REFERENCES

1. J. Schechter and J.W.F. Valle, Phys. Rev. D22 (1980).
2. Cheng and Li, Carnegie-Mellon preprint (1980).
3. V. Barger, P. Langacker, J. Leveille, and S. Pakvasa, Phys. Rev. Lett. 45, 692 (1980). Also Gell-Mann, Stephenson, and SLansky (unpublished). D. Wu, Harvard preprint (1980).
4. M. Gell-Mann, P. Ramond, and R. Slansky in Supergravity, edited by P. Van Nieuwenhaizen and D. freedmore (North-Holland, Amsterdam, 1979).

NEUTRINO MASS IN THE SO(10) MODEL*

Yukio Tomozawa
Randall Laboratory of Physics, University of Michigan
Ann Arbor, MI 48109

ABSTRACT

The question of neutrino mass in the SO(10) grand unified gauge theory is considered. It is pointed out that while the radiative corrections generate the left hand Majorana mass of neutrino, it is smaller than that obtained by diagonalization of the mass matrix consisting of the Dirac mass and a large right hand Majorana mass.

In the standard electroweak theory, there is no particular reason for neutrino not to have mass. A real mystery, however, is apparent smallness of its value, if not zero. Several experiments (though all are merely circumstantial evidences at best) indicates[1] that some of the neutrinos may have masses in the range of $1 \sim 30$ eV. In recent articles,[2,3] a suggestion has been made that a Higgs mechanism in the SO(10) gauge model[4] can generate a large Majorana mass for the right handed (RH) neutrino and then a small mass for the left handed (LH) neutrino results from the diagonalization of the mass matrix.

In this talk, we discuss the problem of the LH neutrino mass by radiative corrections and see whether the above mechanism for the explanation of the observed small neutrino mass is spoiled or not.

Let us start with a general discussion of the Dirac mass and the Majorana mass. A general expression for the mass term for neutrino is given by

$$-\mathcal{L} = \frac{1}{2} M_R(\overline{\nu_R^c}\nu_R + \bar{\nu}_R\nu_R^c) + \frac{1}{2} m_L(\overline{\nu_L^c}\nu_L + \bar{\nu}_L\nu_L^c) + m(\bar{\nu}_L\nu_R + \bar{\nu}_R\nu_L) \qquad (1)$$

where M_R (m_L) and m are the Majorana masses for the RH (LH) neutrino ν_R (ν_L) and the Dirac mass respectively. The suffix c stands for the charge conjugated field and is defined by

$$\psi^c = C\bar{\psi}^T, \qquad \overline{\psi^c} = -\psi^T C^{-1} \qquad (2)$$

*A talk presented in the Neutrino Mass Miniconference and Workshop held at Telemark Lodge, Wisconsin on Oct. 2-4, 1980.

where C is the 4x4 charge conjugation matrix. Hence, it is easy to see the identity

$$\bar{\psi}_1^{\,c}\,\psi_2 = \bar{\psi}_2^{\,c}\,\psi_1 \tag{3}$$

for anti-commuting fields ψ_i.

Defining the Majorana fields

$$\begin{aligned} \xi &\equiv \nu_L + \nu_L^{\,c} \\ \eta &\equiv \nu_R + \nu_R^{\,c} \end{aligned} \tag{4}$$

and using the identities

$$\begin{aligned} \bar{\xi}\,\xi &= \bar{\nu}_L^{\,c}\nu_L + \bar{\nu}_L\nu_L^{\,c} \\ \bar{\eta}\,\eta &= \bar{\nu}_R^{\,c}\nu_R + \bar{\nu}_R\nu_R^{\,c} \\ \bar{\xi}\,\eta &= \bar{\nu}_L\nu_R + \bar{\nu}_L^{\,c}\nu_R^{\,c} = \bar{\nu}_L\nu_R + \bar{\nu}_R\nu_L \\ &= \bar{\eta}\,\xi = \bar{\nu}_R\nu_L + \bar{\nu}_R^{\,c}\nu_L^{\,c} \end{aligned} \tag{5}$$

we can reexpress Eq. (1) as

$$\begin{aligned} -\mathcal{L} &= \frac{1}{2}\, m_L \bar{\xi}\,\xi + \frac{1}{2}\, M_R\, \bar{\eta}\,\eta + \frac{m}{2}\,(\bar{\xi}\eta + \bar{\eta}\xi) \\ &= \frac{1}{2}\,(\bar{\xi},\ \bar{\eta}) \begin{pmatrix} m_L & m \\ m & M_R \end{pmatrix} \begin{pmatrix} \xi \\ \eta \end{pmatrix} \end{aligned} \tag{6}$$

The diagonalization of the mass matrix in Eq. (6) leads to

$$-\mathcal{L} = \frac{1}{2}\, m_1 \bar{\psi}_1\psi_1 + \frac{1}{2}\, m_2 \bar{\psi}_2\psi_2 \tag{7}$$

where the eigenvalues m_i and the eigenfunctions ψ_i are, for m, $m_L \ll M_R$, given by

$$m_1 = m_L - \frac{m^2}{M_R}, \qquad \psi_1 = \xi\cos\theta - \eta\sin\theta \tag{8}$$

and

$$m_2 = M_R + \frac{m^2}{M_R}, \qquad \psi_2 = \xi\sin\theta + \eta\cos\theta$$

with

$$\tan\theta = \frac{m}{M_R} \tag{9}$$

The Majorana field ψ_1 (ψ_2) is predominantly the LH (RH) neutrino and the inverse of Eq. (8) is

$$\begin{aligned} \xi &= \nu_L + \nu_L^{\,c} = \psi_1\cos\theta + \psi_2\sin\theta \\ \eta &= \nu_R + \nu_R^{\,c} = -\psi_1\sin\theta + \psi_2\cos\theta \end{aligned} \tag{10}$$

and therefore

$$\begin{aligned} \nu_L &= (\psi_1)_L\cos\theta + (\psi_2)_L\sin\theta \\ \nu_R &= -(\psi_1)_R\sin\theta + (\psi_2)_R\cos\theta \end{aligned} \tag{11}$$

With these preparation, we now consider the neutrino mass in the SO(10) model.

The fundamental representation, ψ_{16}, of the SO(10) gauge group is given by

$$\psi_{16} = (\nu, u_1, u_2, u_3, e, d_1, d_2, d_3, -d_3^c, d_2^c, d_1^c, -e^c, u_3^c, -u_2^c, -u_1^c, \nu^c)_L \tag{12}$$

The RH neutrino, ν_R, belongs to a SU(5) singlet in $\psi_{\overline{16}} \simeq \psi_{16}^c$ (denoted by $(\overline{16},1)$ or $(\overline{16},1)_R$). Then the Majorana mass for the RH neutrino, $\bar{\nu}_R^c\,\nu_R$, has the transformation property of $(\overline{16},1)\times(\overline{16},1) = (\overline{126},1)$. Therefore, we need either a 126 representation of Higgs fields or a combination of Higgs fields that trnasforms as a 126 representation, in order to have the HR Majorana mass generated by spontaneous symmetry breaking. The idea of Ref. 2 is to endow M_R with the unification mass of SO(10), M($\simeq 10^{15}$-10^{19}GeV) and for $m_L = 0$ and $m \simeq m_q$(quark mass) we have

$$m_1 = -\frac{m^2}{M} \tag{13}$$

which is quite small ($10^{-12} \sim 10^{-16}$ ev for m = 1 Mev).

Witten[3] considered the SO(10) model with the minimum Higgs system, i.e., a ϕ_{16} to break the SO(10) symmetyr to SU(5), a ϕ_{45} to further reduce the symmetry[6] to $SU_c(3)\times SU(2) \times U(1)$ and several ϕ_{10}'s to give a final residual symmetry U(1). These 10's are also necessary to give masses to leptons and quarks including the Dirac mass for neutrinos. In this model, therefore, the RH Majorana mass term $(\overline{126},1)$ must be generated by a multi-Higgs diagram. Witten showed that this can be done by the two loop diagram shown in Fig. 1. This diagram introduces the relevant vacuum expectation values that behave like $(\overline{16},1) \times (\overline{16},1) = (\overline{126},1)$

In order to understand possible vertices allowed by the gauge group, we list the multiplication table and the SU(5) contents of some of the irreducible representations of SO(10) group in Table 1 and 2. Table 2 is a direct transcription of Ref. 7. The multiplication Table 1 is obtained by comparing the SU(5) contents of both sides and/or by using the index rule explained in Ref. 7 (page 14). [In the reduction of a product of the irreducible representations ϕ and ϕ' into the sum of ϕ_i, $\phi \times \phi' = \sum_i \phi_i$, we have the relationship $N(\phi)+N(\phi') = \Sigma N(\phi_i)$ and $\ell(\phi)N(\phi') + \ell(\phi')N(\phi) = \Sigma\ell(\phi_i)$, where $N(\phi)$ and $\ell(\phi)$ are the dimensions and the index of the representation ϕ, respectively].

The estimated Majorana mass for the RH neutrino for the diagram of Fig. 1 is shown to be[3] $g^4\varepsilon\, m/M_W\, M$, or including an appropriate factor of π

$$M_R = \left(\frac{\alpha}{\pi}\right)^2 \varepsilon \frac{m}{M_W} M, \tag{14}$$

where M and M_W stand for the SO(10) unification mass (10^{15} GeV $\sim$ 10^{19} GeV) and the SU(2) x U(1) unification mass ($\sim$ 100 GeV). ε is the mixing angle of $\phi_{(10,\overline{5})}$ and $\phi_{(16,\overline{5})}$, which originates from the trilinear Higgs coupling term $\phi 10 \phi 16 \phi 16$. (The symmetry breaking by the vacuum expectation value of (16,1) causes a mixing of $(10,\overline{5})$ and $(16,\overline{5})$. For the value of $\varepsilon \simeq 1/10$ and $M \simeq 10^{15}$ GeV, Eq. (14) leads to a considerable amount of reduction of M_R and consequently an enhancement of m^2/M_R. Namely

$$M_R \simeq 10^{-11} M \text{ for } m = 1 \text{ MeV}$$

and

$$\frac{m^2}{M_R} = \frac{mM_W}{(\frac{\alpha}{\pi})^2 \varepsilon M} \doteq 10^{-7} m \tag{15}$$

We will show now that a diagram similar to that of Fig. 1 can give rise to a Majorana mass for the LH neutrino, provided that all the symmetry breakings are taken into account. Such an example is shown in Fig. 2, where the symmetry breakups are caused by two of the $(16,\overline{5})$'s obtaining vacuum expectation values. Here we have used the fact that the trilinear Higgs interaction term $\phi 10 \phi 16 \phi 16$ +h.c., which was described above, can cause a mixing among $(10,5)$ and $(16,10)$ when the $(16,\overline{5})$ has a nonzero vacuum expectation value. (As noticed earlier, $\phi_{(10,\overline{5})}$ and $\phi_{(16,\overline{5})}$ are mixed by the SO(10) symmetry breaking).

An estimate of Fig. 2 can be done in a similar manner to Witten's analysis and is given by

$$m_L = (\frac{\alpha}{\pi})^2 \frac{m}{M_W} \varepsilon' M_W \tag{16}$$

The difference between Eq. (3.4) and Eq. (3.2) is twofold. Firstly, the mass scale of the tadpole $(16,\overline{5})$ in Fig. 2 is given by M_W instead of M for $(\overline{16},1)$ in Fig. 1. Secondly, the mixing angle ε' (from the tadpole on the left hand side of Fig. 2) is dictated by the ratio of the mass sclae of the two symmetry breakings, i.e.

$$\varepsilon' = \frac{M_W}{M} \tag{17}$$

Hence, we obtain

$$m_L = (\frac{\alpha}{\pi})^2 \frac{mM_W}{M} \simeq (10^{-18} \sim 10^{-22})\, m \ll \frac{m^2}{M_R} \tag{18}$$

For $m = 1$ MeV, however, this value is comparable to that of Eq. (13), m^2/M, since $(\alpha/\pi)^2 M_W \simeq 1$ MeV.

An alternative diagram which leads to the m_L term is given in Fig. 4, where the double line represents the repeated insertions of an effective $(\overline{126},1)$ tadpole, an example of which is Witten's diagram shown in Fig. 1 (See Fig. 3). We notice that the double line in Fig. 3 and 4 amounts to a propagator

$$(-1)^3 \frac{-1}{i\gamma p} M_R \frac{-1}{i\gamma p} \sum_{n=0}^{\infty} (M_R \frac{-1}{i\gamma p} M_R \frac{-1}{i\gamma p})^n = \frac{-i\, M_R}{p^2+M_R^2} \tag{19}$$

This is precisely the propagator for the Majorana particle with mass M_R sandwiched between the two projection operators $(1+\gamma_5)/2$,

$$\overline{\nu_L^c} \frac{-1}{i\gamma p + M_R} \nu_L = \overline{\nu_L^c} \frac{-iM_R}{p^2+M_R^2} \nu_L \tag{20}$$

An estimate of the order of magnitude of the diagram in Fig. 4 is given by $g^2 \frac{m^2}{M_R}$: m^2 comes from the two tadpoles and the one loop integration is dictated by the mass M_R of the doublet lined propagator. Thus we have

$$m_L = (\frac{\alpha}{\pi}) \frac{m^2}{M_R} \tag{21}$$

In conclusion, the LH Majorana mass induced by radiative corrections is smaller than that obtained by diagonalization of the mass matrix, m^2/M_R.

ACKNOWLEDGEMENTS

The author is greatly indebted to David Unger for valuable discussions, collaboration and also for reading the manuscript. He is grateful to Paul Langacker for useful comments. The work is supported in part by the US Department of Energy.

REFERENCES

1. e.g., S.M. Bilenky and B. Pontecorvo, Physics Reports 41C, 225 (1978).
2. M. Gell-Mann, P. Ramond and R. Slansky - unpublished; P. Ramond, CALF-68-709 (1979); R. Barbieri, D.V. Nanopoulos G. Morchio and F. Strocchi, TH 2776-CERN (1979); R. Barbieri, J. Ellis and M.K. Gaillard, LAPP-TH-10 & TH 2787-CERN (1979).
3. E. Witten, HUTP-79/A076 (1979).
4. H. Georgi and D.V. Nanopoulos, Nuclear PHys. B155, 52 (1979).
5. Note that $C^{-1}\gamma_\mu C = -\gamma_\mu^T$, $C^\dagger C = 1$, $C^T = -C$.
6. H. Georgi and S.L. Glashow, Phys. Rev. Lett. 32, 438 (1974); A.J. Buras, J. Ellis, M.K. Gaillard and D.V. Nanopoulos, Nuclear Phys. B135, 66 (1978).
7. J. Patera and D. Sankoff, Tables of Branching Rules for Represnetations of Simple Lie Algebras, University of Montreal Press, Canada (1973).
8. H. Georgi and C. Jarlskog, HUTP-79/26 (1979).
9. M. Magg and Ch. Wetterich, TH 2829-CERN (1980).

FIGURE CAPTIONS

1. Two loop diagram generating the Majorana mass for the right handed neutrino (Witten's diagram)
2. Two loop diagram generating the Majorana mass for the left handed neutrino.
3. Propagator for the Majorana neutrino with mass.
4. One loop diagram generating the Majorana mass for the left handed neutrino.

Table I. SO(10) multiplication table

$$10 \times 10 = 1 + 45 + 54$$

$$10 \times 16 = \overline{16} + 144$$

$$16 \times \overline{16} = 1 + 45 + 210$$

$$16 \times 16 = 10 + 120 + 126$$

$$10 \times 45 = 10 + 120 + 320$$

$$16 \times 45 = 16 + \overline{144} + 560$$

$$45 \times 45 = 1 + 45 + 54 + 210 + 770 + 945$$

$$10 \times 120 = 45 + 210 + 945$$

Table II. SU(5) contents and index of SO(10) irreducible representation (taken from Ref. 7)

Representation ϕ	Dimension $N(\phi)$	SU(5) contents	Index $\ell(\phi)$
(00000)	1	1	0
(10000)	10	5, $\overline{5}$	2
(00001)	16	10, $\overline{5}$, 1	4
(00010)	$\overline{16}$	$\overline{10}$, 5, 1	4
(01000)	45	24,10,$\overline{10}$,1	16
(20000)	54	24, 15, $\overline{15}$	24
(00100)	120	45,$\overline{45}$,10,$\overline{10}$,5,$\overline{5}$	56
(00002)	126	50,$\overline{45}$,$\overline{15}$,10,$\overline{5}$,1	70
(00020)	$\overline{126}$	$\overline{50}$,45,15,$\overline{10}$,5,1	70
(10001)	144	$\overline{45}$,40,24,$\overline{15}$,$\overline{10}$,5,$\overline{5}$	68
(10010)	$\overline{144}$	45,$\overline{40}$,24,15,10,5,$\overline{5}$	68
(00011)	210	75,40,$\overline{40}$,24,10,$\overline{10}$,5,$\overline{5}$,1	112
(11000)	320	70,$\overline{70}$,45,$\overline{45}$,40,$\overline{40}$,5,$\overline{5}$	192
(01001)	560	175,50,$\overline{70}$,75,45,$\overline{45}$,$\overline{40}$,24, 10,10,$\overline{10}$,$\overline{5}$,1	364
(01010)	$\overline{560}$	conjugate of above	364
(02000)	770	200,175,$\overline{175}$,50,$\overline{50}$,75,24 10,$\overline{10}$,1	616
(10100)	945	126,$\overline{126}$,175,$\overline{175}$,75,45 $\overline{45}$,40,$\overline{40}$,15,$\overline{15}$,24,24,10,$\overline{10}$	672

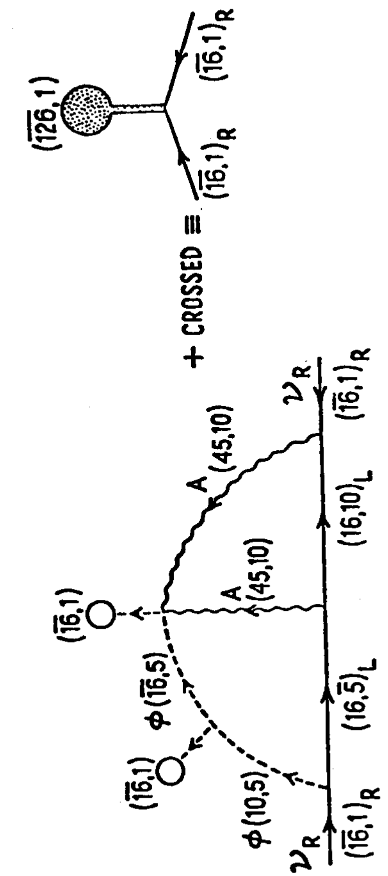

Fig. 1. Two loop diagram generating the Majorana mass for the right-handed neutrino (Witten's diagram).

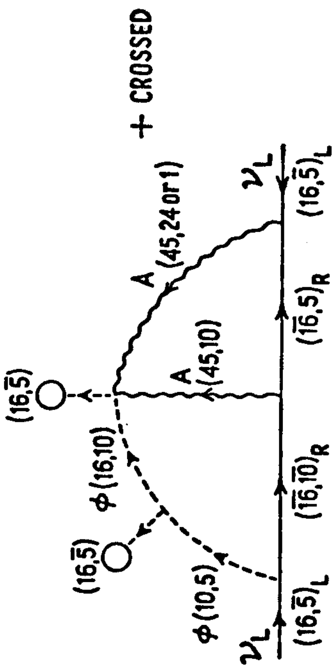

Fig. 2. Two loop diagram generating the Majorana mass for the left-handed neutrino.

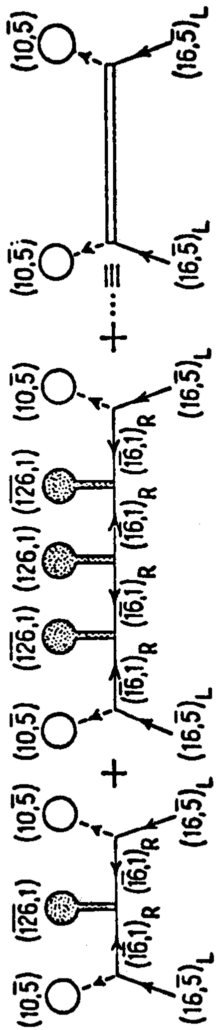

Fig. 3. Propagator for the Majorana neutrino with mass.

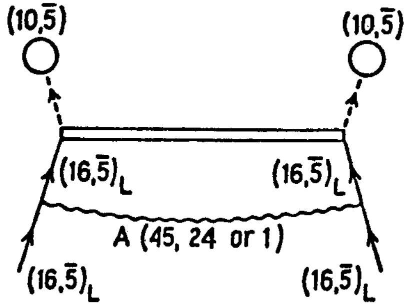

Fig. 4. One loop diagram generating the Majorana mass for the left-handed neutrino.