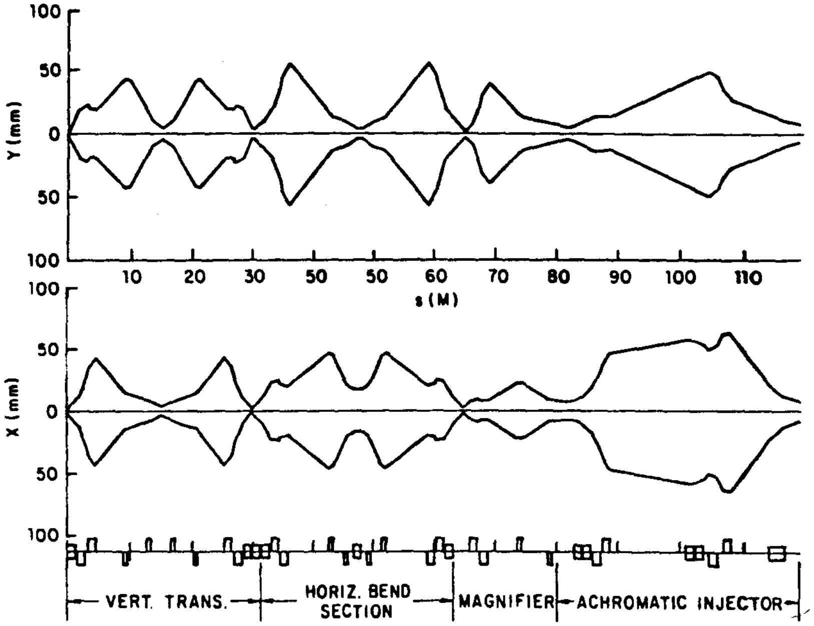

FIGURE 2

DESIGN CONSIDERATIONS FOR A MUON STORAGE RING

David Neuffer
Fermi National Accelerator Laboratory*, Batavia, ILL 60510

ABSTRACT

It was noted earlier[1] that a muon (μ) storage ring can provide neutrino (ν) beams of precisely knowable flux and therefore suitable for ν oscillation experiments. In that paper it was suggested that parasitic use of the Fermilab $\bar{p}$ precooler could provide a useful μ storage ring. In this paper design possibilities for μ storage rings are explored. It is found that a low energy (~1 GeV) ring matched to a high intensity proton source (8 GeV Booster) is most practical and can provide ν beams suitable for accurate tests of ν oscillations.

INTRODUCTION

In an earlier paper[1] it was noted that a muon storage ring can provide electron neutrino and muon neutrino beams for the study of neutrino oscillations. In that paper the Fermilab $\bar{p}$ precooler[2] was studied as a possible μ storage ring and flux estimates for the resulting ν are obtained.

The Fermilab $\bar{p}$ precooler must perform functions of $\bar{p}$ collection at 4.5 GeV/c $\bar{p}$ cooling and deceleration to 600 MeV/c, followed by $\bar{p}$ acceleration from 600 MeV/c to 8 GeV/c. The ring is designed for the maximum momentum of 8 GeV/c and is not optimum for μ storage at 4.5 GeV/c. In this paper we explore the possibility of designing a ring specifically for μ storage, and compare the resulting neutrino fluxes to the earlier precooler estimate.

The operation of a muon storage ring to provide neutrinos has been described previously by Cline and Neuffer[1], and is illustrated in Figure 1. A proton beam is focussed with high intensity on a target of ~1-2 interaction lengths, producing large numbers of secondary pions. The pions are separated from the non-interacting protons and transported to injection in a storage ring. Pion decay within the injection transport and after injection provides muons ($\Pi\to\mu\nu_\mu$), which are stored within the ring for the muon lifetime. Muon decay in the ring straight sections ($\mu\to e_\nu\nu_\mu$) provides collimated ν_e and $\bar{\nu}_\mu$ beams. A detector is placed along a beam line at some distance L from the ring to observe ν-interactions.

In reference 1, it was estimated that ~5×10^8 - 5×10^9 ν_e and ν_μ per pulse of 2×10^{13} 80 GeV/c protons could be obtained using the $\bar{p}$ precooler as a 4.5 GeV/c μ storage ring.

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*Operated by the Universities Research Association, Inc., under contract with the United States Department of Energy.

Choice of Storage Ring Energy

We first comment on the constraints governing the choice of an optimum storage ring energy. Neutrino oscillations are supposed to occur at a rate given by

$$P(\nu_1 \rightarrow \nu_2) \cong \delta_{12} + \sum_{i,j} 4\, U_{1i} U^*_{2i} U^*_{1j} U_{2j} \cdot \sin^2(1/2\ \Delta_{ij})$$

where the U_{ij} are ν - mixing matrix elements and

$$\Delta_{ij} = 1.27\ M^2_{ij}(\mathrm{eV}^2) \cdot \frac{L}{E}\frac{(\mathrm{m})}{(\mathrm{MeV})}$$

where M_{ij} is the neutrino mass difference, L is the distance from source to detector, and E is the neutrino energy. A sensitive search for ν-oscillation requires L/E relatively large.

The transverse momentum of neutrinos in the decay $\mu \rightarrow e \nu_e \bar{\nu}_\mu$ has an average value $\bar{p}$ = 30 MeV, so the mean angle of ν production is $\bar{p}/E$ and the ν intensity at the detector is proportional to $(L\theta)^{-2}$ or E^2/L^2. The ν-N cross section increases linearly with E, so the detector event rate varies as E^3/L^2. These factors favor higher energy and small distance, opposing the constraints of the previous paragraph. Cost of storage ring is roughly proportional to radius and therefore to energy E; this provides a counterfactor favoring small E.

Event rate also depends on the number of neutrinos which depends directly on the number of capturable π's produced at the target. Following empirical formulae of Wang this production has a broad maximum at $E_\pi \cong 0.1\ E_{\mathrm{proton}}$, similar to the broad maximum for $\bar{p}$ production at $E_{\bar{p}} \cong 0.1\ E_p$ used in $\bar{p}$ accumulators.

Our design strategy is to choose $E_\pi \cong E_\mu$ (ring energy) within this maximum and then to place the detector at the largest value of L consistent with adequate event rate.

We design below sample storage rings with E_μ = 8 GeV, 4.5 GeV and 1.5 GeV, and note that these can be matched to several possible proton beams such as the Fermilab 80-GeV extraction line, the CERN PS (30 GeV) or the Fermilab booster (8 GeV).

Storage Ring Injection

In the precooler/storage ring of reference[1] most of the π decays occurred within the transport from target to ring injection. This procedure was favored because: (1) the momentum acceptance of the $\bar{p}$ precooler (±2%) was much smaller than the injection transport (±10%), (2) single turn injection was demanded for $\bar{p}$ accumulation, (3) the initial precooler design contained sufficient transport length.

In a redesigned muon storage ring the momentum acceptance can be more directly matched to the injection acceptance, and to the pion decay momentum width, so pi decays can occur within the storage ring.

We suggest that better injection schemes are shown in Figure 2. The injection optics is provided by a short section containing a lithium lens (or "horn") immediately following the target, bending magnet to separate positives from negatives, and a few quadrupoles and kicker magnets to match injected beam to the desired orbit. Only a few meters of magnet length are necessary for matching.[3]

Pi decay permits use of what we call "stochastic injection" in which injected π's decay to circulating muon orbits, which is similar to a previous scheme[4] for collecting $\bar{p}$'s from decaying Λ's. Injected pions naturally decay to lower energy muons within ~1 turn in the ring, and this circulating muon beam will naturally follow different orbits from the injected beam. "Stochastic injection" can continue indefinitely without changing the injection optics; the proton pulse length need not be fitted to the storage ring length as in the $\bar{p}$ precooler.

We find two alternative designs for stochastic injection and these are shown in Figures 2A and 2B. In Figure 2A the injection region beam optics matches injected π beam (at a higher momentum $p_\mu + \Delta p$) and circulating μ beam to identical orbits in an $\eta = 0$ (zero dispersion) straight section. The surviving π beam separates from the circulating beam in the curved section $(\eta \neq 0)$ and is lost. π decays in the straight section, which is $\sim L_\pi/2$ in our designs, contribute to the circulating beam. Separate injection and circulating orbits need only be provided in the injection area.

In Figure 2B, the pions are injected into an off-momentum orbit centered at $p_0 + \delta p/2$ with a circulating muon orbit at $p_0 - \delta p/2$ where p_0 is the central ring momentum and $\pm\delta p$ is the ring momentum acceptance. Pions circulate for a full turn of the ring (length L_T), but separate π and μ orbits must be provided for the full ring circumference, which limits μ acceptance.

A first appraisal indicates similar muon current accumulated by both designs and we have followed design B in the estimates below. A more detailed design study is necessary to develop a specific choice.

The important advantages of "stochastic injection" are

1. A separate transport for π-decay is unnecessary.
2. The proton pulse length need not be shortened to the storage ring circumference but can be the full proton synchrotron length. This allows greater proton intensity.
3. Stochastic injection is particularly attractive because the storage ring circumference is naturally matched to the pion decay length (see below).

Muon Storage Ring Design Constraints

In this section we outline some general design requirements of a μ storage ring. In Figure 1 we show a model μ ring with two long straight sections of length S and two half circle sections of length πR.

The possible values of R are limited by the bending requirement

$$R = B\rho/\bar{B}$$

where $B\rho$ is the magnetic regidity, which can be found from the formula: $B\rho$ (T-M) = 3.3 P (GeV/c) and P is the momentum. $\bar{B}$ is the mean bending field which is limited to $\lesssim$2 T for conventional magnets.

The mean path length L_π for relativistic pions of momentum P_π before decay is

$$L_\pi = 53.6\ P_\pi \text{ meters.}$$

Stochastic injection demands that

$$L_\pi \lesssim 2\pi R + 2S$$

since it is desired that π's decay within one turn. If we choose $S > \pi R$ we find a requirement $\bar{B} \lesssim .8$T. This is reasonably well matched to the field strength limit on $\bar{B}$ and will be used in our sample designs.

The mean path length of relativistic muons is:

$$L_\mu \simeq 6000 \text{ P (GeV/c) meters}$$

or ~110 turns of the ring if the ring circumference is L_π. To separate π decay ν_μ's from μ decay neutrinos we require that injection occur within a fraction of this lifetime, say ≲10 turns of the μ ring or one proton synchrotron circumference; this is a requirement that is naturally fulfillable.

ν beam intensity depends directly on the straight section length S, which should be chosen large relative to R. We will typically choose $S \simeq 4R$.

The ring should be designed with a large transverse acceptance and a large momentum acceptance. This seems to require a strong focussing ring lattice (FODO) with a relatively short FODO period. Sample designs are discussed in the next section.

Sample μ-Storage Ring Designs

In this section we present possible parameters for μ rings with E_μ = 8, 4.5, 1.5 GeV. The general ring design is a FODO lattice and we assume that beam acceptances are dominated by the lattice parameters. In designing the lattices we follow general design constraints outlined elsewhere by Collins.[5] For the 4.5 and 8 GeV rings we will use precooler magnets which are described elsewhere.[2] For the 1.5 GeV ring these magnets are not suitable and suggested parameters are outlined below.

As noted by Collins,[5] there is a broad optimum in lattice design at ~90^0 particle phase shift per FODO period, and we have conservatively chosen maximum fields of ~1 T. The aperture limitations are set by the maximum beta function value $\beta_{max} \cong 3.4\ L_{FO}$ and the maximum dispersion function $\eta_{max} \cong 2.7\ L_{FO} \cdot \theta_0$, with L_{FO} = 3.8 m with 2 1.5 m 1 T bending magnets and one .6 m quad per period. The ring contains two long straight sections of 1.5 πR length (see Figure 1).

For the 4.5 GeV design we have used one precooler quad and one bend per half period (2.2 m). For the 1.5 GeV case we have scaled to a 1 m length, envisaging a bending magnet of ~.7 m length and a .3 m quad. Acceptance parameters are shown in Table 1.

To calculate neutrino beam intensities we have used the formula of C.L. Wang[6] to calculate pion production in a one interaction length target. We have assumed transverse acceptance is dominated by the lens aperture immediately following the target, which we assume to be a Li lens with the same parameters as in the precooler design report. This sets the acceptance angle in Wang's formula:

$$\frac{d^2N_\mu}{d\,P_\pi\,d\Omega} = A\,P_p\,x\,(1-x)\,e^{-BX^C\,-DP_\pi\theta} \qquad \frac{\text{pions}}{\text{GeV/c - interacting proton}}$$

with A = 1.57, B = 5.73, C = 1.33, D = 4.25, P_p the incident proton momentum, P_π the product pion momentum, $x = P_\pi/P_p$ and θ is the production angle with θ_{max} = 25 mR at 8 GeV P_π/c and 50 mR at 4.5 GeV, 100 mR at 1.5 GeV.

The acceptance of the storage ring with stochastic injection is limited by the necessity of separate orbits for both injected pions and circulating muons. We assume the momentum acceptance for both orbits is given by momentum half-width $\Delta p/p$.

We finally obtain the number of ν_e and $\bar{\nu}_\mu$ per proton on target by multiplying the target efficiency (0.4), a π-decay efficiency (0.75) and μ decay efficiency (0.3 per beam line). The results are shown in Table 2. We have considered the possibility of 80 GeV, 30 GeV and 8 GeV proton beams. We have also calculated the number of neutrino events per day which would be observed by a 200 ton detector 2 km from the μ-ring. We also include an estimate of the maximum value of L/E at which μ oscillations can be tested.

Discussion of the Test Cases

The above design cases are very conservative possibilities which have not been optimized for maximum neutrino production. Some possible improvements are:

1. An improved target lens with greater acceptance than the precooler lens is probably possible.
2. Higher field quads and somewhat shorter FODO periods are probably possible.
3. Protons on target can probably be increased from ~10^{18}/day to 2-4x10^{18}/day at some accelerators.
4. Using positives ($\pi^+ \rightarrow \mu^+\,\bar{\nu}_\mu \rightarrow e^+\,\nu_e\nu_\mu$) will double flux.

With these four changes a factor of 10 increase in flux may be possible.

5. A larger detector is also possible.

On the cases considered, the 4.5 GeV is preferred, provided a 30-80 GeV proton accelerator is available, and can provide ~10 times more ν's per proton pulse than the parisitic precooler use of reference 1. However the cost of the 350 meter storage ring is not trivial and the proton beam must be provided.

The ~1.5 GeV storage ring appears to have nearly as great a ν-oscillation sensitivity, and is half as long and therefore half the expense. The 8-GeV Fermilab Booster may be available as an injector. This, or a still lower energy, storage ring may be a more practical possibility. A lower energy (0.5-GeV to 1-GeV) storage ring will provide similar sensitivity with somewhat smaller cost.

Summary

We have considered the possibility of constructing a muon storage ring to be used as a source of ν_e and $\bar{\nu}_\mu$ beams for neutrino oscillation experiments. We have found it possible to measure oscillations with L/E ≅ 1-10 m/MeV.

We summarize some of the major advantages of a μ storage ring as a ν source:

1. The decaying muons can be monitored so that the neutrino intensities can be precisely known.
2. There are no impurities from π-decay, K-decay or stray charged particles.
3. The beam pulse is localized in time to a muon lifetime after the p pulse, so cosmic rays and other noise effects can be removed.

Both oscillations of the "first kind" ($\nu_e \to \nu_\mu, \nu_\tau$) and of the "second kind" ($\nu_e \to M$ where M is a "moron", a non-interacting particle) are observable.

ACKNOWLEDGEMENTS

We thank D. Cline for original and encouraging discussions. We also thank F. Mills and E. Colton for informative discussions.

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Table I

SAMPLE MUON STORAGE RINGS

Parameter	Symbol	8 GeV Ring	4.5 GeV Ring	1.5 GeV Ring
Energy	E_μ	8. GeV	4.5	1.5
FODO Half Cell Length	L_{FO}	3.8 m.	2.2	1.0
Bend Per Half Cell	θ	0.1 R	0.1	0.1
Maximum Beta-Function Value	β_{max}	13.0 m.	7.5	3.4
Magnet Aperture (~1 T field)	a_{max}	4.8 cm	4.8	4.0
Maximum Dispersion Function	η_{max}	1.03 m.	0.60	0.27
Momentum Acceptance $\frac{\Delta p}{p} \cong .75 \frac{a_{max}}{\eta_{max}}$	$\pm\frac{\Delta p}{p}$	±3.5 %	±6.0	±10.
Ring Circumference	$2(\pi R + S) \cong 5\pi R$	600 m	350.	160

Table II

ν FLUXES IN SAMPLE CASES

ν ring energy	8 GeV	4.5	4.5	1.5	1.5
Proton injector energy	80 GeV	80	30	30	8
θmax acceptance angle	25 mR	50.	50.	100.	100.
$\frac{dN_\pi}{dp}$ (pions/GeV/c -proton)	3.9×10^{-3}	2.2×10^{-2}	1.4×10^{-2}	1.7×10^{-2}	0.9×10^{-2}
$N_{\nu\mu}$ neutrinos/incident proton/beam line	2.0×10^{-5}	1.9×10^{-4}	1.2×10^{-4}	1.3×10^{-4}	0.6×10^{-4}
ν_μ and $\bar{\nu}_\mu$ per 3×10^{13} proton pulse	6×10^{8}	5.7×10^{9}	3.6×10^{9}	3.9×10^{9}	1.8×10^{9}
Events per day (10^{18} p/day, 200 ton detector, L= 2 km.)	30	54	34	1.3	0.6
L/E measurable (> 4 events/day)	1.4	3.3	2.6	1.5	1.0

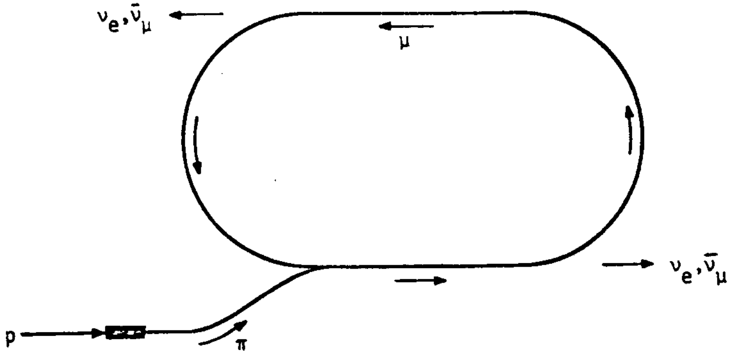


Figure 1. A μ storage ring

FIGURE 2A
"STOCHASTIC INJECTION" WITH NON-CIRCULATING π BEAM.

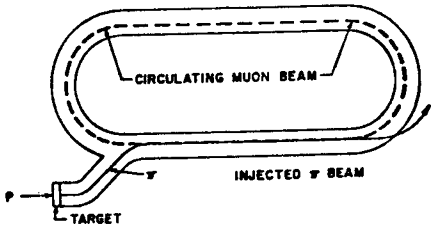
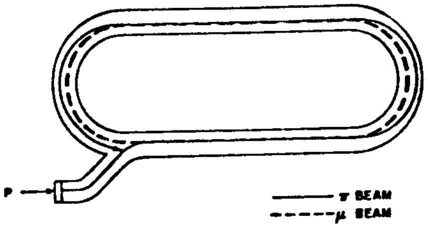


FIGURE 2B
"STOCHASTIC INJECTION" WITH SEPARATE π AND μ ORBITS.



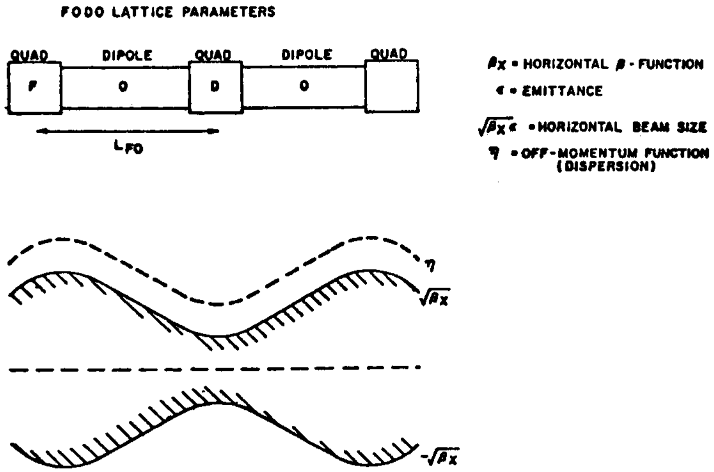


Figure 3. Ring lattice parameters