CP Violation in the Standard Model

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The violation of the CP quantum numbers has been observed in the neutral kaon system. Nonzero values for the quantities $\eta_{+-}$ and $\eta_{00}$, defined by

$$
\eta_{+-} = \frac{\langle n^+ n^- | H^{(3+1)} | K_S \rangle}{\langle n^+ n^- | H^{(3+1)} | K_L \rangle}, \quad \eta_{00} = \frac{\langle n^0 n^0 | H^{(3+1)} | K_S \rangle}{\langle n^0 n^0 | H^{(3+1)} | K_L \rangle}
$$

(1)

are an indication of CP violation. Experimentally $|\eta_{+-}|$ and $|\eta_{00}|$ are about $2 \times 10^{-3}$. In this paper the predictions that the standard model makes for CP violation in the neutral kaon system are discussed.

The standard model for strong, weak and electromagnetic interactions is based on the gauge group $SU(3) \times SU(2) \times U(1)$. There is experimental evidence for three generations of quarks and leptons. The charged $\pi$-bosons couple to the quarks through the weak current

$$
J_{\mu}^{\pi} = \frac{g_2}{E \sqrt{2}} (\bar{q} \gamma_\mu l) \gamma_\mu (1 - \gamma_5) U \frac{\partial}{\partial x}
$$

(2)

Here $U$ is a $3 \times 3$ unitary matrix that arises from the diagonalization of the quark mass matrices. By adjusting the phases of the quark fields, $U$ can be written in the form

$$
U = \begin{pmatrix}
c_1 & -s_1 c_2 & -s_1 s_2 \\
s_1 c_2 & c_1 c_2 s_3 - s_1 c_3 s_3 s_2 & c_1 s_2 s_3 + c_2 c_3 s_2 s_1 \\
s_1 s_2 & c_1 s_2 s_3 + c_2 c_3 s_2 s_1 & c_1 c_3 s_3 + s_1 s_2 s_3
\end{pmatrix}
$$

(3)

where $c_i = \cos \theta_i$, $s_i = \sin \theta_i$, $i \in \{1, 2, 3\}$ and the angles $\theta_1$, $\theta_2$, and $\theta_3$ lie in the first quadrant. Provided $s_1 \neq 0$ for $i \in \{1, 2, 3\}$ and no quarks of the same charge are degenerate in mass, the phase $\delta$ cannot be removed from $U$ by a redefinition.

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of quark fields and $\delta$ is a source of CP violation.\(^3\)

The weak interaction phase $\delta$ is not the only source of CP violation in the standard model. A nonzero value for the strong interaction parameter $\bar{\theta}$ also violates CP. However, the stringent experimental upper limit on the electric dipole moment of the neutron,\(^4\)

$$d_n \lesssim 10^{-26} \text{ cm}$$  \hspace{1cm} (4)

implies that\(^5\)

$$\bar{\theta} \lesssim 10^{-5}. \hspace{1cm} (5)$$

Strong interaction CP violation is too small to be responsible for the CP violation observed in the neutral kaon system.

Experimental information on neutron $\beta$ decay and semileptonic hyperon decays give\(^8\) (for small $s_3$)

$$s_1 \approx 0.22. \hspace{1cm} (6)$$

The weak mixing angles $\theta_w$ and $\theta_s$ are constrained by experimental information on $B$-meson decays. Since the $b$-quark is heavy compared with the typical strong interaction scale, the rate for semileptonic $B$-meson decay can be approximated by the decay rate of a free $b$-quark. Thus

$$\Gamma(B \rightarrow e\nu x) \approx \Gamma(b \rightarrow c e\bar{\nu}) + \Gamma(b \rightarrow u e\bar{\nu}) \hspace{1cm} (7)$$

where

$$\Gamma(b \rightarrow c e\bar{\nu}) = |U_{cb}|^2 \frac{G_F m_b^2}{16\pi^2} f(m_\pi/m_b), \hspace{1cm} (8a)$$

$$\Gamma(b \rightarrow u e\bar{\nu}) = |U_{ub}|^2 \frac{G_F m_b^2}{16\pi^2} f(m_\pi/m_b), \hspace{1cm} (8b)$$

and

$$f(x) = 1 - 8x^2 + 8x^3 - x^4 - 24x^4 \ln x. \hspace{1cm} (9)$$

With $m_b = 4.6$ GeV, and $m_\pi = 1.5$ GeV Eqs. (8a & b) imply

$$\Gamma(b \rightarrow c e\bar{\nu}) = 4.3 |U_{cb}|^2 10^{13} \text{ s}^{-1}, \hspace{1cm} (10a)$$

$$\Gamma(b \rightarrow u e\bar{\nu}) = 8.7 |U_{ub}|^2 10^{13} \text{ s}^{-1}. \hspace{1cm} (10b)$$

Comparing Eqs. (10a & b) with

$$\Gamma(b \rightarrow c e\bar{\nu}) = Br(b \rightarrow c) \frac{Br(B \rightarrow e\nu x)}{\tau_B}, \hspace{1cm} (11a)$$

$$\Gamma(b \rightarrow u e\bar{\nu}) = Br(b \rightarrow u) \frac{Br(B \rightarrow e\nu x)}{\tau_B}, \hspace{1cm} (11b)$$

using the experimental values\(^7\)

$$Br(b \rightarrow c) < 0.04, \hspace{1cm} (12)$$

$$Br(B \rightarrow e\nu x) = 11.6 \%, \hspace{1cm} (13)$$

gives

$$|U_{cb}|^2 \approx 3 \times 10^{-3} (10^{-18} \text{ s} / \tau_B) \hspace{1cm} (14a)$$

$$|U_{ub}|^2 \approx 5 \times 10^{-6} (10^{-18} \text{ s} / \tau_B). \hspace{1cm} (14b)$$

Recent measurements\(^3\) of the $B$-meson lifetime imply $\tau_B$ is about $10^{-12}$ sec and constrains the angles $\theta_2$ and $\theta_3$ to be very small. To leading nontrivial order in the small angles Eqs. (14a & b) become

$$\{s_2^2 + s_3^2 + 2s_2 s_3 c_2\} = 3 \times 10^{-3} (10^{-18} \text{ s} / \tau_B) \hspace{1cm} (15a)$$

$$s_2^2 < 1 \times 10^{-3} (10^{-18} \text{ s} / \tau_B). \hspace{1cm} (15b)$$

Since $-1 < c_3 < 1$ Eq. (10a) gives
The CP violation parameters \( \eta_{\pi} \) and \( \eta_{0} \) depend on CP violation in the kaon decay amplitudes \( A_0 \) and \( A_2 \) defined by

\[
\begin{align*}
\text{Re}(\langle \pi(n = 0) | H_{\text{etDM}}^{(1=1)} | \Lambda^0 \rangle) &= \langle A_0 \rangle A_{\pi} e^{i\delta_2}, \\
\text{Re}(\langle \pi(n = 2) | H_{\text{etDM}}^{(1=1)} | \Lambda^0 \rangle) &= \langle A_2 \rangle A_{\pi} e^{i\delta_2}.
\end{align*}
\]

Here \( \delta_2 \) and \( \delta_2 \) are the \( I = 0 \) and \( I = 2 \) phase shifts. It is convenient to choose the phase of the kaon states so that \( A_2 \) is real. Then an imaginary part for \( A_2 \) violates CP. CP violation in the decay amplitudes \( A_0 \) and \( A_2 \) is usually described by the parameter

\[
\epsilon' = \frac{i}{\sqrt{2}} e^{i\delta_2} \text{Im}(A_0 / A_2).
\]

The CP violation parameters \( \eta_{\pi} \) and \( \eta_{0} \) also depend on CP violation in second order weak (\( \sim \Gamma^0 \)) \( \Lambda^0 - \Lambda^0 \) mixing. The eigenstates \( K_S \) and \( K_L \) are given by

\[
K_S = \frac{1}{2(1 + |\epsilon|^2)^{1/2}} ([1 + \epsilon] \Lambda^0 - (1 - \epsilon) \Lambda^0),
\]

\[
K_L = \frac{1}{2(1 + |\epsilon|^2)^{1/2}} ([1 + \epsilon] \Lambda^0 + (1 - \epsilon) \Lambda^0),
\]

where

\[
\epsilon = \frac{\text{Im} M_{13} - \text{Im} \Gamma_{13} / 2}{\text{Re} M_{13} - \text{Re} \Gamma_{13} / 2}.
\]

In Eq. (21)

\[
M_{13} = \langle \Lambda^0 | H_{\text{etDM}}^{(1=2)} | \Lambda^0 \rangle / (\text{Re} M_{13} - \text{Re} \Gamma_{13} / 2),
\]

\[
\Gamma_{13} = 2n \sum_{F} \langle F | H_{\text{etDM}}^{(1=1)} | F \rangle \langle F | H_{\text{etDM}}^{(1=2)} | \Lambda^0 \rangle / (\text{Re} M_{13} - \text{Re} \Gamma_{13} / 2).
\]

where \( \rho_F \) is the density of final states. The parameter \( \epsilon \) is a measure of CP violation in \( \Lambda^0 - \Lambda^0 \) mixing.

Using the experimental values \( \delta_0 \approx 48^\circ \), \( \delta_2 = -7.2^\circ \), \( A_2/A_0 = +1/20 \),

\[-(m_{K_S} - m_{K_L}) \approx (\Gamma_2 - \Gamma_2)/2 \quad \text{and} \quad \text{Im} \Gamma_{13}/(\text{Im} M_{13}) < 1/10 \]

we have that

\[
\epsilon = \frac{1}{\sqrt{2}} e^{i\delta_2} \frac{\text{Im} M_{13}}{(m_{K_S} - m_{K_L})},
\]

\[
\epsilon' = \frac{1}{\sqrt{2}} e^{i\delta_2} \frac{\text{Im} A_0}{A_2}.
\]

The CP violation parameters \( \eta_{\pi} \) and \( \eta_{0} \) can be expressed in terms of \( \epsilon \) and \( \epsilon' \)

\[
\eta_{\pi} = \epsilon + \epsilon',
\]

\[
\eta_{0} = \epsilon - 2\epsilon'.
\]

Experimentally

\[
|\epsilon| = 0.3 \times 10^{-3}
\]

\[
\epsilon'/\epsilon = (-6.6 \pm 5.3 \pm 2.4) \times 10^{-9}.
\]

In order to compare the experimental result for \( \epsilon \) with the prediction of the standard model we need to know the effective Hamiltonian for \( \Lambda^0 - \Lambda^0 \) mixing (or at least its imaginary part) and the matrix element of this Hamiltonian between \( \Lambda^0 \) and \( \Lambda^0 \) states. The effective Hamiltonian density for \( \Lambda^0 - \Lambda^0 \) mixing has been computed in the leading logarithmic approximation by successively treating the \( W \)-boson, \( t \)-quark, \( b \)-quark, and \( c \)-quark as heavy and integrating them out of the theory. The result, to leading nontrivial order in the large masses, is

\[
\text{(23a)}
\]
Here $\eta_1$, $\eta_2$ and $\eta_3$ are QCD correction factors. They are roughly independent of the top quark mass and have the values $\eta_1 = 0.7$, $\eta_2 = 0.6$ and $\eta_3 = 0.4$ (using $m_t = 1.5 \text{ GeV}$, $m_b = 4.8 \text{ GeV}$, $M = 80 \text{ GeV}$, $A_{QCD} = 0.1 \text{ GeV}$ and $\alpha_s(\mu^2) = 1$).

In Eq. (27) long distance contributions to the effective Hamiltonian have been neglected since they do not contain a factor of a heavy quark mass squared. A typical long distance contribution is the time ordered product of two effective Hamiltonians for $|\Delta S| = 1$ nonleptonic kaon decays. It is likely that the $\Delta S = 1$ matrix element of this operator dominates $\text{Re}M_{12}$. The contribution of the matrix element of this operator to $\text{Im}M_{12}$ involves CP violation in $1\Delta S = 1$ amplitudes. Since these amplitudes are not necessarily suppressed by the $|\Delta I| = 1/2$ rule, their contribution to $\epsilon$ is expected to be of order $20\epsilon'$. Anticipating a modest improvement in the experimental limit on $\epsilon'$ I shall ignore long distance contributions to $\text{Im}M_{12}$. Then using Eqs. (27) and (30)\footnote{There is a short distance contribution from $|\epsilon|$ arising from the redefinition of kaon fields to make $A_1$ real. It has magnitude $O(\epsilon')|\text{Re}M|^2/|m_t/m_b - m_\pi|$, where $\text{Re}M$ is the short distance contribution to the $K^0 - \bar{K}^0$ mass mixing. Anticipating a modest improvement in the limit on $\epsilon'$ this term will be neglected.}.

$$\epsilon = \frac{s_{2\epsilon} s_{2\epsilon} m_t^2}{16 \sqrt{2} \pi f^2 (m_t - m_\pi)} s_{2\epsilon} s_{2\epsilon} \eta_3 (m_t^2/m_b^2)$$

$$+ \frac{\eta_2 (m_t^2/m_\pi^2) (s_{2\epsilon}^2 + s_{2\epsilon}^2 c_{2\epsilon}^2)}{\epsilon''} \epsilon'^{1/4}.$$  

(28)

to leading nontrivial order in small angles. Here $f$ is the pion decay constant and the dimensionless parameter $B$ is defined by\footnote{One particle states are normalized to $2\pi V$, where $V$ is the volume of space and $E$ is their energy. Factors of $V$ are set to unity.}

$$\langle \bar{K}^0 |\mathcal{O}(\epsilon) | K^0\rangle = \epsilon f m_\pi^2.$$  

(29)

where

$$O' = (\bar{s}_u (1 - \gamma_5) d_u) (\bar{d}_u (1 - \gamma_5) d_u).$$  

(30)

The magnitude of the parameter $B$ is determined in chiral perturbation theory.\footnote{In chiral perturbation theory the up, down and strange quark masses are treated as small compared with the typical hadronic scale ($\sim 4\pi f$). The operator $O'$ transforms as $(2\frac{3}{2}, 1)$ under SU(3)$_L \times SU(3)_R$. The operator $O$ transforms as $(2\frac{1}{2}, 1)$ under SU(3)$_L \times SU(3)_R$. The operator $O$ also transforms this way. The effective Hamiltonian density for $|\Delta S| = 1$, $|\Delta I| = 3/2$ weak nonleptonic decays is}

$$H_{0u} = -\frac{f}{2 \sqrt{2}} s_{2\epsilon} c_{2\epsilon} CD^0.$$  

(32)

Here $C$ is a factor that takes into account strong interaction corrections. With the same parameters used previously $C \approx 0.4$. To leading order in derivatives and quark masses there is a unique operator involving the pseudo-Goldstone boson fields that transforms as $(2\frac{1}{2}, 1)$, incorporating these fields in the special unitary matrix

$$\begin{align*}
H_{0u} &= -\frac{f}{2 \sqrt{2}} s_{2\epsilon} c_{2\epsilon} CD^0.
\end{align*}$$  

(32)
\[
\Sigma = \exp \left( \frac{2\pi}{f} \begin{pmatrix}
\frac{\pi^0 + \pi^-}{\sqrt{2}} & \pi^+ & K^+
\pi^- & \frac{\pi^0 + \pi^+}{\sqrt{2}} & K^0
K^- & K^0 & -\sqrt{3} \eta
\end{pmatrix} \right)
\] (33)

that transforms under \(SU(3)_c \times SU(3)_w\) as

\[
\Sigma \rightarrow \Delta \Sigma \Sigma^* ,
\] (34)

the leading operator that transforms as \((27_L, 1_R)\) is

\[-\alpha T^a_H \{\bar{\Sigma}_0 \Sigma\} \{\bar{\Sigma}_0 \Sigma\} \} .
\] (35)

Here \(T^a_H\) is a traceless symmetric tensor and \(\alpha\) is a constant. For the operator \(O\)
\(T^a_H\) has nonzero components

\[T^{a_1 a_2}_H = T^{a_2 a_1}_H = T^{a_2 a_2}_H = 1/2, \quad T^{a_1 a_1}_H = -1/2 .
\] (36)

while for \(O\) the only nonzero component is

\[T^{a_1 a_1}_H = 1 .
\] (37)

Expanding the operator in Eq. (35) gives

\[
\langle K^0 | \int d^4x \mathcal{O}^c(x) | K^0 \rangle = \frac{8\alpha m_K f^2}{f} ,
\] (38)

\[
\langle \pi^+ \pi^- | \int d^4x \mathcal{O}^c(x) | K^0 \rangle = \frac{1}{\sqrt{2}} \frac{8\alpha m_K f^2}{f} (m_K^2 - m_{\pi}^2) .
\] (39)

Thus

\[B = 8\alpha m_K f^2
\] (40)

and

\[\langle n^+ n^- | H_{\text{eff}}^{1 \ast} | K^0 \rangle = -\frac{3G_F}{8\pi} B_{s1} c_1 c_2 m_K (m_K^2 - m_{n}^2) .
\] (41)

Therefore from the measured \(K^0 \rightarrow n^+ n^-\) decay rate we extract

\[|B| \approx 0.37 .
\] (42)

Here a small correction, arising from the fact that isospin violating quark masses allow the \((\Delta I = 1/2)\) weak Hamiltonian to contribute to the \(K^0 \rightarrow n^+ n^-\) decay amplitude, has been included. Unfortunately, the order \(m_{\pi}^2 m_K^2\) corrections to Eq. (30) are large indicating a breakdown of chiral perturbation theory for \(D\).\(^{12}\)

Therefore \(B\) shall be kept as a free parameter.

The long \(B\) meson lifetime implies that the angles \(\theta_2\) and \(\theta_3\) are small. Then the top quark mass may have to be heavy to have Eq. (28) accommodate the measured value of \(\epsilon\). Therefore there may be a useful lower bound on the top quark mass as a function of the \(B\) meson lifetime.\(^{13}\) Substituting the numerical values for some of the quantities that appear in Eq. (28) for \(\epsilon\) gives\(^{16}\)

\[0.11 = (B/0.37)s_s s_t (s_s + s_t) \left[ -0.7 + 0.4\ln(m_t^2/m_{\pi}^2) + 0.6(m_t^2/m_{\pi}^2)(s_s + s_t) \right] .
\] (43)

Since \(s_s < s_t\) it is clear that \(B s_s > 0\). Using the identities

\[s_s s_t^2 \leq s_s^2 + s_t^2 + 2s_t s_s |c_4|\] (44a)

and

\[|s_t c_4| < 1/2 .
\] (44b)

together with Eqs. (15) and (16) the bound for \(c_s < 0\) is a.

\(^{15}\) The subtraction point dependence of \(B\) cancels against the subtraction point dependence of the \(a_0\) in the expression for \(a\).

\(^{16}\) The constraint \(c_s > 0\) gives a more stringent bound on the top quark mass.
Note that if the second term in the numerator is negligible this gives a lower bound on $m_1$ that is linear in the $B$ meson lifetime

$$m_1 \geq 70(\tau_B/10^{-12}s)B/0.37f^{1/2} \text{GeV}. \quad (48)$$

However, the second term in the numerator is not completely negligible. For $\tau_B = 10^{-12}s$ and $|R| = 0.37$ the second term in the numerator is about 40% of the first, and the lower bound is $m_1 \geq 55 \text{ GeV}$. Therefore, if the prediction for $B$ based on lowest order chiral perturbation theory is off by more than a factor of two, the term proportional to $m_f^2$ is not needed to account for the measured value of $\varepsilon$.

The expression for $\varepsilon$ can be used to derive a lower bound on the combination of angles $|\delta_{S_2S_3S_4}|$ as a function of the $B$-meson lifetime and the top quark mass. For $c_4 < 0$ it is:

$$|\delta_{S_2S_3S_4}| > \frac{0.06 |0.37/B|}{\sqrt{0.7 + 0.4 \ln(m_f^2/m_f^0) + 0.8f^{1/2}(10^{-12}s/B)(m_f^2/m_f^0)}} \quad (47)$$

Even for $|0.37/B| = 1/2$, if $\tau_B = 10^{-12}s$ and $m_1 = 45 \text{ GeV}$, we have $|\delta_{S_2S_3S_4}| \geq 5 \times 10^{-5}$.

The bound on $|\delta_{S_2S_3S_4}|$ coming from $\varepsilon$ has implications for the CP violation parameter $c'$. This is a measure of CP violation in kaon decay amplitudes and arises from “Penguin-type” diagrams involving a heavy quark loop. Here the electromagnetic contribution will be included. "Electromagnetic "Penguin-type" diagrams give rise to local operators that transform under SU(3)$_L \times SU(3)_R$ as $(\bar{q}_L, \bar{q}_R)$ and their $K \to \pi\pi$ matrix elements do not vanish in the chiral limit $m_{u,d,s} \to 0$. Therefore they can make a significant contribution to $c'$ despite their suppression by the factor $a_{em}/a_e(m_f^2)$. The effective Hamiltonian density for $|AS| = 1$ weak nonleptonic decays is

$$H_{\Delta S}^{\Delta F=1} = -\frac{G_F}{\sqrt{2}} \sum_{f_i} \bar{c}_1 \sum_{c_2} C_1 Q_1 + \text{h.c.} \quad (48)$$

where

$$Q_1 = (\bar{q}_L (1 - \gamma_5) d_L) (\bar{q}_R \gamma^\mu (1 - \gamma_5) u_R), \quad (48a)$$

$$Q_2 = (\bar{q}_L (1 - \gamma_5) d_L) (\bar{q}_R \gamma^\mu (1 - \gamma_5) u_R), \quad (48b)$$

$$Q_3 = (\bar{q}_L (1 - \gamma_5) d_L) (\bar{q}_R \gamma^\mu (1 - \gamma_5) u_R) + (\bar{q}_L \gamma^\mu (1 - \gamma_5) d_L) + (\bar{q}_R \gamma^\mu (1 - \gamma_5) u_R), \quad (48c)$$

$$Q_4 = (\bar{q}_L (1 - \gamma_5) d_L) (\bar{q}_R \gamma^\mu (1 - \gamma_5) u_R) + (\bar{q}_L \gamma^\mu (1 - \gamma_5) d_L) + (\bar{q}_R \gamma^\mu (1 - \gamma_5) u_R), \quad (48d)$$

$$Q_5 = (\bar{q}_L (1 - \gamma_5) d_L) (\bar{q}_R \gamma^\mu (1 + \gamma_5) u_R) + (\bar{q}_L \gamma^\mu (1 + \gamma_5) d_L) + (\bar{q}_R \gamma^\mu (1 + \gamma_5) u_R), \quad (48e)$$

$$Q_6 = (\bar{q}_L (1 - \gamma_5) d_L) (\bar{q}_R \gamma^\mu (1 + \gamma_5) u_R) + (\bar{q}_L \gamma^\mu (1 + \gamma_5) d_L) + (\bar{q}_R \gamma^\mu (1 + \gamma_5) u_R). \quad (48f)$$
\[ Q_5 = (g_s F^\mu(1 - \gamma_5) d_5) \left[ (u_s F^\mu(1 + \gamma_5) u_s) \right] \]

\[ Q_6 = (g_s F^\mu(1 - \gamma_5) d_5) \left[ (u_s F^\mu(1 + \gamma_5) u_s) \right] \]

The prime on the sum in Eq. (48) means the \( Q_i \) is not included since it is linearly dependent on the other operators. The operators \( Q_4, Q_5 \) and \( Q_6 \) arise from strong interaction, "Penguin-type" diagrams. They transform as \((8_L, 1 \bar{R})\) under \( SU(3)_L \times SU(3)_R \) and give an imaginary part to \( A_0 \). The redefinition of kaon fields to comply with the phase convention \( A_0 \) real induces an imaginary part to \( A_0 \).

The operators \( Q_1, Q_3, Q_8 \) arise from electromagnetic "Penguin-type" diagrams. They transform as \((8_L, 8_R)\) under \( SU(3)_L \times SU(3)_R \) and contribute an imaginary part directly to \( A_0 \). It follows from Eqs. (23b), (48) and (49) that

\[ |\epsilon' / \epsilon| \approx 16 \pi \, e_4 \]

Here the dependence of \( \text{Im} G_j \) on the angles and on \( n_{\alpha \alpha} \) has been made explicit by writing

\[ \text{Im} G_j = \sum_{\alpha \beta} c_\alpha c_\beta e_4 \, e_5 \, \text{Im} \text{Re} c_\alpha c_\beta \]

The \((8_L, 8_R)\) piece of the effective Hamiltonian also gives an imaginary part to \( A_0 \). However, this is suppressed by a factor of \([u_s F^\mu(1 - \gamma_5) u_s] \) in \( \epsilon' / \epsilon \), while the imaginary part of \( A_0 \) is not.

With the strong interaction parameters used previously \(^{16,18}\) (and \( m_s = 30 \text{ GeV} \)), \( \bar{\epsilon}_1 = -0.03, \bar{\epsilon}_2 = 0.006, \bar{\epsilon}_2 = -0.004, \bar{\epsilon}_3 = 0.1, \bar{\epsilon}_4 = 0.007, \bar{\epsilon}_5 = 0.1 \).

The operators \( Q_1, \ldots, Q_6 \) transform under chiral \( SU(3)_L \times SU(3)_R \) as \((8_L, 8_R)\) and \((27_L, 1_R)\). It follows that their \( K \to \pi \pi \) matrix elements are proportional to the strange quark current algebra mass and are expected to be of order \( 0.1 \text{ GeV} \). The operators \( Q_7 \) and \( Q_8 \) transform as \((8_L, 8_R)\) under chiral \( SU(3)_L \times SU(3)_R \). Their \( K \to \pi \pi \) matrix elements do not vanish as the current algebra masses go to zero and are expected to be of order \( 1 \text{ GeV} \).

It has been suggested \(^{17}\) that the \((\gamma - A) \times (\gamma + A)\) chiral structure of \( Q_6 \) and \( Q_8 \) leads to an enhancement of their \( K \to \pi \pi \) matrix elements by an order of magnitude or more \(^{17}\) and that this is responsible for the \( \Delta I = 1/2 \) rule. This is supported by bag model calculations \(^{18}\) which predict that the magnitude of the \( K \to \pi \pi (I=0) \) matrix element of \( \sum \int d^2 x Q_6(x) \) is about \( 1 \text{ GeV} \). If this is the case then \(^{19}\)

\[ |\epsilon' / \epsilon| \approx 16 \pi \, e_4 \times 1 \text{ GeV} \]

Given the bound on \( |8 \epsilon_3 e_4| \) from \( \epsilon' / \epsilon \), the value of \( |\epsilon' / \epsilon| \) should be near the present experimental limit. If \( Q_6 \) is responsible for the \( \Delta I = 1/2 \) rule then \( \epsilon' / \epsilon \) has the same sign as \( e_4 \) provided the sign of \( \text{Re} c_6 \) computed in the leading logarithmic approximation is reliable. The sign of \( e_4 \) is positive if \( H \) has the same sign as in the "vacuum insertion" approximation.\(^{19}\)

\(^{17}\) The value of the Wilson coefficient \( \text{Re} c_6 \) is very uncertain and therefore one cannot say precisely how much enhancement of the matrix elements of \( Q_8 \) is needed for the local \((\gamma - A) \times (\gamma + A)\) operator to be the source of the \( \Delta I = 1/2 \) rule.

\(^{18}\) Here long distance contributions to \( \epsilon' / \epsilon \) are neglected.
An improvement in the experimental limit on $\epsilon'/\epsilon$ could either rule out the order of magnitude or more enhancement of the matrix elements of $\int d^2 \mathbf{Q}_i(z)$ needed to explain the $\Delta f = f$ rule, or perhaps signal a failure of the standard six quark model for CP violation. In order for $\epsilon'/\epsilon$ to serve as a quantitative test of the standard model it is necessary to have reliable computations of hadronic matrix elements of local 4-quark operators. It is not inconceivable that lattice Monte Carlo methods could provide these.\textsuperscript{9} If chiral perturbation theory is valid it is not necessary to compute the matrix element $\langle m \langle f = 0 \rangle | \int d^2 \mathbf{Q}_i(z) | K^0 \rangle$. It is sufficient to calculate both $\langle m | \int d^2 \mathbf{Q}_i(z) | K^0 \rangle$ and $\langle 0 | \int d^2 \mathbf{Q}_i(z) | K^0 \rangle$, where these matrix elements conserve 3-momentum but not energy. Because energy is not conserved, operators that are total derivatives must be included in the chiral Lagrangian. To leading order in derivatives and masses there are only two operators involving $\Sigma$ that transform under $SU(3)_{L} \times SU(3)_{R}$ as $(8, 1)$ and are invariant under $(CP)^*D$, where $D$ is a discrete transformation that interchanges strange and down quarks. Matrix elements of $\mathbf{Q}_i$ between kaon and pion states can be replaced by matrix elements of $\mathbf{Q}_i$ between kaon and pion states

\begin{equation}
\alpha \tau \langle O_\mu \Sigma^\mu \mathbf{P} \rangle + b \tau \langle 0 \mathbf{M} \rangle + \langle 0 \mathbf{M} \rangle^* \end{equation}

(23)

to leading order in chiral perturbation theory. Here

\begin{equation}
m = \begin{bmatrix}
m_0 & 0 & 0 \\
0 & m_s & 0 \\
0 & 0 & m_u
\end{bmatrix}
\end{equation}

(54)
is the quark mass matrix.

\begin{equation}
O = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 1 & 0
\end{bmatrix}
\end{equation}

(55)

and $a$ and $b$ are constants. The term proportional to $b$ is a total derivative (provided the masses $m_u$, $m_d$, and $m_s$ are not all equal) and doesn’t contribute to $\langle m \langle f = 0 \rangle | \int d^2 \mathbf{Q}_i(z) | K^0 \rangle$. However

\begin{equation}
\langle 0 | \int d^2 \mathbf{Q}_i(z) | K^0 \rangle = \frac{2}{\sqrt{3}} b (m_d - m_u)
\end{equation}

(56a)

and

\begin{equation}
\langle m^2 | \int d^2 \mathbf{Q}_i(z) | K^0 \rangle = \frac{4 g^2}{\lambda} m_u m_s - \frac{2 g}{\lambda} (m_u + m_s)
\end{equation}

(56b)

Therefore a computation of the two matrix elements above allows one to extract $a$ and hence predict the $K^0 - \bar{K}^0$ mixing. In summary, the standard model makes predictions for $\epsilon$ and $\epsilon'$ that, at the present time, are compatible with experiment. All the CP violation observed so far is consistent with it arising from $K^0 - \bar{K}^0$ mixing. Since this is a second order weak ($\sim G_F$) process, very short distance physics beyond the standard model could be responsible for most of the CP violation observed there. Measurement of a nonzero value for $\epsilon'/\epsilon$ is very important, since it would eliminate this possibility. Reliable computations of the matrix elements $\langle m \langle f = 0 \rangle | \int d^2 \mathbf{Q}_i(z) | K^0 \rangle$ and $\langle K^0 | \int d^2 \mathbf{Q}_i(z) | K^0 \rangle$ would make $\epsilon$ and $\epsilon'$ sensitive tests of the standard six quark model for CP violation.

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