NEUTRINO OSCILLATIONS IN THE EARLY UNIVERSE

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ABSTRACT

We consider the influence of neutrino mixing and oscillations on the primordial helium abundance. We find appreciable effects only for neutrino chemical potentials of order kT.

The primordial helium abundance in the Universe is determined principally by the temperature at which the reactions involving electron neutrinos inter-converting protons and neutrons go out of equilibrium. In the presence of neutrino mixing and oscillations there is the possibility that the conversion between neutrino flavours will influence this equilibrium and change the helium abundance. We investigate this possibility in this paper.

We begin by noting that the relevant temperatures are about 1 MeV, at which time the radius of the universe is of order 10^{8}m. As long as $\Delta m_\nu^2 \geq 10^{-8}\mathrm{eV}^2$ individual oscillations will not be important, and we can concern ourselves with average values of the density of the different neutrino flavours. We will assume this to be the case.

Next we define our notation for the neutrino mass eigenstates $\nu_1, \nu_2, \ldots, \nu_i, \ldots, \nu_n$ and the neutrino weak interaction eigenstates $\nu_e\ \nu_\mu, \ldots, \nu_\xi, \ldots \nu_\mu$, with the usual unitary transformation between the two

$$\nu_\xi = \sum_i U_{\xi i}\nu_i \tag{1}$$

The neutrino density matrix ρ is assumed to be diagonal in the mass eigenstate basis

$$\rho_{ij} = \delta_{ij}\ \rho_i \tag{2}$$

with

$$\rho_i = \frac{1}{\exp\left[\frac{E_i}{T_\nu} - \xi_i\right] + 1} \tag{3}$$

in a system of units with k = 1, and where $\xi_i = \frac{\mu_i}{T_\nu}$, μ_i being the chemical potential for the neutrinos of mass m_i.

The rates of the relevant reactions, such as

$$\nu_e + n \rightarrow p + e\ , \tag{4}$$

are then determined by replacing n_{ν_e} in the usual formulae with ρ_{ee} – the leading diagonal element of the density matrix in the ξ basis

$$\rho_{ee} = \sum_{i=1}^{n} |U_{ei}|^2\ \rho_i \tag{5}$$

We can then write the rate for $n \rightarrow p$,[2]

$$\lambda(n \rightarrow p) = \sum_{i=1}^{n} \lambda_i(n \rightarrow p)\ |U_{ei}|^2 \tag{6}$$

where

$$\lambda_i(n \rightarrow p) = A \int dq \frac{\left\{1 - \frac{m_e^2}{(Q+m_e)^2}\right\}^{1/2} (Q+q)^2 q^2}{\left[1 + \exp\left\{-\left(\frac{q}{T_\nu} - \xi_i\right)\right\}\right]\left[1 + \exp\left(-\frac{Q+q}{T_\gamma}\right)\right]}$$

Clearly neutrino mixing has no effect if all of the neutrino chemical potentials are equal, since then $\lambda_i(n \rightarrow p)$ and $\lambda_i(p \rightarrow n)$ are independent of i (assuming that all of the neutrino masses are negligible compared to $T \sim 1$ MeV), and $\sum_i |U_{ei}|^2 = 1$. Physically this is to be expected since with equal neutrino densities any electron neutrino flux lost to oscillation into another neutrino is exactly compensated by that neutrino oscillating into electron neutrinos.

In particular to get significant effects we will require $\xi_i \sim 1$ for at least some of the neutrinos. This runs counter to the conventional wisdom that the net lepton number of the Universe should be of order of the net baryon number. As $\frac{n_{\nu_i}}{n_\gamma} \sim \xi_i^3$, the value $\xi_i \sim 1$ requires a net lepton number for the ith lepton of order the number of photons, whereas $n_{\nu_i} \sim n_B \sim 10^{-8} n_\gamma$ requires $\xi_i \sim 10^{-3}$. However, appreciable values of ξ_i have been investigated by Wagoner et al[3], Yahil and Beaudet[4], Linde[5] and Dimopolous and Feinberg[2]. Dimopolous and Feinberg[2] also investigate some scenarios which could lead to net lepton numbers much greater than the net baryon number.

The motivation of Linde and Dimopolous and Feinberg for considering $\xi_{\nu_i} \sim 1$ was that it provides a way of evading the constraint imposed on the number of neutrino types by the helium abundance. If the low values of the primordial helium abundance recently proposed[6] are established then such an evasion may be necessary to accommodate as few as 3 neutrinos. In equilibrium the neutron to proton ratio is

$$\frac{n}{p}_{\text{equil.}} = \frac{e^{\xi_n - m_n/T}}{e^{\xi_p - m_p/T}} = e^{-Q/T - \xi_{\nu_e}}$$

$$\text{since} \quad \xi_{\nu_e} = \xi_n = \xi_e + \xi_p \text{ , and } \xi_e \approx 0. \tag{7}$$

Thus $\frac{n}{p}_{\text{equil.}} < e^{-\xi_{\nu_e}}$

and can be made as small as we please if ξ_{ν_e} is a free parameter, independently of the number of neutrino species and the temperature at which the neutrino reactions go out of equilibrium. To obtain the neutron fraction, and hence the helium fraction, it is necessary to follow the evolution of the ratio through the non equilibrium phase using the rate equation

$$\frac{dX_n}{dT} = \frac{1}{HT}\left[\{\lambda(n \to p) + \lambda(p \to n)\}X_n - \lambda(p \to n)\right] \tag{8}$$

At $T \sim 0.1$ MeV the dentrium bottleneck breaks open and the neutrons are essentially all converted to Helium, so we can set the Helium fraction $Y \approx \frac{2X_n(0.1\text{ MeV})}{1 + X_n(0.1\text{ MeV})}$

We discuss the no mixing case in detail elsewhere.

When neutrino mixing is possible we have many more parameters at our disposal. For illustrative purposes we have taken ξ_{ν_1} and one other neutrino chemical potential, say ξ_{ν_2}, as non zero. Only ν_1 and ν_2 are assumed to mix with ν_e and we set $\alpha = |U_{e1}|^2$. We assume a total of N neutrinos which are of Majorama type, or of Dirac type with $G_R \ll G_L$.

For $\alpha = 0.75$ we show the results in figure 1 for $N_\nu = 3$ and $N_\nu = 6$. Plotted are iso-Helium-fraction lines for $Y = 0.2$ and $Y = 0.3$ on a ξ_1 ξ_2 plot. For large values of ξ_1 and ξ_2 the results are independent of N_ν at fixed (ξ_1,ξ_2) as we expect. However this increase can be compensated by increasing either ξ_1 or ξ_2 or both.

The introduction of neutrino mixing simply increases the number of parameters available to fit the data. The helium abundance is no longer a probe of the number of light neutrinos unless the mixing angles and the chemical potentials can be fixed.

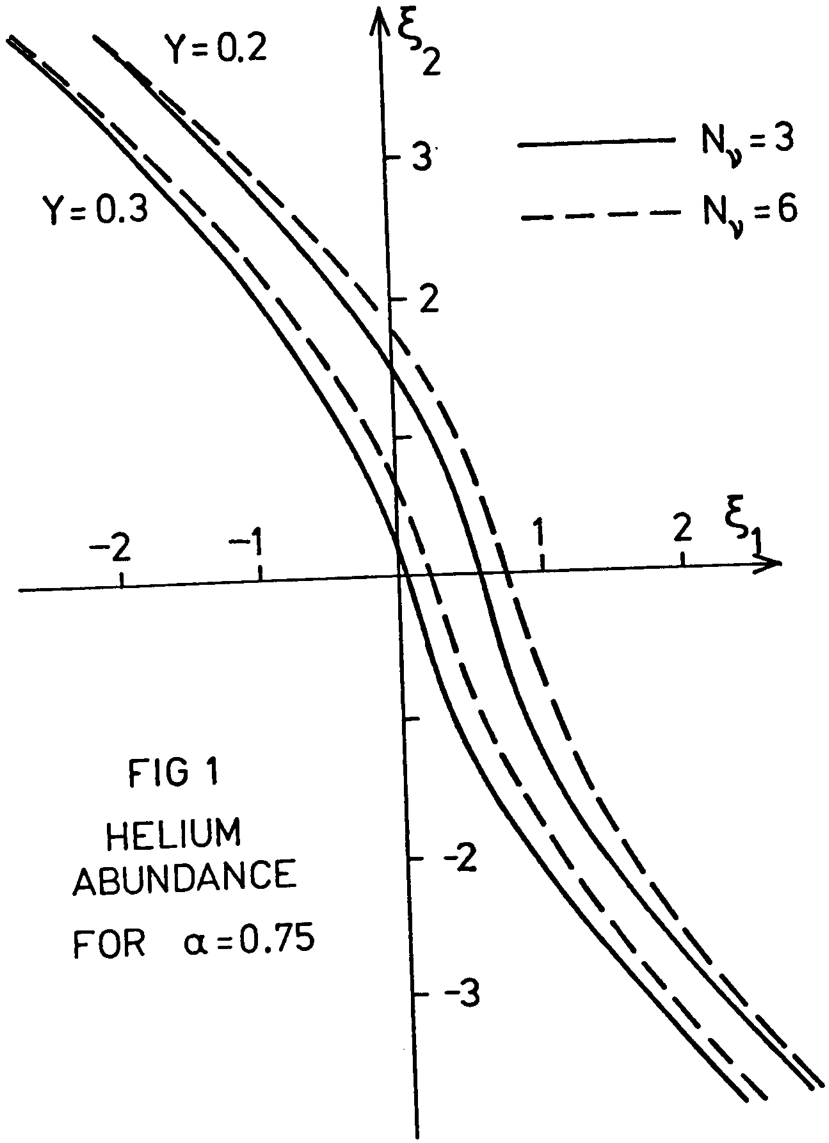

FIG 1
HELIUM
ABUNDANCE
FOR α=0.75

It is possible that the deuterim abundance may further fix the parameters, as it did in ref. 3. We plan to investigate this question further. We also intend to investigate the possibility that neutrino oscillations play a cosmological role at a time when the diameter of the universe is of the same order as the oscillation length.

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BUBBLE CHAMBER NEUTRINO OSCILLATION PROPOSALS

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I. Wisconsin-Athens-Fermilab-Padova Proposal

A. Introduction

We have proposed a search for the oscillation of $\nu_e \rightarrow \nu_\tau$ or $\nu_e \rightarrow \eta$ where η is a new species of neutrino (possibly a singlet) which does not interact. The neutrino detector will be the Fermilab 15' BC with a heavy neon-Hydrogen mix and beam will be enriched with ν_e. The oscillation probability can be written:

$$P(\nu_e \rightarrow \nu_\beta) = \delta_{e\beta} - \sin^2 2\theta \sin^2(1.27 \frac{L}{E}\Delta)$$

where Δ $(eV^2) = m^2_{\nu e} - m^2_{\nu\beta}$, L(m) = length, E(MeV) = ν_e energy, and θ = mixing angle.

estimate this experiment would be sensitive to $\sim$ 2-3% oscillation probability if $\nu_e \rightarrow \nu_\tau$ and $\sim$ 10% if $\nu_e \rightarrow \eta$. Previous results on ν_e oscillations are:

$$P(\nu_e \rightarrow \nu_e) = 1.00 \pm 0.14 \quad \text{BEBC exp}^1$$

$$P(\nu_e \rightarrow \nu_e) = 1.09 \pm 0.40 \quad \text{Los Alamos exp}^2$$

Figure 1 shows a graphical comparison of these results and the results on $\nu_\mu \rightarrow \nu_e$ experiments.

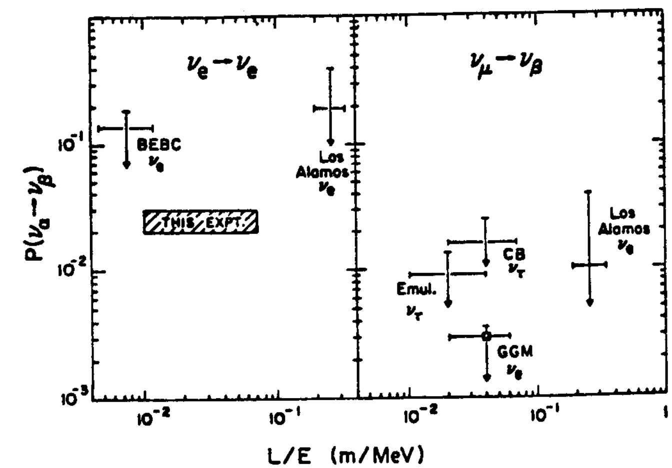

Figure 1) Oscillation probabilities for previous ν_e and ν_μ experiments as a function of L/E.