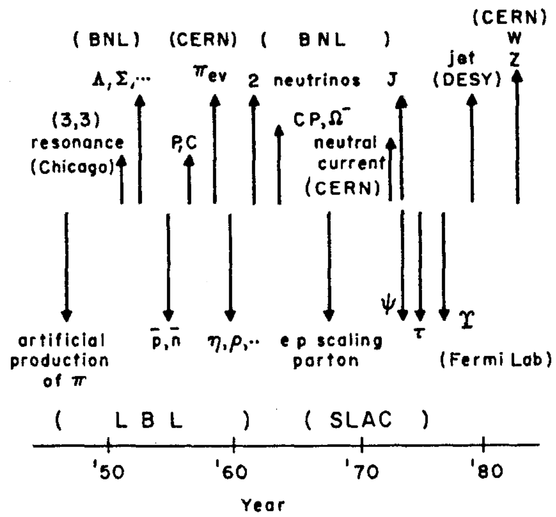



Table 1

We could go on and on, and the same pattern would repeat itself throughout this list. There is no reason for us to believe that it will change. Nor should we expect too much from our present theorists for the prediction of the future. This certainly will make my task much easier.

2) Another interesting feature is the density of great discoveries per unit time. As we can see, this is quite uniform and averages out to about one in two years. There is every reason for us to expect that very soon there should be another major breakthrough beyond the present intermediate boson physics. From CERN we hear reports of several rather puzzling high energy $\bar{p}p$ events, some give a lepton pair plus a hard photon with large probability, and some give only a single jet or a photon with large missing 4-momentum. Perhaps these are indications of the future. Although we do not know how it will eventually shape up, it is just this unpredictability that makes physics so challenging.

While theorists are not very good at predicting the future, we <u>are</u> full of hindsight. The present status of theory may be summarized as follows:

| Interaction | Theory |
|---|---|
| strong | Q C D |
| electro-weak | SU(2) X U(1)<br>Q E D |
| gravitation | general relativity |

The remarkable thing is the simplicity of the theoretical basis of this entire edifice, with renormalizable quantum gauge theory for the strong and electroweak interactions, and Einstein's theory of general relativity for gravitation. The whole foundation was essentially available almost 50 years ago. We could speculate that a bright young theorist might have been able to construct QCD and SU(2) X U(1) gauge theory in the 1930s. Indeed this was attempted by Oscar Klein, who published a remarkable paper in <u>New Theories in Physics</u> (International Institute of Intellectual Cooperation, League of Nations, 1938).

In that paper, Klein started with the Kaluza-Klein theory and proceeded to formulate an SU(2) non-Abelian gauge theory with the neutral member of the gauge field identified as the photon. In addition, Klein proposed one generation of hadrons consisting of the proton and neutron, plus one generation of leptons, the electron and neutrino. There it is! All the essential ingredients are correct. Yet somehow, like the League of Nations itself, the idea was right, but the components did not quite represent the practical reality. From this, maybe we can draw a lesson. Without experimental facts, it is perhaps impossible to foresee all the details of our future theory.

In Klein's time, intelligence alone could not predict the complexity of color and flavor degrees of freedom that we know today. Nevertheless, it was possible to anticipate the broad theoretical basis on which the future might rest. Therefore in this talk I will concentrate on the theoretical foundation, not the detailed structure.

Let us now examine the major problems in today's physics:

1. *Quantization of gravity*

General relativity lies outside the scope of renormalizable theories. Our inability to quantize gravity shows that there should be new physics, at least at the level of the Planck length, $10^{-33}$ cm.

2. New puzzling $\bar{p}p$ events

The new high energy $\bar{p}p$ events observed at CERN, mentioned before, give indications that there might be new physics at $10^{-16}$ cm.

Both of these may lead us into territories outside our present theory.

3. Number of parameters in the present theory

In addition, there are too many parameters in our present theory: the various masses of quarks and leptons, the different angles required for mixing, the CP - violating phase, etc. All these make one feel that our present theory is far from perfect and may only be phenomenological in nature.

From the uncertainty principle, we should associate small distances with large masses. Since at distances larger than $10^{-16}$ cm we know that the present theory works quite well, it is not clear how the unknown physics of small distances can produce the type of small masses that we know today. Now, the product of a small mass times a small distance is an even smaller dimensionless number. This implies that our underlying fundamental theory must either contain or be able to generate some extremely small dimensionless physical parameters.

This is hardly surprising since we already have a few such small parameters on hand: for example, the ratio of the CP violation amplitude to the strong interaction amplitude and the ratio of proton number to photon number in the universe. They are both $\sim 10^{-9}$. Any theory containing parameters of the order of $10^{-9}$ can easily upset some naive estimates. Squaring $10^{-9}$, we can relate the present 100 GeV physics into regions of the Planck length. In that sense, we may not have the right to dissociate gravitation from other physical interactions.

It is not difficult to convince oneself that the concept of local field theory is probably inapplicable to distances of the order of the Planck length. Imagine that someone measures the gravitational field at two nearby space-time points within the Planck length, but outside each other's light cone. Local field theory then assures us that these two experiments can be done independently of each other, no matter how close the points. Yet, just based on uncertainty principle, we expect the disturbance caused by these two measurements to be the creation of a black hole. Thus, it seems unreasonable to accept the local field theory at this level. Furthermore, if one does apply the present local field theory to gravity, incurable infinities arise to render the quantum theory unusable. Now if locality is not satisfied at the Planck length, then the correct physical theory must be nonlocal in character. The fact that the Planck length is small is beside the point. We would like this nonlocal fundamental theory to retain all the good features: Lorentz invariance, Poincaré invariance, non-Abelian gauge symmetries, unitarity and the general coordinate invariance of general relativity. In addition it should not contain divergence difficulties, so that quantization of gravity can be carried out. For the rest of this talk, I will give you one such candidate.

## III. Space and Time as Dynamical Variables (An Overview)

In this new theory that we shall discuss, space and time will be treated as dynamical variables. This is quite different from our traditional way of thinking. For example, in the usual local field theory, only the field is the dynamical variable. Both space and time are regarded as parameters. While the field is embedded in a continuous 4-dimensional space-time manifold, that manifold itself does not represent dynamics, but only kinematic parameters. This view dates back to Newtonian mechanics. We may summarize the role of time in our usual continuum theory as follows:

| | Usual Continuum Theory | |
|---|---|---|
| Classical Mechanics | $\vec{r}(t)$<br>$t$ | dynamical variable<br>parameter |
| Non-relativistic Quantum Mechanics | $\vec{r}(t)$<br>$t$ | operator (observable)<br>parameter |
| Relativistic Quantum Theory | field $\phi(\vec{r}, t)$<br>$\vec{r}, t$ | operator (observable)<br>parameters |

In our usual approach, the position $\vec{r}(t)$ of a particle is a dynamical variable in classical mechanics, but the time $t$ is a parameter. When we go over to the non-relativistic quantum mechanics, the observable $\vec{r}(t)$ becomes an operator while $t$ remains a parameter. In the relativistic theory, $\vec{r}$ and $t$ have to be treated on an equal basis. Two choices are open. Either regard $t$ as an operator or $\vec{r}$ as a parameter. Our traditional course is to opt for the latter: only the fields are operators or observables. The space-time coordinates are merely parameters. An alternative route is to see whether we can regard $t$ as an operator; this is then the essence of this new approach, which I call discrete mechanics, and may be summarized as follows:

| | Discrete Theory | |
|---|---|---|
| Classical Mechanics | $\vec{r}, t$ | both dynamical variables |
| Non-relativistic Quantum Mechanics | $\vec{r}, t$ | both operators (observables) |
| Relativistic Quantum Theory | $\vec{r}, t, \phi$ | all operators (observables) |

Thus, in the discrete version of relativistic quantum theory, the space-time position, as well as the field, is considered a dynamical variable. For example, in a collision experiment of, say, $e^+e^- \to \mu^+\mu^-$ at SLC in 1986, the precise location and time of the collision should be regarded as part of the measurement, on the same footing as the electric field, magnetic field, . . . . In order to incorporate such a view, let us start with the discrete theory in its classical form.

## IV. Classical Mechanics

Consider the example of a non-relativistic point particle of unit mass moving in a potential $V(\vec{r})$. In the table on the next page, we give the familiar formulation of classical continuum mechanics in the left column with $A_C$ as the action. The corresponding discrete version is given in the right column.

Let us consider a large time interval $T$. A fundamental postulate of discrete mechanics is that within such a time interval a particle can only assume $N$ space-time positions

$$(\vec{r}_n, t_n)$$

with $n = 1, 2, \cdots, N$. The ratio

$$\frac{N}{T} \equiv \rho$$

is a fundamental constant of the theory. For convenience and without loss of generality we have arranged $t_1, t_2, \cdots, t_N$ in ascending order. The discrete action $A_D$ is then given in the table. Unlike the continuum case (where only $\vec{r}(t)$ is the dynamical variable) we regard $\vec{r}_n$ and $t_n$ both as dynamical variables. Consequently there are two sets of equations:

$$\frac{\partial A_D}{\partial \vec{r}_n} = 0 \qquad (1)$$

which gives the discrete version of Newton's law, and in addition

$$\frac{\partial A_D}{\partial t_n} = 0 \qquad (2)$$

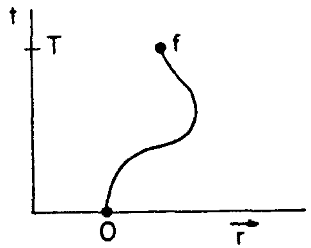
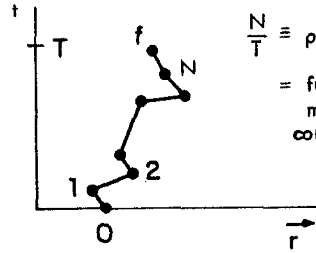
| Continuum Mechanics | Discrete Mechanics |
|---|---|
|   |   $\frac{N}{T} \equiv \rho$ = fundamental constant |
| $A_C = \int_0^T (\frac{1}{2}\dot{\vec{r}}^2 - V)\, dt$ | $A_D = \sum_n \{\frac{1}{2}\vec{v}_n^2 - \frac{1}{2}[V(\vec{r}_n) + V(\vec{r}_{n-1})]\} \cdot (t_n - t_{n-1})$<br>$\vec{v}_n = \frac{\vec{r}_n - \vec{r}_{n-1}}{t_n - t_{n-1}}$ |
| fix $\vec{r}(0) = \vec{r}_0$<br>$\vec{r}(T) = \vec{r}_f$ | fix $(\vec{r}_n, t_n) = \begin{cases} (\vec{r}_0, 0) & \text{when } n = 0 \\ (\vec{r}_f, T) & \text{when } n = N+1 \end{cases}$ |
| $\frac{\delta A_C}{\delta \vec{r}(t)} = 0$ gives $\ddot{\vec{r}} = -\nabla V$ | $\frac{\partial A_D}{\partial \vec{r}_n} = 0$ gives $\frac{\vec{v}_{n+1} - \vec{v}_n}{\frac{1}{2}(t_{n+1} - t_{n-1})} = -\vec{\nabla} V(\vec{r}_n)$ |
| $\vec{r}(t)$ = dynamical variable | $\frac{\partial A_D}{\partial t_n} = 0$ gives |
| $t$ = parameter | $E_n \equiv \frac{1}{2}v_n^2 + \frac{1}{2}[V(\vec{r}_n) + V(\vec{r}_{n-1})]$<br>$= E_{n+1}$ |

Table 2

which yields the energy conservation. There are altogether 4N unknowns:

$$\vec{r}_n = (x_n, y_n, z_n) \quad \text{and} \quad t_n \ .$$

Exactly the same number of equations is supplied by (1) and (2). In continuum mechanics conservation of energy is a consequence of Newton's equation. This is not so in discrete mechanics.

Had we treated time merely as a predetermined discrete parameter, the system would then violate time-translational invariance, and lead to energy non-conservation. Historically this has always been the difficulty encountered in *any attempt to treat time as a discrete parameter.* Here, by viewing $\vec{r}_n$ and $t_n$ as dynamical variables we bypass this problem.

For a free particle $V = 0$, Eq. (1) gives $\vec{v}_n$ = constant, which also satisfies (2). The trajectory of the particle is always a straight line, the same as in the continuum version. Therefore Newton's first law remains unaltered.

For a nonzero $V$, both $\vec{r}_n$ and $t_n$ are determined by the difference Equations (1) and (2). Both sets of variables respond to the potential and both are integral parts of the dynamics.

In the above discrete action, we use

$$\tfrac{1}{2}\left[V(\vec{r}_n) + V(\vec{r}_{n-1})\right] (t_n - t_{n-1}) \qquad (3)$$

as the discretized version of $V(\vec{r})\,dt$ in the usual continuum theory. Of course, we may also use an alternative form; e.g.,

$$V(\vec{r}_n)\,(t_n - t_{n-1}) \ , \qquad (4)$$

or some other choice. When $\rho = \frac{N}{T} \to \infty$, these different choices all approach the same continuum limit. However, when $\rho$ is finite, they correspond to different discrete dynamical systems. *We may ask: is there a principle to guide us so that the choice can be unique? This will be discussed in the next section.*

V. The Discrete Limit of Continuum Theory

vs

The *Continuum Limit of Discrete Theory*

In our usual approach the continuum theory is regarded as fundamental. The motion of a particle is then described by, say, a smooth continuous curve C in the x , t plane. As an approximation we may replace C by a continuous but piece-wise linear lattice curve D ; as illustrated in Figure 3(a). We shall call the vertices on D the lattice sites and the straight lines between the consecutive sites the links. Let $\ell$ be the average length of the links. When $\ell \to 0$ ; $D \to C$ and the discrete action $A_D$ for the different choices (3), (4), . . approaches the same continuum limit $A_C$ , as discussed before.

*In discrete mechanics we assume that the particle can only take on a discrete* number of space-time positions. If we represent these points by lattice sites in the x , t plane, and if we connect the neighboring sites by straight lines, a discrete path can be formed. The result is a continuous but piece-wise linear curve D . Let us keep its link length $\ell$ nonzero by fixing D. Consider a sequence of smooth continuous curves C which approach D as a limit, as shown in Figure 3(b). When $C \to D$ , the continuum action

$$A_C = \int (\tfrac{1}{2}\dot{x}^2 - V)\, dt \qquad (5)$$

approaches a unique limit $A_D$ . By taking the discrete limit of the usual continuum theory, we arrive at a unique discrete action $A_D$ .

If we accept this approach, then we may regard the discrete theory as a limit of the usual continuum theory. By considering only those orbits

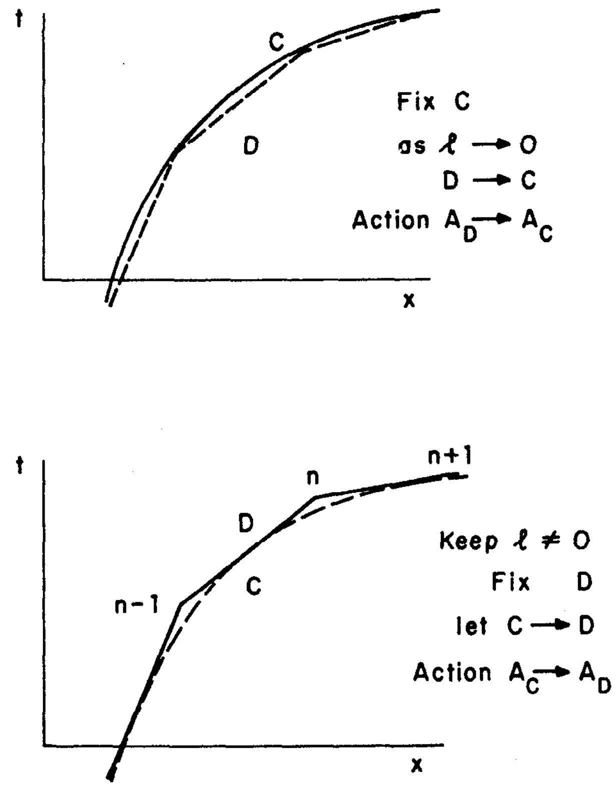



Figure 3.

$$x(t) = x_D(t) \tag{6}$$

in which $x_D(t)$ is continuous but piece-wise linear with the constraint of a fixed density $\rho$ of lattice sites (vertices), we can derive the discrete mechanics from the usual continuum theory. Because $x_D(t)$ is uniquely specified by the positions $x_i$ and $t_i$ of its lattice sites $i = 1, 2, 3, \cdots$, a variation in $x_D(t)$ is identical to a variation in $x_i$ and $t_i$. Therefore the usual differential equation

$$\frac{\delta A}{\delta x(t)} = 0$$

becomes simply the difference equations

$$\frac{\partial A}{\partial x_i} = 0 \quad \text{and} \quad \frac{\partial A}{\partial t_i} = 0$$

in discrete mechanics. Furthermore, since the constraint of a fixed density of lattice sites is an invariant concept, all symmetries of the usual continuum theory remain valid in its discrete limit. This is why our discrete theory can retain translational invariance in both space and time, and therefore conserves energy and momentum. The same idea can be generalized to the quantum theory, and the resulting S-matrix can be shown to satisfy unitarity.

## VI. Relativistic Quantum Field Theory

As an example, let $\phi(x)$ be a scalar field in the usual continuum theory with $x$ denoting the space-time coordinates. In the path integration formulation the operator $e^{-iHT}$ is given by $(\hbar = c = 1)$

$$e^{-iHT} = \int e^{iA_C} \left[ d\phi(x) \right] \tag{7}$$

where $H$ is the Hamiltonian operator, $A_C$ the usual continuum action and $T$ the total time interval. Because in the usual continuum theory the space-time coordinates $x$ are parameters, and only $\phi(x)$ are dynamical variables, the functional integration in (7) is over $\left[ d\phi(x) \right]$, not $\left[ dx \right]$.

In the discrete version, we impose a maximal* number $N$ of experiments that can be performed within any given space-time volume $V$, with

$$\frac{N}{V} \equiv \ell^{-4} = \text{fundamental constant.} \tag{8}$$

Each measurement determines the field $\phi_i$ as well as the space-time position $x_i$ with $i = 1, 2, \cdots, N$. The $i$ will be referred to as lattice sites, as illustrated by Figure 4(a). The Green's function (7) is now replaced by

$$\int e^{iA_D} \left[ dx_i \right] \left[ d\phi_i \right] \quad . \tag{9}$$

Because $\phi_i$ and $x_i$ are all dynamical variables, in the discrete theory we

* If more than $N$ measurements are performed, then one would discover from the results of the measurements that the space-time volume they cover would automatically be larger than the prescribed $V$.

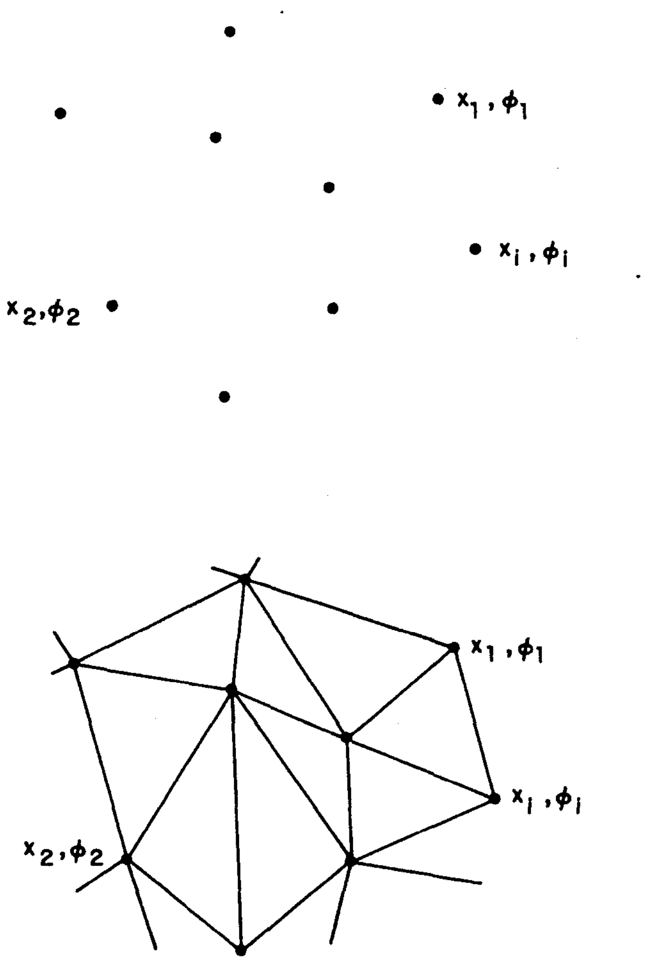



Figure 4.

integrate over $[d\phi_i]$ as well as $[dx_i]$. *The latter integration makes it obvious that rotational and translational symmetries can be maintained in the discrete theory.*

To simulate the local character of the usual continuum theory, each site in the discrete theory is coupled only to its neighboring sites, as illustrated in Figure 4(b). The whole volume is then divided into triangles if the dimension $d = 2$, tetrahedra if $d = 3$, etc. The algorithm of linking the sites into a simplicial lattice for any $d$ is given by Christ, Friedberg and Lee.[1] The result is a simplicial lattice D.

To derive $A_D$, we may regard the discrete theory as the limit of the usual continuum theory. Again we consider a sequence of smooth continuous surfaces C in the $(d + 1)$- dimensional space (the field $\phi$ plus the $d$ components of the space-time coordinate $x$). In this sequence the surface C acquires sharper and sharper ridges, so that in the limit $C \to D$. The limiting lattice surface D is continuous but piece-wise flat. Just as in the previous section, $A_C \to A_D$ which gives a unique expression for the discrete action. Since D is characterized by the positions of its lattice sites $\phi_i$ and $x_i$ in the $d+1$ space, a variation over the functional space $\phi(x)$ in the usual continuum theory becomes

$$[d\phi(x)] \to [dx_i][d\phi_i] .$$

As in Sect. V, the discrete theory can be regarded as a limit of the usual continuum theory under the constraint of a fixed density of lattice sites, Eq. (8). Because the site density is an invariant, Lorentz invariance and Poincaré invariance can both be preserved in this discrete limit.

## VII. Fundamental Length $\ell$

Assuming that the discrete theory is the fundamental one, the length $\ell$ defined in (8) can be determined experimentally. The presence of a nonzero $\ell$ can produce many physical effects, among them energy level changes and propagator modifications.

1) Hydrogen-like atoms

There will be a shift between the energy $E_D$ in the discrete theory from $E_C$ of the usual continuum theory for a hydrogen-like atom [2]:

$$E_D - E_C \cong - \frac{6\pi}{5m_e R} \alpha^2 \ell^2 \, |\psi(0)|^2 \qquad (10)$$

where $R$ is the nuclear radius, $m_e$ the electron mass, $\alpha$ the fine-structure constant and $\psi(0)$ the usual non-relativistic Schroedinger wave function of the electron at the origin. By using the present Lamb-shift result[3] we determine an upper limit for the fundamental length

$$\ell < 1.6 \times 10^{-14} \text{ cm.}$$

A better bound, $< 10^{-15}$ cm, can be obtained by using mu-mesic atoms, g - 2, and other atomic results.[4]

2) Photon propagator modification

In a discrete theory, the photon propagator $\mathcal{D}(k)$ differs from its usual form $k^{-2}$ where $k$ is the wave number. Because Lorentz invariance is maintained in the discrete theory, $\mathcal{D}(k)$ depends only on $k^2$. At the origin, $k = 0$, the propagator is finite (whereas in the usual continuum theory, $k^{-2} \to \infty$). At the long wavelength limit we have

$$\mathcal{D}(k^2) \cong \left[k^2 - \frac{1}{12}(k^2)^2 \ell^2 + O(\ell^4)\right]^{-1} . \qquad (11)$$

From high-energy $e^+e^-$ data[5] we find

$$\ell < 5 \times 10^{-16} \text{ cm} \quad . \qquad (12)$$

A similar bound can also be set by using the forward dispersion relation[6] in $\pi p$.

## VIII. Lattice Gravity

The usual Einstein action in general relativity is

$$A_C(S) = \int_C \sqrt{|g|}\; R\,dx \tag{13}$$

where $C$ is a $d$-dimensional smooth continuous surface, $|g|$ is the absolute value of the determinant of the matrix of the metric tensor $g_{\mu\nu}$ on $C$, $R$ is the scalar curvature and $dx$ is the $d$-dimensional volume element in the space-time coordinate $x$.

Consider now a simplicial $d$-dimensional discrete lattice $D$ with $i = 1, 2, \cdots$ *denoting the lattice sites and* $\ell_{ij}$ the link lengths between sites $i$ and $j$. The $\ell_{ij}$ are assumed to satisfy all simplicial inequalities, so that each $d$ simplex of $d+1$ linked sites, by itself, can be realized in a flat $d$-dimensional space. This entire lattice $D$ can be embedded in a $d+n$ flat space provided $n$ is large enough, with $\ell_{ij}$ the Cartesian distance between $i$ and $j$ in this flat $(d+n)$-dimensional space.

Next we consider a sequence of smooth continuous $d$-dimensional surfaces $C$ which have sharper and sharper ridges, and approach $D$ in the limit. At first sight, *it might appear difficult to approach this limit because the metric* $g_{ij}$ would change discontinuously from simplex to simplex, the Christoffel symbol would then acquire $\delta$-functions and the scalar curvature $\delta'$-functions. Since Einstein's action is nonlinear in $g_{ij}$, one might expect the resulting action to be totally unmanageable. It turns out that this is not so. In the discrete limit of the continuum theory, Einstein's action approaches Regge's action.[7] In Regge's original approach, he considers the discrete action as an approximation to Einstein's continuum action. Here we are reversing the role and regarding the discrete action $A_D$ as more fundamental. It is therefore satisfying to realize that Regge's action $A_D$ is precisely the discrete limit of Einstein's continuum action.[8]

The discrete action $A_D$ can also be formulated in terms of Regge's deficit angle $\epsilon_s$ around a $d-2$ simplex $s$:

$$A_D = \sum_s \bar{s}\,\epsilon_s \tag{14}$$

where $\bar{s}$ is the volume of $s$. [See References 7 and 8 for details.] Because $A_D$ is the discrete limit of Einstein's action, $A_D$ enjoys the same symmetry property under a general coordinate transformation. In addition to all invariance properties of general relativity, $A_D$ the lattice action has further symmetries. We may fill in the space between the lattice sites by a $d$-dimensional smooth surface $S$ embedded in a flat $(d+n)$-dimensional space. Given $S$ and the relative position of the $i^{th}$ site $z_i$ on the manifold, we can figure out the link length $\ell_{ij}$ between the $i^{th}$ and $j^{th}$ lattice sites. Therefore the lattice action $A_D$ can also be regarded as a function of the manifold $S$ and the coordinates $z_i$:

$$A_D = A_D(S, z_i) \; . \tag{15}$$

We can now consider transformations:

$$\begin{aligned} & S \to S\,, \quad z_i \to z_i' \\ \text{or}\quad & S \to S'\,, \quad z_i \to z_i \; . \end{aligned} \tag{16}$$

$A_L$ can be invariant under either of these two types of transformations provided the link length $\ell_{ij}$ is unchanged. These symmetries do not exist in the usual Einstein theory of general relativity. Therefore the discrete theory has all the invariance character of the usual theory of general relativity; in addition it is invariant under these new transformations (16).

## IX. Concluding Remarks

By regarding space and time as dynamical variables, a fundamental length $\ell$ can be introduced which removes all ultraviolet divergences, and therefore makes quantization of gravity possible. As we have shown, such a discrete theory can also be viewed as a limit of the usual continuum theory but with a fixed density of lattice sites. Because this is an invariant constraint, the discrete theory shares the same symmetries of the usual continuum theory. In this way, we have succeeded in the creation of theories with finite degrees of freedom, but which retain all the good properties of the usual continuum theory. At the minimum, this gives a cut-off procedure that satisfies all symmetries including the general coordinate-transformation symmetry of general relativity which is otherwise extremely difficult to achieve. At the maximum, the discrete theory may change our fundamental concept of space-time. The discrete formulation is more basic, and our usual differential formulation is only an approximation.

The fundamental length $\ell$ can be $10^{-16}$ cm or smaller. If it is about $10^{-16}$ cm, then in a few years it will be measured here at SLC. But if it is much smaller, then a direct measurement may require an accelerator of a scale shown in the next figure. We might call such an accelerator SSSPFC (Super-super-super PieF-Collider), but otherwise it is also known as Quasar 3C273. Of course there may also be some more clever way to determine the fundamental length, and we shall leave this as a challenge to Pief.

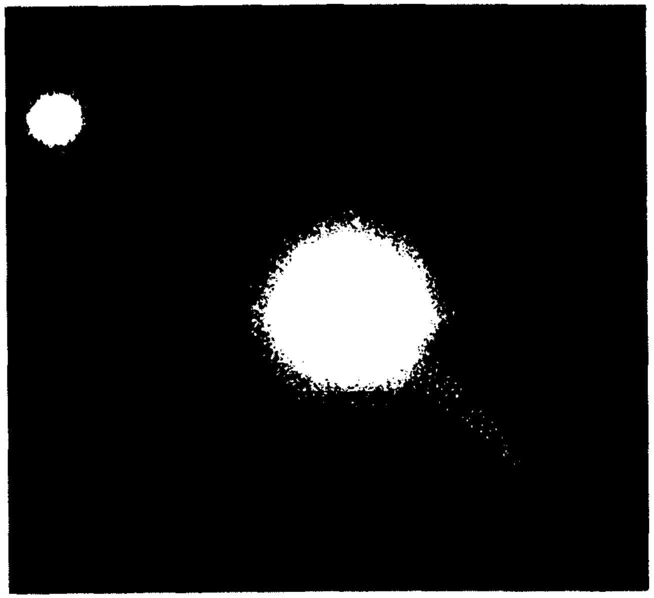


Figure 5.

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