18. S. Nandi (in preparation). K. Kanaya in Ref. 14 has discussed neutrino mixing in SO(10) GUT based on Witten model. However, he does not take into account realistic quark-lepton mass relations, (3). Also, he uses two $\underset{\sim}{10}$ of Higgs representations, both of which cannot couple to $\underset{\sim}{16}_H$ in a natural way. The results on neutrino masses and mixing reported here are very different from that given in Kanaya's paper.
19. See talk by T. Kondo in this conference. For a detailed discussion of the neutrino oscillation phenomenology, see talk by V. Barger in this conference, and also the references cited there.

NEUTRINO OSCILLATIONS AND NEUTRINO-ELECTRON SCATTERING†

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ABSTRACT

We point out that neutrino flavor oscillations can significantly alter the cross section for neutrino-electron scattering. As a result, such oscillations can affect the comparison between existing reactor data and theories of neutral-current processes. They may also lead to strikingly large effects in high-energy accelerator experiments.

One expects that, in general, neutral-current processes will be completely unaffected by the oscillation of neutrinos among their various possible flavors ($\nu_e, \nu_\mu, \nu_\tau, \ldots$). After all, these processes presumably preserve neutrino flavor, and are independent of that flavor. Thus, oscillation of neutrinos from one flavor to another will not change any neutral-current cross sections. Indeed, even if the neutral weak interactions do change neutrinos ν_f of one flavor (e.g., ν_e) into those of another, if they do so through amplitudes of the form

$$a(\nu_f A \to \nu_{f'} B) = N_{f'f}\bar{a} \quad , \tag{1}$$

where $N_{f'f}$ is a unitary matrix and $\bar{a}$ is a universal amplitude, neutrino oscillations will still have no effect. (The unitarity of $N_{f'f}$ guarantees that the interactions remain independent of flavor in the sense that the total cross section for a neutrino ν_f to interact, $\sum_{f'} \sigma(\nu_f A \to \nu_{f'} B)$, does not depend on the incoming flavor.)

There is one exception to all this; namely, neutrino- (or antineutrino-) electron scattering.[A] For all neutrino flavors but ν_e, this reaction is a purely neutral-current process. However, for this one flavor, the reaction receives both neutral- and charged-current contributions, as illustrated in Fig. 1. Since the charged-

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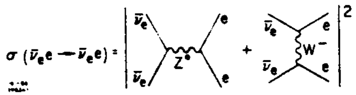

Fig. 1. Contributions to $\sigma(\bar{\nu}_e e \to \bar{\nu}_e e)$. Here Z^0 and W^- represent the neutral and charged weak bosons.

current amplitude is appreciably larger than the neutral-current one, the cross section $\sigma(\nu_e e \to \nu_e e)$ is much larger than $\sigma(\nu_\mu e \to \nu_\mu e)$, $\sigma(\nu_\tau e \to \nu_\tau e)$, etc. (Assuming universality, the latter cross sections are all equal.) Consequently, at a reactor, where the beam is initially $\bar{\nu}_e$, oscillations into other flavors will decrease the (anti) neutrino-electron event rate. At an accelerator, where the beam usually starts out as ν_μ or $\bar{\nu}_\mu$, the development of a ν_e or $\bar{\nu}_e$ component will increase the neutrino-electron event rate.

To make this more quantitative, suppose there are N physical neutrinos (mass eigenstates) ν_m with masses M_m. Suppose further that the neutrinos of definite flavor, ν_f, are not the mass eigenstates, but linear combinations of them, given by

$$\nu_f = \sum_m U_{fm}\nu_m \quad , \tag{2}$$

where U is an orthonormal mixing matrix. Then, as everyone at this Workshop knows very well, a neutrino ν_f born with one flavor will evolve after time t into a linear superposition of all the flavors,

$$\nu(t) = \sum_{f'} \alpha_{f'}\nu_{f'} \quad . \tag{3}$$

During this time the neutrino will have travelled a distance $x \simeq ct$. The probability of observing it as a neutrino of flavor f' at this distance from its source is[1]

$$\alpha^2_{f'}(p_\nu,x) = \sum_m U^2_{f'm}U^2_{fm} + \sum_{m'\neq m} U_{f'm}U_{fm}U_{f'm'}U_{fm'}\cos 2\pi\frac{x}{\ell_{mm'}} \quad , \tag{4}$$

where p_ν is the neutrino momentum, and the oscillation length $\ell_{mm'}$ is given by

$$\ell_{mm'} = \frac{4\pi p_\nu}{|M^2_m - M^2_{m'}|} \quad . \tag{5}$$

For various reasons,[2] recent discussions of neutrino oscillation have frequently centered on values of $M^2_m - M^2_{m'}$ of order $(1\text{ eV})^2$. If $M^2_m - M^2_{m'} = (1\text{ eV})^2$, then $\ell_{mm'} = 2.5$ m for $p_\nu = 1$ MeV (a typical value for reactor antineutrinos), and 2.5 km for $p_\nu = 1$ GeV (a typical value at some high-energy accelerators, such as the one at Brookhaven). Thus, if $M^2_m - M^2_{m'}$ is indeed $\sim(1\text{ eV})^2$, then in a high-energy experiment one must turn down the energy of the neutrino beam, or else locate the detector farther from the neutrino source than is customary, in order to give the neutrinos a chance to oscillate significantly between their source and the detector. Experiments involving these special steps are planned, and we shall return to them. However, reactor experiments will be sensitive to oscillations without any special steps being taken, and we shall focus for the moment on these experiments.

If a neutrino which is a coherent superposition of flavors, $\sum_{f'} \alpha_{f'}\nu_{f'}$, scatters from an electron, the cross section for producing an outgoing electron with kinetic energy T_e is given (suppressing spins and phase space factors) by

$$\frac{d\sigma}{dT_e}\left(\left[\sum_{f'} \alpha_{f'}\nu_{f'}\right]e\right) = \sum_{\substack{\text{flavors}\\ g}} |\langle\nu_g e|H|\left[\sum_{f'} \alpha_{f'}\nu_{f'}\right]e\rangle|^2 \quad . \tag{6}$$

Assuming that H, the weak interaction Hamiltonian, preserves flavor, this simplifies to

$$\frac{d\sigma}{dT_e}\left(\left[\sum_{f'} \alpha_{f'}\nu_{f'}\right]e\right) = \sum_{f'} \alpha^2_{f'} \frac{d\sigma}{dT_e}\left(\nu_{f'}e\right) \quad . \tag{7}$$

This relation applies, of course, to a neutrino (or antineutrino) of definite momentum. At a reactor emitting a spectrum $N(p_\nu)$ of antineutrinos, the event rate $R(T_1<T_e<T_2)$ for $\bar{\nu}e$ scattering with outgoing electron energy in the bin $T_1<T_e<T_2$ will be

$$R(T_1<T_e<T_2) = \int_{T_1}^{T_2} dT_e \int_{p_\nu^{min}}^{\infty} dp_\nu\, N(p_\nu) \cdot\left[\sum_{f'} \alpha^2_{f'}(p_\nu,x)\frac{d\sigma}{dT_e}(\bar{\nu}_{f'}e\ ;\ p_\nu,T_e)\right] \quad . \tag{8}$$

Here p_ν^{min} is the minimum neutrino momentum that can lead to an electron recoil energy T_e. Note that because of the x-dependence in $\alpha^2_{f'}$, eq. (4), a rate computed via eq. (8) will be specific to a particular distance between neutrino source and detector. In an effort to present results that are more general than that, we notice from eqs. (4) and (5) that if the source to detector distance x is big enough, the oscillatory term in $\alpha^2_{f'}(p_\nu,x)$ will oscillate very rapidly as the integral over p_ν in eq. (8) is performed, and thus will cancel out. To be specific, this will occur if

$$x \gg \frac{\langle p_\nu\rangle}{\Delta p_\nu}\,\ell_{mm'}(\langle p_\nu\rangle) \quad , \tag{9}$$

where $\langle p_\nu \rangle$ is the average momentum of the antineutrinos, and Δp_ν is their momentum spread. When condition (9) is met, the quantities $\alpha^2_{f'}$ may be replaced in eq. (8) by their mean values,

$$\overline{\alpha^2_{f'}} = \sum_m U^2_{f'm} U^2_{fm} \quad , \tag{10}$$

which depend neither on x nor on p_ν. Since we expect the $\bar{\nu}e$ cross sections to be equal for all flavors of antineutrino other than $\bar{\nu}_e$, it then follows from eq. (8) that

$$R(T_1<T_e<T_2) = A + B\overline{\beta^2} \quad , \tag{11}$$

where A and B are constants, and $\overline{\beta^2} \equiv 1 - \overline{\alpha^2_e}$ is the average fraction of the $\bar{\nu}$ flux that has gone into flavors other than the original $\bar{\nu}_e$.

If $M^2_m - M^2_{m'} = (1\ eV)^2$, a reactor with $\langle p_\nu \rangle$ and $\Delta p_\nu \sim 4$ MeV will have $\ell_{mm'}(\langle p_\nu \rangle) \sim 10$ m, and condition (9) will be met if $x >> 10$ m. The published reactor $\bar{\nu}e$ measurements of Reines, Gurr, and Sobel[3] correspond to x = 11.2 m, so this condition is not well-satisfied, and the results we shall present do not apply in detail to the published data. However, these results are intended mainly to illustrate how large the effects of oscillation can be. From calculations of R by Barger, Whisnant, Cline, and Phillips,[4] performed for a particular oscillation scheme but without making the approximation $\alpha^2_{f'} \approx \overline{\alpha^2_{f'}}$, we know that this approximation becomes accurate when x = (20 - 30)m. This distance is not that much larger than 10 m, so we do expect our results to give reasonably good estimates of the effects of oscillation, valid for most values of x of practical interest, and for any model of oscillation.

The Glashow-Salam-Weinberg (GSW) model with $\sin^2\theta_W = \frac{1}{4}$ predicts that

$$\frac{d\sigma}{dT_e}(\bar{\nu}_e e) = \frac{G^2 m_e}{8\pi}\left[1 + 9\left(1 - \frac{T_e}{p_\nu}\right)^2 - 3\,\frac{m_e T_e}{p_\nu^2}\right] \tag{12a}$$

and

$$\frac{d\sigma}{dT_e}(\bar{\nu}_\mu e) = \frac{G^2 m_e}{8\pi}\left[1 + \left(1 - \frac{T_e}{p_\nu}\right)^2 + \frac{m_e T_e}{p_\nu^2}\right] \quad . \tag{12b}$$

Here G is the Fermi coupling constant, m_e is the electron mass, and eq. (12b) actually applies for any flavor of antineutrino except $\bar{\nu}_e$. The last term in eqs. (12) is negligible for high-energy antineutrinos, and even for reactor antineutrinos it is only a (10 - 20)% correction. The dominant terms are the first two, and it is apparent that, as stated earlier, the $\bar{\nu}_e e$ cross section is much bigger than that for $\bar{\nu}_\mu e$.

However, let us not assume the GSW model, but try to be quite general. In the $\bar{\nu}_e e$ channel, $\bar{\nu}e$ scattering involves the two diagrams of Fig. 1. Assuming μ-e universality, we already know how big the W^- diagram is from the muon lifetime. Under the same assumption, we will soon know quite accurately how big the Z^0 diagram is from high-energy studies of $\nu_\mu e$ and $\bar{\nu}_\mu e$ scattering. What we do not know is the sign of the interference between the W and Z diagrams. This sign is the new information which can be revealed by experiments involving the $\bar{\nu}_e e$ or $\nu_e e$ channel.

Before all the recent attention to the possibility of neutrino oscillations, we and our colleagues[5] had computed the expected $\bar{\nu}_e e$ event rates for three cases, using the 1977 Avignone-Greenwood reactor $\bar{\nu}$ spectrum.[6] In all three cases, the sizes of the weak couplings were taken to be as in the GSW model with $\sin^2\theta_W = \frac{1}{4}$, but the W-Z interference was alternatively assumed to be destructive (as in the model), constructive, and absent. The results of those calculations are compared to the $\bar{\nu}_e e$ data in Table I.

TABLE I

Theoretical and experimental reactor $\bar{\nu}_e e$ event rates as fractions of R_{V-A}, the event rate for a pure charged-current interaction.

Case \ Electron Energy	Event rate/R_{V-A}: $1.5 \le T_e \le 3$ MeV	$3.0 \le T_e \le 4.5$ MeV
Destructive	0.83	1.20
Constructive	2.2	2.8
No Interference	1.5	2.0
Experimental[3]	0.87 ± 0.25	1.7 ± 0.44

Reines, Gurr, and Sobel[3] presented data for two T_e bins. From Table I we see that in the low-T_e bin the experimental result favors the destructive (GSW) case, but is only 2½ standard deviations (σ) from the no coherent interference case. In the high-T_e bin, the experimental result is closest to the no interference case, but is only a bit more than 1σ from the destructive case, and only 2½σ from the constructive one. The evidence that the interference is destructive, as predicted by GSW, is not very strong, but the data do mildly favor this case.

Now let us see what happens when we allow for the possibility of oscillations. Table II gives the approximate event rates, eq. (11), in the presence of oscillations, for the same three cases as before: destructive, constructive, and no W-Z interference. We see from this Table that the effects of oscillation can be appreciable, and that they are even bigger for constructive or no interference than for the destructive GSW case.

TABLE II

Theoretical reactor $\bar{\nu}e$ event rates as fractions of R_{V-A}, the pure charged-current event rate without neutrino oscillations. The symbol $\overline{\beta^2}$ denotes the average probability of finding a flavor other than $\bar{\nu}_e$ in the reactor beam, due to the oscillations.

Case \ Electron Energy	$1.5 \le T_e \le 3$ MeV	$3 \le T_e \le 4.5$ MeV
Destructive Interference (Weinberg-Salam, $\sin^2\theta_W = \frac{1}{4}$)	$0.83 - 0.32\ \overline{\beta^2}$	$1.21 - 0.21\ \overline{\beta^2}$
Constructive Interference	$2.20 - 1.68\ \overline{\beta^2}$	$2.76 - 1.77\ \overline{\beta^2}$
No Coherent Interference	$1.51 - 1.0\ \overline{\beta^2}$	$1.97 - 0.98\ \overline{\beta^2}$

(For the case of destructive interference, Halls and McKellar[7] have recently obtained the oscillation-modified event rates by following essentially the same procedure as we did. They confirm our results nicely when they use one or another of the published reactor antineutrino spectra in their calculations.[8] However, they have also inferred an additional spectrum by working backwards from the results quoted in Ref. 3.[8] They find that when this inferred spectrum is used, the top row of Table II becomes $0.83 - 0.06\ \overline{\beta^2}$ for the low-T_e bin, and $1.21 - 0.03\ \overline{\beta^2}$ for the high-T_e bin. These results indicate much less sensitivity to oscillations than do our own. That they are so different from our results is very surprising, since in the absence of oscillations the inferred spectrum and the 1970 Avignone spectrum, which is one of the published ones used by Halls and McKellar, lead to $\bar{\nu}_e e$ event rates which differ by only ~ 6% for a given theoretical $\bar{\nu}_e e$ cross section. Therefore, we believe it is reasonable to rely upon the results based on the published spectra, but this matter should obviously be clarified.)

In Table III, the event rates of Table II are evaluated for specific illustrative values of $\overline{\beta^2}$. Let us consider, for example, $\overline{\beta^2} = \frac{1}{2}$. From Table III, we see that for this degree of oscillation, the low-T_e data point lies closest to the predicted event rate for no coherent interference. However, to within 2σ, it is consistent with all three possible types of interference. For this same degree of oscillation, the measured rate in the high-T_e bin lies closest to the predicted rate for constructive interference. However, to within 2σ, it too is consistent with any type of interference. We see that if $\overline{\beta^2} \simeq \frac{1}{2}$, the data do not provide any evidence for the Glashow-Salam-Weinberg prediction for the interference.

An alternative approach is to assume that the GSW model is right, and then to ask whether the data tell us anything about the degree of oscillation. From Table III, they clearly do not. In neither T_e bin do they provide very much discrimination between one value of $\overline{\beta^2}$ and another.

TABLE III

Experimental and theoretical event rates as fractions of R_{V-A}. The theoretical rates are given for various values of the oscillation parameter $\overline{\beta^2}$.*

Case \ Electron Energy	$1.5 \le T_e \le 3$ MeV			$3 \le T_e \le 4.5$ MeV		
$\overline{\beta^2}$ =	1/3	1/2	2/3	1/3	1/2	2/3
Destructive Interference (Weinberg-Salam $\sin^2\theta_W = 1/4$)	0.72	0.67	0.62	1.13	1.1	1.06
Constructive Interference	1.63	1.35	1.07	2.17	1.88	1.58
No Coherent Interference	1.17	1.01	0.84	1.64	1.48	1.32
Experimental Data[3]	0.87 ± 0.25			1.7 ± 0.44		

*If the oscillation involves two families, Ref. 2 favors $0.25 \le \overline{\beta^2} \le 0.40$. A maximal oscillation involving three families would correspond to $\overline{\beta^2} = 2/3$.

Thinking of the future, we note that if $\overline{\beta^2} \simeq \frac{1}{2}$, oscillations reduce the event rate by ~ 20% in the low-T_e bin, and by ~ 10% in the high-T_e bin, assuming destructive interference. For the other types of interference, the reduction is greater. Thus, a reactor experiment with 5% accuracy would be able to measure, at least crudely, the effect of oscillations.

Let us turn now to accelerator νe experiments. If

$M_m^2 - M_{m'}^2 \sim (1\ \mathrm{eV})^2$ and $p_\nu = 150$ MeV, then the oscillation length $\ell_{mm'}$ is 375 m. Thus (remembering that $\ell_{mm'}/2$ is the distance in which the effect of oscillation becomes maximal), a detector ~ 100 m away from the neutrino source would be quite sensitive to the oscillation.[9]

At a high-energy accelerator, the beam starts out as a ν_μ (or $\bar{\nu}_\mu$) beam, and the event rate will increase if the flux develops a ν_e (or $\bar{\nu}_e$) component. For neutrinos of a given momentum, the effect of oscillations involving any number of flavors follows from eq. (7). The total cross section for an evolved ν_μ beam, $\sigma(\nu(t)e)$, relative to that for the pristine beam, $\sigma(\nu_\mu e)$, will be

$$\frac{\sigma(\nu(t)e)}{\sigma(\nu_\mu e)} = 1 + \alpha_e^2 \left[\frac{\sigma(\nu_e e) - \sigma(\nu_\mu e)}{\sigma(\nu_\mu e)}\right] . \qquad (13)$$

Here α_e^2 is the probability of observing a ν_e in the beam. Evaluating the expression in brackets, assuming weak couplings with the GSW sizes but allowing for the three possible types of W-Z interference, we find that

$$\frac{\sigma(\nu(t)e)}{\sigma(\nu_\mu e)} = 1 + \alpha_e^2 \times \begin{cases} 6, & \text{destructive interference} \\ 18, & \text{constructive interference} \\ 12, & \text{no interference.} \end{cases} \qquad (14)$$

We see that the amplification of the event rate can be very large. For GSW (destructive) interference, an oscillation coefficient $\alpha_e^2 = 1/3$, which is not particularly big, will still lead to a tripling of the event rate!

Of course, there will be no amplification at all unless the oscillation produces electronic neutrinos. Note that large $\nu_\mu \leftrightarrow \nu_e$ oscillation with $M_m^2 - M_{m'}^2 \sim (1\ \mathrm{eV})^2$ is allowed by the Gargamelle,[10] Los Alamos,[11] and Grenoble[12] $\nu_\mu \leftrightarrow \nu_e$ oscillation limits. However, such oscillation with, say, $M_m^2 - M_{m'}^2 \sim (4\ \mathrm{eV})^2$ is not allowed. If $M_m^2 - M_{m'}^2 \sim (1\ \mathrm{eV})^2$, a detector at Fermilab located ~ 1 km from the neutrino source, as is usual for that laboratory, will not see a greatly modified νe cross section unless the neutrino momentum is down around 2 GeV. This is an atypically low momentum for Fermilab. However, a relatively intense neutrino beam with momentum in this range could be generated using the protons in the Fermilab booster.[13]

In addition to oscillation among the various neutrino flavors, the possibility of mixing between "normal" neutrinos, which have weak isospin $I = ½$, and hypothetical particles with $I = 0$ has been considered.[14] Since the latter particles do not participate in the usual weak interactions, this mixing would turn "live" neutrinos in a beam into non-interacting matter. Event rates for neutrino-induced reactions would necessarily decrease. Thus, an increased event rate for νe scattering at an accelerator would be unambiguous evidence for the more commonly considered oscillation among "live" neutrino flavors.[A,15]

In summary, among neutral-current processes there is one reaction - neutrino electron scattering - whose event rate will be affected by neutrino flavor oscillations. This comes about because for one neutrino flavor - ν_e - this reaction has a charged-current piece. If $M_m^2 - M_{m'}^2 \sim (1\ \mathrm{eV})^2$, then reactor $\bar{\nu}e$ experiments will be sensitive to the oscillations without any special steps being taken. Oscillations can reduce the event rates in these experiments by (10 - 20)%, so that, if oscillations are known to be present, they must be taken into account in comparisons between the data and the Glashow-Salam-Weinberg model.

In accelerator experiments, if $M_m^2 - M_{m'}^2 \sim (1\ \mathrm{eV})^2$, oscillations will have large effects only if the neutrino beam momentum is made quite low, or the detector is placed a healthy distance from the neutrino source, or both. However, if these steps are taken, the effects of $\nu_\mu \leftrightarrow \nu_e$ oscillations can be dramatically large.

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