

Production and Uses of Heavy Quarks\*

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ABSTRACT

These lectures give a pedagogical treatment of the perturbative production of heavy quarks, mainly in  $e^+e^-$  and hadronic collisions. The current experimental situation for charm production is discussed but not resolved, and the reader is alerted to what considerations will be important to watch as understanding improves. A number of examples are considered where we can learn from the interactions and decays of heavy quarks about tests of the Standard Model or possible physics beyond the Standard Model, e.g. right-handed currents, measurement of weak isospin quantum numbers, flavor changing neutral currents, polarization phenomena, and restrictions from heavy quark decays (e.g. the t-quark) on existence of proposed particles such as supersymmetric partners, charged Higgs bosons, etc.

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## Contents

Introduction

$e^+e^- \rightarrow Q\bar{Q}$

Determining the Charge of a New Heavy Quark

Quarkonium

Heavy Quark Lifetimes

Perturbative Production of Heavy Quarks in Hadronic Reactions

Structure Functions

Scaling Violations

Constituent Cross Sections

Experimental Situation

Spin and Polarization Effects

(A) Looking for Mirror Fermions

(B) Uses of Polarized t-Quarks

(a) Polarized t's from W decays

(b) True polarization test of QCD

Flavor Changing Z Decays

If  $t \rightarrow b\gamma$  Then ...

Implications of Flavor Changing Neutral Current Decays of

Heavy Quarks

## Introduction

The first two-thirds of these lectures are mainly pedagogical. Their purpose, in the context of the SLAC Summer Institute, is to give a pedagogical treatment of the perturbative theory of heavy quark production in hadronic reactions;  $e^+e^-$  results, which are straightforward, are summarized for comparison. The emphasis is designed to be on explaining the physics rather than compiling results. As is well known, the experimental data for charm production are not in agreement with the perturbative theory, suggesting the presence of additional contributions. The discrepancy is reviewed but not resolved here; we content ourselves with mentioning some of the criteria by which choices can be made among presently suggested alternatives.

The remainder of the lectures discusses a number of uses to which heavy quarks can be put to learn new physics. These are in two categories:

1. to check and study the Standard Model -- examples are:
  - (a) There may be further short distance tests of QCD.
  - (b) It is possible to check predicted lifetimes for b and maybe for t.
  - (c) It is possible to test QCD polarization predictions involving phase and helicity structure.
  - (d) There are implications of spin for other measurements.
2. heavy quark decays or interactions may contain new information about physics beyond the Standard Model -- examples are:
  - (a) The absence of neutral current b decays implies a t quark must exist, as does the size of the forward-backward asymmetry in

$e^+e^- \rightarrow b\bar{b}$ , and implies that the  $t$  has V-A couplings.

(b) It is possible to determine whether a new quark has substantial couplings that are not V-A, as might be expected for mirror fermions.

(c) If  $Q \rightarrow \nu X$  is observed, then any charged Higgs has mass  $M(H^\pm) > M_Q$ .

(d) Generalizing (c), any two-body decay allowed by quantum numbers must be kinematically excluded or it will dominate, so if nature is supersymmetric, then  $\bar{m}_Q + \bar{m}_\gamma > m_Q$ , etc.

(e) Flavor changing neutral decays into heavy quarks may be induced by new interactions, e.g.  $Z^0 \rightarrow t\bar{c}$ .

In the pedagogical aspects of the lectures, I will most closely follow the two books by Collins and Martin [1] and by Halzen and Martin [2] which in turn are based on the original literature. The pedagogical goal is to help the student understand the literature, not to provide a compendium of results or a recipe for calculation. The latter are available elsewhere.

Since a variety of topics are covered, it is difficult to be sure they have all been referenced fully. I have done some searching, and incorporated all suggestions given to me during and after the lectures. I apologize to anyone whose work has not been included, and I would be glad to be informed of any omissions.

$e^+e^- \rightarrow Q\bar{Q}$

For completeness, and partly for later mention, we summarize [3] the results for  $e^+e^- \rightarrow Q\bar{Q}$ . The transition can occur via a  $\gamma$  or via a  $Z^0$ . As always, unless otherwise specified, we work in the Standard Model (SM). The photon contribution is, for production of any fermion  $f$ ,

$$d\sigma/d\Omega = \frac{\alpha^2 Q_f^2}{4s} \beta (2 - \beta^2 \sin^2\theta) N_C$$

where  $\beta = (1 - 4m_f^2/s)^{1/2}$ , and  $N_C$  is the number of colors of  $f$ .

Integrating over angles gives

$$\sigma = 2\pi\alpha^2 Q_f^2 \beta (3 - \beta^2)/3s$$

$$\xrightarrow{s \gg 4m_f^2} \frac{4\pi\alpha^2}{3s} = \frac{87\text{nb}}{s(\text{GeV}^2)} = \frac{10^{-37}\text{cm}^2}{s(\text{TeV}^2)} = \sigma_{\text{point}}$$

$$N_C = Q_f = 1$$

For any process, it is customary to define  $R = \sigma/\sigma_{\text{point}}$ .

The  $Z^0$  contribution is

$$\sigma = \frac{G_F^2 s}{6\pi} \frac{m_Z^4}{(s-m_Z^2)^2 + \Gamma_Z^2 m_Z^2} (V_e^2 + A_e^2) (V_f^2 + A_f^2).$$

Since the electron coupling to the photon is vector, while its coupling to the  $Z^0$  is mainly axial vector (because the vector coupling is  $1-4s m^2 \theta_w \ll 1$ ), there is not much interference.

For  $s = m_Z^2$  this contribution is about 50 nb, but radiative corrections reduce the peak by about 2/3. Neglecting phase space, the branching ratios at the  $Z^0$  for final fermions are 0.13 for a

$Q_f = -1/3$  quark, 0.10 for  $Q_f = 2/3$ , 0.03 for  $Q_f = -1$ , and 0.06 for  $Q_f = 0$ .

When  $s \gg m_Z^2$ , the combined  $\gamma + Z$  contribution is

$$R(Q_f = -1/3) = 1.1$$

$$R(Q_f = 2/3) = 2$$

$$R(Q_f = -1) = 1.2$$

$$R(Q_f = 0) = 0.3$$

and do not scale as  $Q_f^2$ .

For comparison,

$$\sigma(e^+e^- \rightarrow W^+W^-) = \frac{\pi\alpha^2}{2s} \frac{1}{\sin^4\theta_W} \left( \ln \frac{s}{m_W^2} - \frac{5}{4} \right)$$

$$R_{WW} = \frac{3}{8} \frac{1}{\sin^4\theta_W} \left( \ln \frac{s}{m_W^2} - \frac{5}{4} \right) = 40$$

at  $\sqrt{s} = 1$  TeV. This is a substantial background to any signals for new fermions; it can be largely cut away since it is strongly forward/backward peaked, but it cannot be ignored.

At very high energies, suppose there is one unit of  $R$  for producing a new fermion, and put  $s = 0.25$  TeV<sup>2</sup>. Then

$$\sigma = 4 \times 10^{-37} \text{ cm}^2$$

and in  $10^7$  sec to see  $N$  events would require a luminosity ( $N = \sigma \mathcal{L}$ )

$$\mathcal{L} > \frac{N}{4} \times 10^{30} \text{ cm}^{-2} \text{ sec}^{-1}.$$

A good way to compare  $e^+e^-$  and hadron colliders is simply to check that both machines in question can find a new heavy quark  $Q$  if it is present. We will discuss below how one proceeds to do that

for a hadron collider, and see that it is somewhat model dependent but not much. Having determined the mass  $M_Q$  that a given hadron collider can achieve, one must simply require  $\sqrt{s}$  for  $e^+e^- > 2M_Q$ . Then, the luminosity must be large enough as above. In addition, since one of the main reasons we will build future colliders is to try to understand Higgs physics better, it is necessary to also check that both machines are sensitive to the same scale of heavy  $H^0$  or Higgs interactions. And, eventually a cost comparison is also necessary.

#### Determining the Charge of a New Heavy Quark

Suppose a new heavy quark is discovered. Can we decide if it has  $Q = 2/3$  or  $Q = 1/3$  (or -----)? In principle, they have different  $\Delta R$  values, but the situation is not so simple as at low energies, because (see the numbers above) (i)  $\Delta R/R \ll 1$ , and (ii)  $\Delta R_{2/3} = \Delta R_{-1/3}$ . It turns out that a good way to decide is available, the analysis of the forward/backward asymmetry in the semileptonic decay. The differential cross section is of the form

$$\frac{d\sigma}{d\Omega} (\bar{f}f \rightarrow \gamma, Z \rightarrow Q\bar{Q}) = A + B \cos^2\theta + C \cos\theta \, d\Omega$$

where  $\cos\theta = \hat{p}_f \cdot \hat{p}_Q$ . The coefficient  $C$  determines the asymmetry and it is

$$C \sim 2V_f A_f V_Q A_Q |X|^2 + e_Q e_f A_Q A_f R_e X$$

$$X = s / (s - m_Z^2 + im_Z\Gamma_Z).$$

If we interchange a  $t$  and a  $b'$  quark, for example, all of  $V_Q, A_Q, e_Q$  change sign so  $C$  does not change sign, and  $t$  and  $b'$  have the same forward/backward asymmetry. The size of the asymmetry changes a

little, but it is hard to measure accurately. However, when they decay

$$t \rightarrow \ell^+ \nu \chi$$

$$b' \rightarrow \ell^- \bar{\nu} \chi.$$

So the lepton forward/backward asymmetry is opposite and allows us to easily distinguish the two cases.

### Quarkonium

Let us briefly summarize the quarkonium results. For a  $Q\bar{Q}$  bound state  $\Theta$  we can have

$$e^+e^- \rightarrow \Theta \rightarrow \ell^+\ell^-, \gamma H^0, Z^0 H^0, \dots$$

and

$$\sigma(e^+e^- \rightarrow \Theta) = \frac{12\pi \Gamma_{ee} \Gamma_{TOT} M_\Theta^2}{s (s-M_\Theta)^2 + \Gamma_{TOT}^2 M_\Theta^2}.$$

In the narrow width approximation,

$$\int R_\Theta d\sqrt{s} = \frac{9\pi}{2\alpha^2} \Gamma_{ee}.$$

The peak R value will be approximately

$$R_{\text{peak}} = \frac{9\pi}{2\alpha^2} \frac{\Gamma_{ee}}{\gamma_{\text{expt}}},$$

where  $\gamma_{\text{expt}}$  is the experimental resolution (assuming  $\gamma_{\text{expt}} > \Gamma_{ee}$ ),

$$R_{\text{peak}} = 2.6 \times 10^5 \Gamma_{ee}/\gamma_{\text{expt}}$$

so for  $\Gamma_{ee} = 3 \text{ KeV}$  and  $\gamma_{\text{expt}} = 300 \text{ MeV}$  one has

$$R_{\text{peak}} = 2.6.$$

This is not a large peak at higher energies -- it will be hard to see it. Radiative correction make it worse, reducing R by  $\sim 1/2$ .

When the Z contributions are included,[4]

$$\Gamma_{J=\Gamma}^{ee} = 8\pi\alpha^2 \frac{|\psi(0)|^2}{M_\Theta^2} \left\{ \left| Q_f - \frac{M_\Theta^2}{M_\Theta^2 - M_Z^2} g_L^e V_f \right|^2 \right.$$

$$\left. + \left| Q_f - \frac{M_\Theta^2}{M_\Theta^2 - M_Z^2} g_R^e V_f \right|^2 \right\}$$

where

$$g_{L,R}^e = \frac{T_{L,R}^{3,e} + \sin^2\theta_w}{\sin\theta_w \cos\theta_w}$$

and

$$V_f = \frac{\frac{1}{2} T_L^{3,f} - Q_f \sin^2\theta_w}{\sin\theta_w \cos\theta_w}.$$

To derive this, note that if the Z contribution is dropped, it gives the standard result for  $\Theta \rightarrow e^+e^-$  via a photon. Since the  $e^+e^-$  couples to the  $\gamma$  with equal left- and right-handed couplings, we can split  $2Q_f^2$  into  $Q_f^2 + Q_f^2$ . The  $Z^0$  contribution adds to each of those, and left- and right-handed couplings do not interfere. Since the  $\Theta$  is a vector state, only the vector part  $V_f$  of the Z coupling enters.

### Heavy Quark Lifetimes

Apart from hadronic corrections such as enhanced non-leptonic decay modes, any heavy fermion is expected in the Standard Model to decay via  $W^\pm$  in the usual way, giving per channel

$$\Gamma(Q \rightarrow qff') = \epsilon G_F^2 M_Q^5 / 192 \pi^3,$$

where  $\epsilon$  is a possible mixing angle factor when decays cross generations. Corrections of order  $\alpha_s/\pi$  will always be expected, just as in  $R(e^+e^- \rightarrow q\bar{q})$ . For the  $\tau$  lifetime this prediction is verified to within a few % now. For the  $b$  there is a mixing angle factor  $\epsilon \approx 1/400$ , which fortuitously puts the lifetime of the  $b$  in a region where it can be measured. For all heavier quarks one expects that the decays will be too fast for the lifetime to be observed even in the most high resolution detector.

Fortunately, again for the  $t$ -quark, we may be lucky and the lifetime may be measurable, in a different way. It has been noticed [5] that for an object with mass  $m = 45$  GeV the partial width due to weak decays,

$$\Gamma_W = N G_F^2 m^5 / 192\pi^3$$

for  $N = 9$  channels is  $\Gamma_W \approx 50$  KeV, while from strong and electromagnetic annihilation decays one expects  $\Gamma_{S, EM}^0 = 25$  KeV by extrapolation from lower quarkonium states. Thus, if the weak decays are observed it amounts to putting a bound on the lifetime,

$$\Gamma_W > \Gamma^0$$

or approximately

$$\tau_t \leq 10^{-19} \text{ sec.}$$

If both weak and annihilation decays are observed, and one trusts the extrapolation of the quarkonium results (as expected) to  $t\bar{t}$ , then the branching ratios provide an actual measurement of the  $t$ -quark lifetime! So far no one has suggested how to measure the

lifetime of quarks heavier than about 50 GeV, or of leptons heavier than about 2.5 GeV.

#### Perturbative Production of Heavy Quarks in Hadronic Reactions

It is standard to use the parton model to calculate heavy quark production in hadronic reactions. For short distance interactions a firm theoretical basis is provided by QCD, but to carry out an actual calculation some ingredients of a phenomenological nature are required. In this section I will follow in a few places the useful books of Halzen and Martin [2], and of Collins and Martin [1]. See also Refs. [6] and [7].

We imagine a collision of hadrons  $A, B$  containing partons  $a, b$  (which can be  $q, g, \gamma, W, \dots$ ). A heavy quark pair  $Q + \bar{Q}$  is produced at a large  $P_T$ . We expect perturbation theory to be useful if the time  $t \sim 1/P_T$  over which the collision occurs is small to the confinement time  $T \sim \gamma R$ , where  $\gamma$  is the time dilation factor ( $\gamma = P_T/M_Q$ ) and  $R$  is a typical hadronic distance ( $R \sim 1/m_\pi$ ). This gives  $T/t \sim P_T^2/M_\pi M_Q \gg 1$  as hoped. Because of the asymptotically free nature of QCD, short distance collisions should be well described by the lowest order perturbative contribution. We assume the masses of  $A, B, a, b$  can all be neglected.

The differential cross section for  $Q\bar{Q}$  production is then

$$d\sigma(A+B \rightarrow Q\bar{Q}X, s) = \sum_{a,b} \int_0^1 dx_a dx_b F_{a/A}(x_a) F_{b/B}(x_b) d\hat{\sigma}(a+b \rightarrow Q\bar{Q}, \hat{s});$$

$F_{a/A}(x_a)$  is the probability, or "structure function" for finding parton  $a$  in hadron  $A$ , with a carrying a fraction  $x_a$  of the hadron

momentum. The constituent cross section for  $a+b \rightarrow Q+\bar{Q}$  is  $\hat{\sigma}$ ; we will discuss below how to calculate it. The full cross section is evaluated at

$$s = (P_A + P_B)^2 = 2P_A \cdot P_B$$

and the constituent cross section at

$$\hat{s} = (p_a + p_b)^2 = 2p_a \cdot p_b = 2x_a x_b P_A \cdot P_B = x_a x_b s .$$

The results of the UA1 and UA2 experiments at the CERN  $\overline{SppS}$  collider on production of  $W^\pm$ ,  $Z^0$ ,  $q\bar{q}$  jet pairs,  $gg$  jet pairs, and 3 jet events, establish that to an accuracy of a factor of 2 or so the above procedure indeed gives results consistent with experiment. However, as we will see below, for charm particles, which are relatively light, the observed rate may be larger than predicted.

### Structure Functions

Much of the difficulty in carrying out such calculations arises from the structure functions. They measure properties which are non-perturbative, so they will not be calculable from first principles for a long time. Furthermore, the probability of a parton carrying a momentum fraction  $x$  depends on the distance scale being probed, because of the possibility of gluon radiation, so the structure functions really depend on two variables,  $x$  and  $Q^2$ , where  $Q^2$  measures the distance scale involved; e.g.  $Q^2 = 4p_T^2$  for production of a large  $p_T$  quark pair, or  $Q^2 = M^2$  for production of a heavy particle of mass  $M$ .

In practice, these problems are dealt with by imposing all of

the constraints possible on the structure functions, including some differential equations they must satisfy, measuring them over a limited range of  $x$ ,  $Q^2$ , and extrapolating to other  $x$ ,  $Q^2$ .

As  $x \rightarrow 1$ , a single parton would have to carry all of the momentum of the hadron, which should be very improbable, so presumably

$$F(x) \xrightarrow{x \rightarrow 1} 0 .$$

To arrange that one parton carried all of the momentum, all of the others would have to transfer all of their momentum to that one, which suggests that  $F$  should vanish faster as the number of partons involved increases. This has been embodied in "dimensional counting rules", [8]

$$F(x) \xrightarrow{x \rightarrow 1} (1-x)^{2n_s - 1} ,$$

where  $n_s$  = minimal number of partons whose momentum would have to vanish. For example, the structure function for a valence quark requires two spectators to give up their momentum as  $x \rightarrow 1$  so

$$F_{val} \sim (1-x)^3 ,$$

the structure function for a gluon requires 3 valence quarks to participate so

$$F_g(x) \sim (1-x)^5 ,$$

and the structure function for a sea-quark requires the three valence quarks plus the sea-antiquark so

$$F_{sea} \sim (1-x)^7 .$$

As  $x \rightarrow 0$  the situation is much more subtle.[9] No rigorous QCD analysis can be done, and one proceeds by comparing deep inelastic electron scattering in an appropriate high energy limit with the expected behavior of  $\sigma_{T\gamma}(\gamma p)$ . The result is that one expects

$$F \xrightarrow{x \rightarrow 0} x^{-\alpha}$$

where  $\alpha=1$  for sea quarks, and  $\alpha=1/2$  for valence quarks.

Thus, we assume a parameterization at a given  $Q^2$

$$F(x) = C x^{-\alpha} (1-x)^{\beta}$$

where approximate expected values for  $\alpha, \beta$  are known. Then (see below) the  $Q^2$  variation can be calculated. A major and valuable constraint is that the same structure functions should be used everywhere, so one can use ep and vp deep inelastic data, lepton pair production  $pp + e^+e^-X$ , and large  $P_T$  jet pairs to measure  $C, \alpha, \beta$ .

Further, useful constraints come from consistency conditions. There are two up quarks in a proton plus  $u\bar{u}$  pairs, one down quark +  $d\bar{d}$  pairs, and only  $s\bar{s}$  pairs. So

$$\int_0^1 dx [F_{u/p}(x) - F_{\bar{u}/p}(x)] = 2 \quad ,$$

$$\int_0^1 dx [F_{d/p}(x) - F_{\bar{d}/p}(x)] = 1 \quad , \text{ and}$$

$$\int_0^1 dx [F_{s/p}(x) - F_{\bar{s}/p}(x)] = 0 \quad .$$

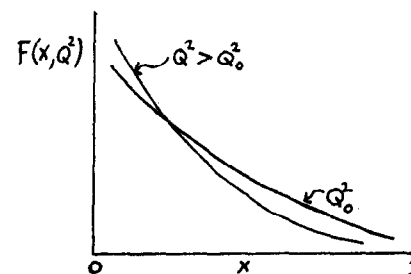
Similarly, adding up all partons must give the total momentum, so,

$$\sum_a \int_0^1 x F_{a/p}(x) dx = 1 \quad .$$

It turns out that quarks contribute about 1/2 of this and gluons the other half; the need for something that carried about 1/2 of the momentum of a proton but did not interact with e or  $\nu$  provided the first indications for gluons.

#### Scaling Violations

Now we must incorporate the effects of quarks and gluons being able to radiate gluons with a significant probability. Because a quark can split its momentum between a quark and a gluon, and does so virtually, the probability of finding a quark at a given  $x$  depends on the  $P_T$  scale involved. A larger  $x$  quark is more likely to have radiated some momentum away and dropped to smaller  $x$  than a small  $x$  quark. Qualitatively we expect an effect as shown,

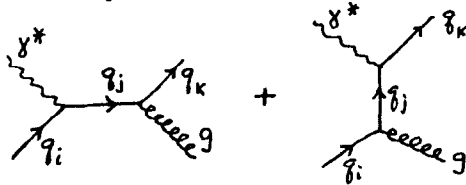


This is observed to happen for  $1 \text{ GeV}^2 \leq Q^2 \leq 200 \text{ GeV}^2$ . To predict high energy cross sections we need to be able to calculate for  $Q^2 \sim 4M_Q^2 > 16 \text{ TeV}^2$ , a large extrapolation. One way to proceed is as follows.[10],[1],[2] By calculating the rate for gluon emission when



a virtual particle (e.g. a virtual photon) hits a quark, we can extract the probability of gluon emission, and use it to see how the momentum is shared.

To find the probability of gluon emissions, we want to calculate the diagrams



where  $\gamma^*$  is a virtual photon of momentum  $K$ ,  $K^2 = -Q^2$ ,  $q_i$  is a quark of momentum  $p$  and color  $i$ ,  $q_j$  a quark of color  $j$ ,  $q_k$  a quark of color  $k$  and momentum  $p'$ , and  $g_a$  a gluon of color  $a$  and momentum  $k'$ . All masses are taken as zero except for the virtual  $\gamma$ . The matrix element squared and summed and averaged over spins is (a variable always denotes one in the constituent collision)

$$|M|^2 = 32\pi^2 e_q^2 \alpha_s^2 [-\hat{u}/\hat{s} - \hat{s}/\hat{u} + 2\hat{t}Q^2/\hat{s}\hat{u}] .$$

In color space the matrix element is  $M = \delta_{ij} \lambda_{jk}^a / 2$  where  $\lambda^a$  are the SU(3) generators. Then averaging and summing our colors gives a color factor

$$\begin{aligned} \frac{1}{3} \sum_{i,j,k,a} |M|^2 &= \frac{1}{12} \delta_{ij} \delta_{jk} \lambda_{im}^a \lambda_{jm}^a = \frac{1}{12} \delta_{jm} (\lambda^a \lambda^a)_{jm} \\ &= \frac{1}{6} \delta_{aa} = \frac{4}{3} . \end{aligned}$$

If the initial and final C.M. momenta are  $K, K'$ , then the outgoing quark has a transverse momentum

$$p_T = K' \sin\theta ,$$

and

$$\hat{t} = -2kk'(1-\cos\theta)$$

$$\hat{u} = -2kk'(1+\cos\theta) .$$

For small angle collisions,

$$p_T^2 = -\hat{u}\hat{s}/(\hat{s}+Q^2),$$

$$dp_T^2 = \hat{s}d\Omega/4\pi, \text{ and}$$

$$\frac{d\hat{\sigma}}{dp_T^2} = \frac{8\pi}{3} e_q^2 \alpha_s^2 \left[ \frac{-\hat{s}}{\hat{u}} - \frac{2Q^2(\hat{s}+Q^2)}{\hat{s}\hat{u}} \right] .$$

If we define

$$z = \frac{Q^2}{\hat{s}+Q^2} = \frac{Q^2}{(k+p)^2 - k^2} = \frac{Q^2}{2pk} ,$$

then

$$\frac{d\sigma}{dp_T^2} = \left( \frac{4\pi^2 e_q^2 \alpha_s^2}{\hat{s}} \right) \frac{\alpha_s}{2\pi} P_{qq}(z) \frac{1}{p_T^2}$$

where

$$P_{qq} = \frac{4}{3} \frac{1+z^2}{1-z} .$$

The factor in brackets is the virtual photon absorption probability without gluon emission. The factor  $P_{qq}$  is singular at  $z=1$  since it is possible to emit soft massless gluons; this singularity is cancelled by diagrams with virtual gluons in a full treatment, but they have little effect in the kinematic domain of interest to us so

we will ignore them. We still need to integrate over  $P_T$ , with physical cutoffs

$$(P_T^2)_{\max} = \hat{s}/4 = Q^2 (1-z)/4z$$

and some  $(P_T^2)_{\min} \equiv \mu^2$  from hadronic structure considerations;  $\mu^2$  will set the scale in a log factor that arises. Then

$$\int dP_T^2/P_T^2 = \ln Q^2/\mu^2 + \text{-----}$$

so

$$\sigma(\gamma^* q + gq) = \sigma_0 \left[ \frac{\alpha_s}{2\pi} P_{qq}(z) \ln \frac{Q^2}{\mu^2} \right].$$

Since  $\sigma_0$  is the probability of virtual photon absorption without gluon emission, we identify the factor in brackets as the probability of gluon emission.

A disturbing feature has appeared -- we know that  $\alpha_s \sim 1/\ln Q^2$  as  $Q^2$  increases, leading to the increasing validity of perturbative analysis for shorter distance collisions -- this is the aspect of QCD called asymptotic freedom. But here  $\alpha_s \ln Q^2$  enters, and that does not fall as  $Q^2$  increases. It turns out, somewhat miraculously, that when a full analysis is made for the structure functions, the  $\ln Q^2$  factor gets absorbed in just such a way as to restore the increasing validity of perturbative theory at larger  $Q^2$ , resulting in the so-called Altarelli-Parisi equations.

To understand how this happens we proceed as follows. Recall that we could write the structure functions, e.g.  $F_2(x)$ , as

$$\begin{aligned} F_2(x) &= \sum_q e_q^2 x F_{q/p}(x) \\ &= x \int_q e_q^2 \int_x^1 \frac{dy}{y} F_{q/p}(y) \delta(1-x/y). \end{aligned}$$

Note  $y > x$  in the integral. Now add the gluon emission term, which must be present physically,

$$F_2 = x \int_q e_q^2 \int_x^1 \frac{dy}{y} F_{q/p}(y) \left[ \delta(1-x/y) + \frac{\alpha_s}{2\pi} P_{qq}(x/y) \ln Q^2/\mu^2 \right].$$

Let  $z = x/y$ . Then  $x < z < 1$ . In the first term  $z=1$ , i.e. the quark keeps all of its momentum. In the second term the quark can emit a gluon and end up with  $x < z$ . We integrate because we must add up all the ways to end up with momentum fraction  $x$ . The above expression can be rewritten

$$F(x) = x \int_q e_q^2 \int_x^1 \frac{dy}{y} \delta(1-x/y) \left\{ F_{q/p}(y) + \frac{\alpha_s}{2\pi} \ln \frac{Q^2}{\mu^2} \int_y^1 \frac{dy'}{y'} F_{q/p}(y') P_{qq}(y/y') \right\}.$$

We call the entire integral the quark distribution function

$$q(x, Q^2) = q_0(x) + \Delta q(x, Q^2).$$

It can be written as a first order differential equation, whose solution is the above integral:

$$\frac{dq(x, Q^2)}{d \ln Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} q(x, Q^2) P_{qq}(x/y).$$

This is called the Altarelli-Parisi evolution equation. Note that the  $\ln Q^2$  factor is now absorbed in the definition of the structure function, and that now we have a useful perturbative series with higher terms smaller by at least  $1/\ln Q^2$ . The basic physics that is built in is that a quark with momentum fraction  $x$  could have come

from one with momentum fraction  $y > x$  if it radiated a gluon.

When all ways of producing such a quark are included, we have at the same order the possibility of a gluon giving a  $q\bar{q}$  pair, so  $g \rightarrow q\bar{q}$ . Then we need to include a term with not only the probability  $q \rightarrow q$ ,  $P_{qq}$ , but also the probability for  $g \rightarrow q$ ,  $P_{gq}$ , and an associated, coupled, differential equation for the gluon distribution function.

The full procedure is then to measure the coupled distribution functions at one  $Q^2$  as functions of  $x$ , and use the Altarelli-Parisi equations to compute them at other  $Q^2$ . Fortunately, the job has been done for us in two valiant efforts which give compatible, useful structure functions. One set is by Duke and Owens (00), [11] the other by Eichten, Hinchliffe, Lane, and Quigg (EHLQ). [12] The latter is more comprehensive, including distribution functions for the heavier quarks  $c, b, t$  as well as light ones, and extensively checking numerical stability for very small  $x$  and for very large  $Q^2$ . The heavier quarks arise because of the possibility of  $g \rightarrow b\bar{b}$ , etc. At  $Q^2 = 10^8 \text{ GeV}^2$  and  $x = 0.01$ , they occur in the ratios  $t/b/c/s/u = 0.21/0.33/0.36/0.89/1$ .

For most applications both sets are useful, and often it is a useful check to calculate with both to see the range of cross sections predicted. Both groups take all existing data for  $ep$ ,  $en$ ,  $\nu p$ ,  $\nu n$ , lepton pair production, jet production, etc., and either use it in their extrapolation procedure or check that the results are consistent with it.

Caution is required at very small  $x$ , which can be needed at very high energy machines when  $m/\sqrt{s} \ll 1$ . The Altarelli-Parisi

equations, when solved with an initial parameterization such as  $x^{-\alpha}(1-x)^{-\beta}$  are known to diverge as  $\exp(\ln 1/x)^{1/2}$ , so at some small  $x$  the existing parameterizations must go bad. EHLQ suggest their results should be stable down to  $x \approx 10^{-4}$ , which is sufficient for most applications, because the value of a structure function at some small  $x$  arises from integrating over more reliable structure function values from larger  $x$ .

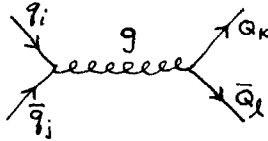
An important question is how to separate the various structure functions, since many processes contribute to each observable reaction. Enough information is available to readily fix the various quark distributions. The process  $\nu + \mu^-$  at large  $x$  selects out a  $d$  quark in the target, and also a  $\bar{u}$  at small  $x$ , while  $\bar{\nu} + \mu^+$  selects a  $u$  at large  $x$ , a  $\bar{d}$  at small  $x$ . Hard photons arise at large  $p_T$  from the constituent process  $gq + \gamma q$  and constrain  $F_{g/p}$ . The CERN large  $p_T$  jet-jet events arise in part from  $gg \rightarrow gq$  and constrain  $F_{g/p}$ ; the contribution of quark jets is already constrained by using  $F_{q/p}$  from other data. A full analysis can determine all of the parton distributions, but to do so it must carefully check that the relevant data do tightly constrain each distribution used in the analysis.

#### Constituent Cross Sections

The other ingredient needed to get the full cross section is the constituent cross section for the process at hand. For our purposes this is mainly pair production of heavy quarks,  $gg \rightarrow Q\bar{Q}$ . The rate for this has been computed in Ref. 13. Here we just

illustrate a piece of the calculation.

For the diagram



where  $a = 1, 2, \dots, 8$  and  $i, j, k, l = 1, 2, 3$  are color indices, the matrix element is

$$M = \frac{g_s^2}{\hat{s}} (\bar{v}_i \gamma^\mu u) (\bar{v}_j \gamma_\mu u) \sum_a \frac{\lambda_{ij}^a}{2} \frac{\lambda_{kl}^a}{2}.$$

Using  $\text{Tr } \lambda^a \lambda^b = 2\delta^{ab}$ , the color factor is, averaging over initial color and summing over final ones,

$$\frac{1}{9} \frac{1}{4} \sum_{\substack{ijkl \\ ab}} \lambda_{ij}^a \lambda_{ji}^b \lambda_{kl}^a \lambda_{lk}^b = \frac{1}{9} \sum_{ab} \delta^{ab} \delta^{ab} = \frac{8}{9}.$$

Then the full average and sum over colors and spins gives

$$|M|^2 = \frac{64}{9} \pi^2 \alpha_s^2 \frac{(M_Q^2 - \hat{t})^2 + (M_Q^2 - \hat{u})^2 + 2M_Q^2 \hat{s}}{\hat{s}^2}$$

so, integrating over angles,

$$\hat{\sigma} = \frac{8\pi}{27} \frac{\alpha_s^2}{\hat{s}} \left(1 + 2M_Q^2/\hat{s}\right) \sqrt{1 - 4M_Q^2/\hat{s}}.$$

If one wants to examine a differential rate in angle or  $p_T$  of  $Q$ , one stops the integrations at an earlier stage. One can compute the

full constituent cross section for any process, or take it from Ref. 13 for  $QQ$  production, and convolute it with structure functions as needed.

### $p_T$ Distributions

The distribution in  $p_T$  of the heavy quark production has to be viewed in two regions, or even three regions. At  $p_T \gg m_Q$  and  $p_T \gg 1 \text{ GeV}$ , the distribution should be predicted by the perturbative QCD calculation we are considering; to get fully quantitative agreement, effects from recoil due to gluon radiation need to be included.

For  $p_T \leq m_Q$ , binding effects can matter so the result must be a nonperturbative one, and is therefore not calculable. A parameterization can be used, such as

$$\frac{d\sigma}{dp_T^2} \sim e^{-a\sqrt{p_T^2 + m_Q^2}} \quad \text{or} \quad e^{-bp_T}.$$

The quantities  $a, b$  can be fitted to data in a given experiment, or taken from a similar early experiment to make predictions. For charm production typical values are  $a \approx 2 \text{ GeV}^{-1}$ ,  $b \approx 1 \text{ GeV}^{-1}$ . [14]

Note that there is some ambiguity in the size of  $\sigma_{TQ} = \int F(x_a)F(x_b)\hat{\sigma}$ , since the integral should begin at  $\hat{s} = 4M_Q^2$ , and be dominated by  $\hat{s}$  near threshold since  $F(x)$  falls rapidly. Therefore,  $Q, \bar{Q}$  are produced with small relative velocity, so binding effects are important. When comparing with experiment it is probably best to avoid this problem by defining both theory and experiment for  $p_T^{\text{min}} \geq m_Q$ .

### Experimental Situation

The theory is nice and clean, to the needed accuracy. How well does it agree with data? Most data is for production of charmed quarks, and the data from FNAL, the CERN SPS, and the ISR, is (a) not internally consistent, and (b) not in agreement with the predictions of the theory. It is found that  $\sigma(\text{expt}) > \sigma(\text{theory})$ , and that the excess is concentrated at larger  $x$  values, i.e. the emerging  $c$  or  $\bar{c}$  carries a larger fraction of the hadron momentum than expected.

There are a number of helpful discussions, by Halzen,[15] Reucroft,[16] Bellini,[17] Halzen and Martin,[18] Gurtu,[19] and Collins and Martin [1]; they show data, explore the conflict in detail, and discuss proposed theoretical alternatives. Data comes from  $pp + c\bar{c}X$ ,  $\pi N + c\bar{c}X$ ,  $\gamma N + c\bar{c}X$ ,  $\nu N + cX$ ,  $\mu N + c\bar{c}X$ . Charmed particles are detected by (i) semileptonic decays  $c + \ell\nu X$  giving prompt leptons, (ii) by narrow peaks in exotic combinations of hadrons (i.e. combinations not arising from  $q\bar{q}$  or  $qqq$ ) in the channels  $D, D^* + k\pi$ ,  $k\pi\pi$ ;  $\Lambda_c + k^-p\pi^+$ ;  $A^+ + \Lambda k^- \pi^+ \pi^+$ , and (iii) by separate production and decay vertices in high resolution detectors. All of these are discussed in the above articles.

An important piece of data to keep in mind for any final resolution is that cosmic ray data, at  $E_{\text{beam}} = 10$  TeV, is reported [20] to give about one charm pair every 20-40 events, which requires a cross section  $\sigma_{c\bar{c}} = 2-3$  mb; this is at  $\sqrt{s} = 140$  GeV. This would correspond to about  $3 \times 10^8$  charm pairs for an integrated luminosity of  $10^{35}$   $\text{cm}^{-2}$  at the CERN Spps!

We will not go into detail here about mechanisms to provide extra charm. They mainly fall into two categories, called "flavor excitation" and "intrinsic charm". [21], [22] Tests exist to distinguish them. Some of the data needed are the  $x$  dependence of  $\sigma$  for  $c$ , and separately for  $\bar{c}$ ; the comparison of  $s$ ,  $c$ ,  $b$ ,  $t$ ; the  $A$  dependence; the results for  $\nu N + cX$ , and for  $\mu N + \mu c\bar{c}X$ . As data improves at least the data will get consistent.

Many observers expect the following picture to emerge. Large  $p_T$ , central region production of any heavy quark,  $c$  included, will agree with the QCD predictions. A non-perturbative source of charm production exists which mainly populates large  $x$ , small  $p_T$ , and accounts for the extra charm at ISR. There is presently some controversy [23] about how this non-perturbative contribution will fall with increasing  $m_Q$ ; it might only be significant for charm, or it might fall as slowly as  $1/m_Q^2$  in which case it would contribute to  $b$  and  $t$  production [ $(m_c/m_t)^2 = 10^{-3}$  so  $10^8$  charm at Spps would allow  $10^5$   $t\bar{t}$ , though there would be some further phase space suppression], and even perhaps to production of more rare components of a nucleon such as supersymmetric scalar partners of quarks. Either theoretical consensus or better data will tell us fairly soon.

### Spin and Polarization Effects

#### A. Looking for Mirror Fermions

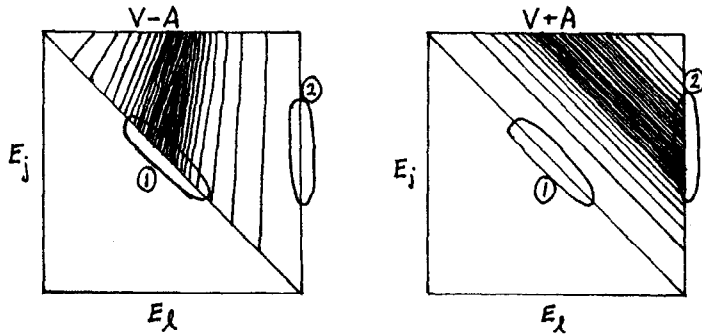
Suppose we produce a new quark. How can we tell if its decay is a normal V-A one or if new interactions are present? For example, many theories require the presence of "mirror fermions" that decay to lighter fermions via a V+A interaction.

Note that this question is not applicable to the t-quark, because the arguments given below that b-quark decays and interactions require the existence of a t-quark imply that the b and t have the same space-time properties, and there is already evidence that the b has V-A decays.

The answer is that one good way to tell is from the semileptonic decay distributions looked at on a Dalitz plot. A similar argument would hold for the b-quark. Consider an  $e_Q = 2/3$  quark, decaying in its rest frame,  $Q \rightarrow q\ell\nu$ , and assume all final state masses can be neglected. Then we can make a table,

	V-A	V+A
$ M ^2 \sim$	$(P_Q \cdot P_\ell)(P_Q \cdot P_\nu)$	$(P_Q \cdot P_\nu)(P_Q \cdot P_\ell)$
$d\Gamma/dE_j dE_\ell \sim$	$E_\ell(m_Q/2 - E_\ell)$	$E_\nu(m_Q/2 - E_\nu)$
		$E_\nu = m_Q - E_j - E_\ell$

where j stands for the quark jet. Then the Dalitz plots look very different:



The density of lines is qualitatively proportional to the expected density of events on the Dalitz plot. To be quantitative, one could choose regions such as those labeled 1, 2 above. Then  $N_1 - N_2$  is positive for V-A and negative for V+A.

For an  $e_Q = -1/3$  quark, V-A  $\rightarrow$  V+A. These arguments could be applied to b decay too. For a new quark the charge may have to be determined by the forward/backward asymmetry of the lepton in the semileptonic decay, as discussed a few sections above.

#### B. Uses of Polarized t-quarks (or Heavier Quarks)

We will consider two important uses or effects of polarized heavy quarks, expressing them in terms of t-quarks although they would apply to any new quark too.

Consider  $t(p) \rightarrow b(p') + \ell^+(k) + \nu(k)$  where momenta are shown in parentheses. The matrix element is

$$M \sim (\bar{u}(p') \gamma_\lambda P_L u(p)) (\bar{u}(k) \gamma^\lambda P_L \nu(k)) ,$$

so the width is proportional to

$$\begin{aligned} d\Gamma &\sim \text{Tr}[\gamma \cdot p' \gamma_\lambda P_L (\gamma \cdot p + m_t) (1 + \gamma_5 \gamma \cdot s) \gamma_\sigma P_L] \text{Tr}[\gamma \cdot k \gamma^\lambda P_L \gamma \cdot \ell \gamma^\sigma P_L] \\ &\sim \text{Tr}[\gamma \cdot p' \gamma_\lambda (\gamma \cdot p - m_t \gamma \cdot s) \gamma_\sigma P_L] \text{Tr}[\gamma \cdot k \gamma^\lambda \gamma \cdot \ell \gamma^\sigma P_L] \\ &\sim p' \cdot k (p \cdot \ell - m_t s \cdot \ell) . \end{aligned}$$

In the t rest frame,  $s = (0, \vec{s})$ ,  $p = (m_t, \vec{0})$ , and  $\ell_0 = |\vec{\ell}|$  so

$$d\Gamma \sim m_t E_\ell (1 + \vec{s} \cdot \hat{n}_\ell) \sim 1 + \cos\theta_{s\ell}$$

where  $\hat{n}_k$  is a unit vector in the direction of  $\vec{k}$ , so  $\theta_{sl}$  is the angle between the lepton momentum and the  $t$  spin directions. This result assumes 100% polarization of the present quark, but is otherwise general. It is different from the result for muon decay, when the  $\vec{s} \cdot \hat{n}_k$  correlation depends on the lepton energy (vanishing when  $E_k = M_\mu/4$ ). This result can have important implications, since it requires a strong correlation, with leptons favorably emitted in the direction of the  $t$  spin, the rate going to zero when the lepton is antiparallel to the  $t$  spin.

For a  $\bar{l}$  the appropriate projection operator is

$$(\gamma \cdot p - m) (1 + \gamma^5 \gamma \cdot s)$$

so effectively the sign of  $m$  changes, and the correlation is

$$1 - \frac{\hat{s} \cdot \hat{k}}{s}$$

In any particular application it is necessary to check whether both  $t$  and  $\bar{l}$  are present and whether the effects can cancel if charges are not measured or  $t$ ,  $\bar{l}$  cannot be distinguished. Let us consider two applications.

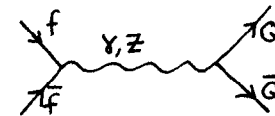
(a) Polarized  $t$ 's from  $W^\pm$  decays

This provides an application with possible significant practical implications. When  $u\bar{d}$  make a  $W^+$ , only left-handed  $u$ 's and right-handed  $\bar{d}$ 's can interact so the  $W^+$  is fully aligned in the direction of  $\vec{d}$ . Then in the decay the  $t$  must be left-handed and the  $b$  right-handed, so the  $t$  must go in the direction of the  $u$ , and its spin is mainly opposite to its momentum. Then the  $k^+$  is emitted mainly in the  $t$  spin direction so it is softer than it would be if the  $t$  were not polarized. For a  $\bar{l}$  from  $W^-$  decay the lepton goes

opposite to the spin but the spin is now mainly along the  $\bar{l}$  momentum so the  $k^-$  is again softer. It is very important to take this effect into account in calculating efficiencies and rates and masses when interpreting data on possible  $t$ -quark signals.

(b) True polarization test of QCD

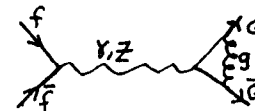
We can also use this effect to try for a true QCD polarization test.[24] Most QCD spin tests are really alignment tests and do not depend either on the phase structure or the helicity structure of QCD. If a heavy quark pair is produced in lowest order at an  $e^+e^-$  collider,



where  $f$  is any lepton or quark, the  $Q$  is necessarily unpolarized, since

$$Pol \sim \text{Im}NF^*$$

where  $N$  is the helicity, non-flip amplitude and  $F$  the helicity, flip amplitude, and they must not be relatively real to give nonzero  $Pol$ . But in the lowest order there is no helicity flip and the amplitudes are real. By going to one loop and including QCD corrections,



a polarization is generated [24] which tests both the relative phase of the amplitude and the QCD helicity couplings; the result is

$$P_{01} = \frac{4}{3} \alpha_s \frac{m_Q}{\sqrt{s}} \frac{\sin\theta \cos\theta}{1 + \cos^2\theta},$$

where  $\theta$  is the production angle of  $Q$ . This is a transverse polarization, normal to the scattering plane defined by  $\vec{k} \times \vec{Q}$ . The result is rigorous in the same sense as QCD jet formulas, and is obviously only valid to order  $\alpha_s$ .

How can we test this in practice? Since hadronization is not understood, and is fundamentally nonperturbative, it is always possible that the polarization is diluted during the hadronization; if possible tests are made and no effect is observed, then theories of hadronization would have to produce the depolarization. Hadronization may occur without depolarizing so the effect may be large. However, it is possible to do even better. First, one can make a general test. If a quark jet is not polarized, there can be associated with it only one direction, its momentum  $\vec{p}$ . If any directional observable, such as

$$\sum_{q \text{ in jet}} \vec{p}_T^q \times \vec{p}$$

gives a non-zero result, one has been able to associate another direction with the jet and therefore one has demonstrated that the jet was polarized! If a light quark jet were polarized, it would violate a basic QCD prediction, since  $P_{01} \leq m_Q/\sqrt{s}$ . A heavy quark jet can have at most the polarization given above.

Second, for  $t$ -quarks, as pointed out by Bigi and Kraseman,[25] if most  $t$ 's are produced as  $T^*$ 's and if the weak decay dominates, most polarization might be retained. Since we have seen above that

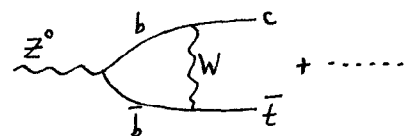
the  $\vec{L}^+$  goes in the direction of the spin in a  $t$  decay, we have a simple polarization analyzer! Simply look for an up-down asymmetry of charged leptons from  $t$ -decay relative to the production plane,

$$\langle \vec{f} \times \vec{t} \cdot \vec{L} \rangle \neq 0$$

and polarization will have been observed. Since the effect is parity violating no background can simulate it so long as experimental cuts do not introduce directions into the problem. For  $b$  or  $c$  the expected polarization is smaller; an optimum measurement will occur enough above threshold to produce jets but at as low a  $\sqrt{s}$  as possible, so  $m_Q/\sqrt{s}$  is as large as possible. Observation of a lepton asymmetry relative to the production plane is clear evidence for  $Q$  polarization.

#### Flavor Changing Z Decays

In the standard model,  $Z^0 \rightarrow t\bar{c}$  is induced at one loop but is expected to be too small to be observed, since all FCNC are forbidden at tree level by the GIM mechanism. It occurs via



so

$$(BR)_{SM} \leq (G_F m_t^2)^2 U_{bc}^2 \approx 10^{-6}.$$

Thus such a decay is an important window to look for new physics, for new interactions which could induce such a decay at a rate  $\geq 10^{-5}$  which is observable at SLC or LEP or TRISTAN.



For example, recently Duncan [26] has argued that it is possible in supersymmetric models for this branching ratio to be as large as about  $10^{-4}$  depending on detailed aspects of the structure of models.

Or a neutral Higgs or a horizontal gauge boson could have couplings so that the BR  $\sim (m_t^2/m_\chi^2)^2 \sim 10^{-4}$  for new bosons with masses of order 1/2 a TeV.

Alternatively, the absence of any such modes to a level of  $10^{-6}$  eventually will constrain new ideas about physics beyond the SM in important ways.

Recognizing such a mode may not be too hard. At  $e^+e^-$  colliders it gives a very asymmetric event with a fat jet opposite a thin one. The semileptonic mode of  $t$  will give an isolated lepton. Even at hadron colliders the signature is probably good enough to recognize, though the event rate may be too small.

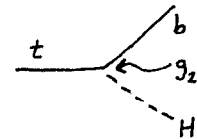
If such events are detected, the  $t$  polarization can be analyzed as discussed in earlier sections, giving even more information about the underlying mechanism which operates.

If  $t \rightarrow b\ell\nu$  Then ...

If it is observed that  $t$  (or any heavy quark or lepton) has a semileptonic decay, then we know that other decays do not dominate, and that tells us a lot about possible physics beyond the standard model.

(a) For example, if there exists a charged Higgs boson, or composite equivalent such as a technipion, then an allowed decay is

$t \rightarrow bH^+$



with strength  $g_2$ . This would give a partial width

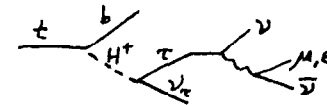
$$\Gamma_H = G_F m_t^3$$

which should be compared with the usual decay rate of  $G_F^2 m_t^5 / 192\pi^3$  per channel. The ratio is

$$\Gamma_H / \Gamma_0 = 192\pi^3 / N G_F m_t^2 \gg 1.$$

Thus if  $t \rightarrow bH^+$  is allowed, it will dominate.

If  $t \rightarrow bH^+$ , how do leptons arise? Assuming couplings are mainly proportional to mass,  $H^+ \rightarrow \tau\nu_\tau$  is the only leptonic channel; it should be smaller than  $H^+ \rightarrow c\bar{s}$  by a factor  $m_\tau^2/3m_c^2$  approximately, and possibly  $H^+ \rightarrow c\bar{b}$  could be important. If  $H^+ \rightarrow \tau^+\nu_\tau$ , then  $\tau^+ \rightarrow e^+\nu_e\nu_\mu$  giving finally  $\mu$  or  $e$ . Since the  $\mu$  or  $e$  arise at the 3rd decay stage, they are much softer and in general not isolated from an associated jet. They would not behave at all like the standard  $t$  decays. Consequently, if the standard  $t$  decays are observed, the  $H^+$  decay must be forbidden, which presumably implies  $m_{H^+} > m_t - m_b$ . Since a virtual decay is also allowed,



one can extend the numbers a little, depending on how accurately the  $t$  semileptonic branching ratio is known. For  $m_t = 45$  GeV, probably one can conclude at present that  $m_{H^\pm} \geq 50$  GeV.

(b) One can extend the argument to other channels. Suppose that nature were supersymmetric, which implies that every particle has a supersymmetric partner differing only by 1/2 unit of spin. Since these partners are not observed, they must be heavier than the known particles at least for the lighter ones--the partners of  $t$ ,  $W$ ,  $Z$  could be relatively light. Call the particle  $X$  and the partner  $\tilde{X}$ .

Then several decays could occur,

$$\begin{aligned} t &\rightarrow \tilde{t}\tilde{\gamma}, \\ t &\rightarrow \tilde{t}\tilde{g}, \\ t &\rightarrow \tilde{b}\tilde{W}, \text{ and} \\ t &\rightarrow \tilde{t}\tilde{Z}. \end{aligned}$$

For any of them the width would be of order

$$\Gamma \geq \alpha m_t$$

since they are two-body decays with coupling  $e$  or  $g_2 = e/\sin\theta_w$ .

Again they totally dominate the three-body decays, so they must be kinematically forbidden,

$$\begin{aligned} m(\tilde{t}) + m(\tilde{\gamma}) &> m_t, \\ m(\tilde{b}) + m(\tilde{W}) &> m_t \end{aligned}$$

etc.

Whatever light particles can occur in any other new theory are similarly constrained.

#### Implications of Flavor Changing Neutral Current Decays of Heavy Quarks

If we have a real theory, any observed behavior will provide lots of indirect information about possible new objects. A good example, which indicates the kinds of arguments that might be made, comes from  $b$  decays.[27]

Suppose that  $b$  were a weak isospin ( $SU(2)$ ) singlet, i.e. that a  $t$ -quark did not exist. Suppose also that we know that  $b$  does decay, and assume that its decays are mediated by the normal gauge bosons  $W^\pm, Z^0$ . We will see that a contradiction emerges from these assumptions so one of them must be wrong.

Since  $b$  is a singlet it cannot couple directly into the charged current, which must be of the form (with appropriate coefficients)

$$J_\mu^\dagger = \bar{u}\gamma^\mu P_L d' + \bar{c}\gamma^\mu P_L s'$$

where

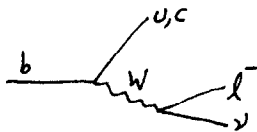
$$d' = c_1 d + c_2 s + c_3 b + \dots$$

is an arbitrary mixture of down type quarks, with  $s'$  a similar mixture. The neutral current will have similar terms

$$J^0 \sim \bar{u}\gamma^\mu P_L u + \bar{c}\gamma^\mu P_L c - \bar{d}'\gamma^\mu P_L d' - \bar{s}'\gamma^\mu P_L s'$$

(again with appropriate coefficients). Then  $b$  decays can result from the  $b$  mixing into  $d'$  and  $s'$ . Therefore the relative sizes of  $b$  decays via induced charge currents and induced neutral currents are determined by the coefficient  $c_3$  above, plus known coefficients, and the similar coefficients in  $s'$ .

If the decay only occurred via the  $d'$  term, the two possible mechanisms,



and



would occur in a definite ratio, since  $c_3$  would cancel out. With  $c_3$  and the equivalent piece from  $s'$ , there are two variables. It turns out that on examination one sees that the ratio of above contributions (neutral current to charged current) has a minimum value. That is very important, since the neutral current decays (giving  $l^+l^-$ ) are very small in the standard model (about  $10^{-5}$ ), while here one can show

$$r = \frac{BR(b \rightarrow X l^+ l^-)}{BR(b \rightarrow X l^- \nu)} > 0.11 .$$

Experimentally  $r < 0.029$  (90% CL) [28] so we know that either the SM does not apply for b decays, or the b is in a doublet with a t-quark. Further, the t must have the same space-time properties as b, since this argument only depends on the SU(2) transformation properties which commute with space-time ones.

Thus we have known for about three years that t exists and has the usual V-A charged current interactions and the usual neutral current interactions; that b does not have SM decays is the alternative, and the data was quite clear in showing that the b decays are indeed as expected in the SM.

More recently, the forward/backward asymmetry due to  $\gamma$ -Z<sup>0</sup> interference in  $e^+e^- \rightarrow \bar{b}b$  has been observed [29] and shows that  $T_3^b = -1/2$ , again indicating that t must exist and have the same space-time properties as b. Neither of these arguments can be used to find  $m_t$ , which hopefully has been determined by now by the UA1 collaboration.

#### Acknowledgements

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