NEUTRINO MASSES IN SU(2)xU(1) THEORIES

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ABSTRACT

SU(2)xU(1) theories in which there are n generations and m singlet neutrino fields are investigated. Natural theories of this type contain massive neutrinos. The resultant gauge boson weak interactions are parametrized. A leptonic GIM mechanism does not generally hold and this leads to the possibility of a "heavy" neutrino decaying into three others as well as oscillations of neutral current interactions in a neutrino beam.

INTRODUCTION

This talk is based on work done in collaboration with J.W.F. Valle. A more detailed discussion of many points(and appropriate references are given in ref. 1. The question of interest here is: what parameters characterize an SU(2)xU(1) theory with massive neutrinos? In other words, what should experimentalists measure in analogy to the K-M parameters of the hadronic weak interactions? We shall require the theory to be natural in the senses that

a) There should be no arbitrary adjustment of coupling constants or masses.

b) There should be no assumptions made about any symmetries other than SU(2)xU(1) and (proper) Poincare invariance. In other words the theory itself should tell us to what degree it respects things like parity, charge conjugation, time reversal, and lepton number conservation.

In SU(2)xU(1) there are the three types of interactions involving neutrinos shown in Fig. 1

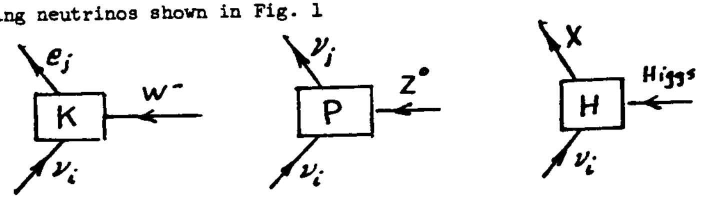

Fig. 1 Neutrino Interactions

The indices i and j differentiate fields of the same charge. We will discuss the matrices K and P here. The Higgs couplings are more model dependent; a recent discussion of them is given by Cheng and Li.[2] We can look upon the parametrization of the matrices K and P as the "kinematics" of the gauge group SU(2)xU(1). Since they give information as to the discrete symmetries they have a "geometric" aspect. Notice that if one imposes a larger symmetry than SU(2)xU(1) (GUT, for example) the parameters will generally get further restricted.

BUILDING BLOCKS OF THE MODEL

Although not the historical approach, the Dirac spinor may be conveniently viewed as the amalgamation of two 2-component relativistic (van der Waerden) spinors for the purpose of obtaining a linear transformation property under the parity operation. Since we don't want to make any assumption about C,P,T, etc. in a natural theory its reasonable to work with the 2-component spinors directly. The 2-component spinors may be considered to be the upper 2 entries, ρ of a four component spinor in a γ_5 diagonal representation:

$$\psi_L = \frac{1+\gamma_5}{2}\psi = \begin{pmatrix}\rho\\0\end{pmatrix}$$

The field ρ will be the basic building block for constructing a theory of neutrinos. The free Lagrangian using ρ and describing a particle of mass m is

$$\mathcal{L} = -i\rho^\dagger\sigma_\mu\partial_\mu\rho - \frac{m}{2}(\rho^T\sigma_2\rho + \text{h.c.}) \qquad (1)$$
$$\sigma_\mu = (\vec{\sigma},-i)$$

Note that this leads to the non-linear equation of motion

$$i\sigma_\mu\partial_\mu\rho = -m\sigma_2\rho^*.$$

Furthermore ρ cannot represent an ordinary (commuting) c-number since then $\rho^T\sigma_2\rho = \rho^T\sigma_2^T\rho = -\rho^T\sigma_2\rho = 0$. There is no problem since we consider ρ as a quantum field operator

$$\begin{pmatrix}\rho\\0\end{pmatrix} = \frac{1+\gamma_5}{2}\sum\left(\frac{m}{EpV}\right)^{1/2}[e^{ip\cdot x}u^{(r)}(\vec{p}) + e^{-ip\cdot x}v^{(r)}(\vec{p})A_r^\dagger(\vec{p})]$$
$$[A_r(\vec{p}),A_{r'}^\dagger(\vec{p}\,')]_+ = \delta_{rr'}\delta_{\vec{p}\vec{p}'} \qquad (2)$$

(Here $u^{(r)}(\vec{p})$ and $v^{(r)}(\vec{p})$ are the ordinary mass m Dirac wavefunctions). Using the canonical procedure and the field expansion (2) we find the energy operator

$$H = \sum\sqrt{\vec{p}^2+m^2}\,A_r^\dagger(\vec{p})A_r(\vec{p}),$$

so the theory has a usual particle interpretation. Note that in general $\mathcal{L}$ of eq.(1) violates lepton number. A collection of terms like (1) is more general than a collection of free Dirac Lagrangians. The Dirac Lagrangian is in fact a special case, being the sum of two with equal masses. Specifically

$$\mathcal{L}_{Dirac}=\mathcal{L}(\rho_1,m)+\mathcal{L}(\rho_2,m)$$

$$\psi_{Dirac}=\begin{pmatrix}\chi\\ \sigma_2\phi^*\end{pmatrix}$$

$$\chi=\frac{1}{\sqrt{2}}(\rho_2+i\rho_1),\ \phi=\frac{1}{\sqrt{2}}(\rho_2-i\rho_1) \tag{3}$$

A WARM UP EXERCISE

For this purpose we consider the parametrization of the hadronic charged current weak interactions given the generalized Cabibbo or Kobayashi-Maskawa matrix, C. The matrix appears in a term

$$W^+\bar{U}_L\gamma C D_L$$

where U_L and D_L are columns of up and down quarks, respectively. C can be taken to satisfy

$$C^+=C^{-1},\ \det C=1.$$

C of course arises in the first place because of the need to bring the mass term of the Lagrangian to diagonal form. It can be parametrized (using all the generators of the group SU(n)) as a product of a diagonal matrix of phases, $\omega_0(\gamma)$ and matrices describing "complex rotations" in each plane, $\omega(\eta_{ab})$:

$$C=\omega_0(\gamma)\ \prod_{a<b}\omega(\eta_{ab}) \tag{4}$$

Here

$$\omega_0(\gamma)=\mathrm{diag}(e^{i\gamma_1},e^{i\gamma_2},\ldots,e^{i\gamma_n})$$

$$\gamma_1+\gamma_2+\ldots+\gamma_n=0\quad, \tag{5}$$

and setting $\eta_{ab}\equiv|\eta_{ab}|e^{i\theta_{ab}}$ we have for the (12) rotation for example

$$\omega(\eta_{12})=\begin{pmatrix}\cos|\eta_{12}| & e^{i\theta_{12}}\sin|\eta_{12}| & 0 & \cdots\\ -e^{-i\theta_{12}}\sin|\eta_{12}| & \cos|\eta_{12}| & 0 & \cdots\\ 0 & 0 & 1 & \cdots\\ \vdots & \vdots & \vdots & \end{pmatrix}$$

A very useful identity is

$$\omega_0(\alpha)\omega(|\eta_{ab}|e^{i\theta_{ab}})\omega_0^{\dagger}(\alpha)=\omega(|\eta_{ab}|e^{i(\theta_{ab}+\alpha_a-\alpha_b)}). \tag{6}$$

Because D and U are Dirac fields equivalent new ones have different phases but leave $\mathcal{L}_{Dirac}$ invariant. Redefining

$$D=\omega_0^{\dagger}(\alpha)D'$$

$$U=\omega_0(\gamma-\alpha)U'\qquad \sum_{a=1}^{n}\alpha_a=0 \tag{7}$$

changes the interaction to $W^+\bar{U}_L'\gamma C_{eff}D_L'$ where, using (6),

$$C_{eff}=\prod_{a<b}\omega[|\eta_{ab}|e^{i(\alpha_a+\theta_{ab}-\alpha_b)}]. \tag{8}$$

The (n-1) independent phases α_a are at our disposal so we may use them to eliminate any (n-1) of the θ_{ab}'s. Thus the resulting matrix in (8) has n(n-1)/2 real angles $|\eta_{ab}|$ and n(n-1)/2-(n-1) CP violating phases. Note that the above procedure gives both a counting and an explicit parametrization.

THE (n,m) MODELS

For the lepton sector of the theory many models of interest are of the class where there are n neutrino fields belonging to SU(2) doublets and m neutrino singlets. In a natural theory the free neutrino part of $\mathcal{L}$ will look like the sum of (n+m) Lagrangians (1) with the additional possibility of non-diagonal mass terms:

$$\mathcal{L}_{Free}=-\sum_{\alpha=1}^{m+n}[i\rho_\alpha^{\dagger}\sigma_\mu\partial_\mu\rho_\alpha+\tfrac{1}{2}(\rho_\alpha^T\sigma_2M_{\alpha\beta}\rho_\beta+\text{h.c.})]. \tag{9}$$

Here the mass matrix M is decomposed as

$$M=M^T=\left(\begin{array}{c|c}M_1 & D\\ \hline D^T & M_2\end{array}\right)\quad. \tag{10}$$

$$\leftarrow n\rightarrow\leftarrow m\rightarrow$$

A natural possibility is to have $M_1=0$; we call this a theory of type II. We would like to introduce physical neutrino fields, ν to bring (9) to the form

$$\mathcal{L}_{Free}=-\sum_{\alpha=1}^{n+m}[i\nu_\alpha^{\dagger}\sigma_\mu\partial_\mu\nu_\alpha+\tfrac{1}{2}(\nu_\alpha^T\sigma_2\nu_\alpha\chi_\alpha+\text{h.c.})],$$

$$\chi_\alpha=\text{real masses}. \tag{11}$$

Since the first term in (9) is of the same form as the first term in (11) we must have

$$\rho=U\nu,\ U^{\dagger}=U^{-1}. \tag{12}$$

For the second term we require

$$U^T M U = x = \text{real, diagonal.} \tag{13}$$

Because U^T rather than $U^\dagger$ appears in (13) this is not the usual diagonalization problem. But (see ref. 1 for details) we can always find a U satisfying both (12) and (13). Note from (11) that phase changes on the ν_α

$$\nu_\alpha \to e^{i\theta_\alpha}\nu_\alpha$$

will not leave $\mathcal{L}_{Free}$ invariant. Thus there is less phase freedom than for Dirac fields.

The interaction terms expressed in terms of the "bare" fields look like

$$W^- \sum_{\alpha=1}^{n} \bar{E}_L \gamma \rho_\alpha \qquad \text{and } Z^0 \sum_{\alpha=1}^{n} \bar{\rho}_\alpha \gamma \rho_\alpha \tag{14}$$

Here E is the column of bare electron type fields and for simplicity we have written ρ_α for $\begin{pmatrix}\rho_\alpha \\ 0\end{pmatrix}$. When the transformations (12) to the physical ν's and $E_L=\Omega_L e$ to the physical e's are made (14) takes the form

$$W^- \bar{e}_L \gamma K \nu \qquad \text{and } Z^0 \bar{\nu}\gamma P \nu. \tag{15}$$

K and P are the matrices of interest shown in Fig 1. K is of the following rectangular form

$$K_{b\alpha} = \sum_{c=1}^{n} (\Omega^\dagger)_{bc} U_{c\alpha} = \underbrace{\boxed{\qquad\qquad}}_{n+m}\; n \tag{16}$$

It satisfies

$$K^\dagger K = 1 = \boxed{\;1\;}\; n \tag{17}$$

but

$$K K^\dagger = \boxed{\qquad}_{\,n+m \times n+m} \neq 1 \tag{18}$$

The matrix P is an (n+m)x(n+m) square matrix given by

$$P_{\alpha\beta} = \sum_{a=1}^{n} U^\dagger_{\alpha a} U_{a\beta} = (KK^\dagger)_{\alpha\beta}, \tag{19}$$

where (16) was used in the last step. We make the following remarks

i) Eq.(19) shows that if we know K, we know P so it is only necessary to parametrize K.

ii) There is no GIM mechanism (this mechanism is the statement P=1) for leptons, in general, unless there are no SU(2) singlet neutrino fields present (m=0).

iii) $P=P^\dagger$

iv) $P^2=P$ so it is a projection matrix.

PARAMETRIZING K

The rows of K are a set of n, m+n dimensional orthonormal complex vectors. The number of real parameters needed is thus equal to:

normalizations orthogonalizations electron phases

$$2n(m+n) - n - n(n+1) - n^2 = n(n+2m-1) \tag{20}$$

K is a truncated unitary matrix but first parametrizing the unitary matrix and then truncating will in general yield too many parameters. For example, consider the rectangular matrix which comprises the first row of a 4x4 real orthogonal matrix. Parametrizing the 4x4 matrix will generally give a first row which depends on six angles. Clearly only four (including the resolution of a sign ambiguity) are needed. We proceed as follows, using the basic "rotations" $\omega(\eta_{ab})$ introduced earlier. Define the unit vectors

$$\vec{e}^{(1)}, \vec{e}^{(2)}, \ldots, \vec{e}^{(n)}$$

by $e^{(\alpha)}_\beta = \delta_{\alpha\beta}$. Take the first row of K to be the transpose of

$$\chi^{(1)} = \prod_{\alpha=2}^{n+m} \omega(\eta_{1\alpha}) e^{(1)}, \tag{21a}$$

the second row of K to be the transpose of

$$\chi^{(2)} = \sum_{\alpha=2}^{n+m} \omega(\eta_{1\alpha}) \prod_{\beta=3}^{n+m} \omega(\eta_{2\beta}) e^{(2)}, \tag{21b}$$

and so on. The counting of parameters is easily verified to agree with (20) and the orthogonality of different rows can be seen, for example, by

$$\chi^{(1)\dagger}\chi^{(2)} = \underbrace{e^{(1)\dagger} \prod_{\beta=3} \omega(\eta_{2\beta})}_{e^{(1)\dagger}} e^{(2)} = 0.$$

We make the following remarks:

i) Eqs.(21) are obtained by multiplying matrices with non-trivial 2x2 sub blocks together. Thus they may be fairly convenient in practice.

(ii) The number of angles $|\eta_{ab}|$ in the parametrization equals the number of phases θ_{ab}.

(iii) In type II models, where M_1 of (10) vanishes, there will be at most $(n-m)^2$ fewer real parameters. This number is the number of generators of $U(n-m)$.

Some examples:

Theory	angles	phases	total
(3,0)II	0	0	0
(2,0)I	1	1	2
(3,0)I	3	3	6
(4,0)I	6	6	12
(3,3)I or II	12	12	24
(3,1)I	6	6	12
(3,1)II	5	3	8

The usual 3 generation model with massless neutrinos is (3,0)II in the present notation. Note that owing to the smaller phase freedom for the spinors ν_a in (11), the (3,0)I case, for example, requires more parameters than does the KM matrix for three quark generations.

CONSEQUENCES OF P≠1

1. The decay of a neutrino into three lighter ones is now permitted if the Q-value is right. As a crude estimate, the parent neutrino is required to be heavier than about 2 MeV if its lifetime is to be less than 1000 sec.

2. Neutral current oscillations in neutrino beams are now possible. This is also discussed by Barger, Langacker, Leveille, and Pakvasa.[3]

A schematic diagram of an experiment to detect oscillations in neutrino reactions mediated by charged W exchange is given in Fig. 2

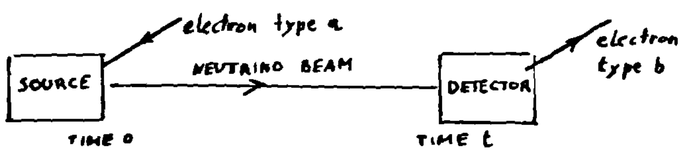

Fig.2 Charged Current Oscillation experiment

Let us denote the probability factor for observing an electron of type b in the above experiment by $I_{cc}(a\to b,t)$. If K is a square matrix (m=0) one will have

$$\sum_b I_{cc}(a\to b,t)=1.$$

However in general we now have

$$\sum_b I_{cc}(a\to b,t)\leq 1. \tag{22}$$

Next consider an experiment designed to detect oscillations in neutrino reactions involving Z^o mesons exchanged from hadrons, shown in Fig. 3

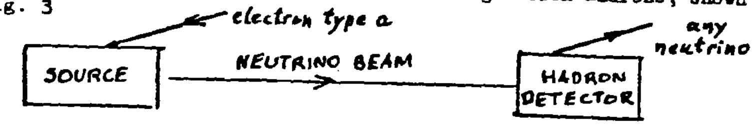

Fig. 3 Neutral Current Oscillation Experiment

This will be described by a probability factor $I_{NC}(a\to\nu's,t)$. Using the properties of $P=KK^\dagger$ we find[1] for real K:

$$I_{NC}(a\to\nu's,t)=\sum_b I_{cc}(a\to b,t) \tag{23}$$

In the usual case the right hand side of (23) is 1 and thus contains no time dependence. However this is no longer true in general. For example take a (1,1) theory with real K given by

$$K=(\cos\theta\ \sin\theta).$$

Then (23) yields

$$I_{NC}(1\to\nu's,t)=I_{CC}(1\to 1,t)=1-\sin^2 2\theta\sin^2\left[\frac{(E_1-E_2)t}{2}\right]. \tag{24}$$

This shows the neutral current oscillation phenomenon. I_{NC}/I_{CC} is constant in time because there is only one term on the r.h.s. of (23). This feature does not hold in general.

Finally, if it is assumed that the submatrix M_2 in (10) is large compared to the other entries (Gell-Mann, Ramond, Slansky mechanism)[4] then P will have the approximate form

$$P\approx\left(\begin{array}{c|c}1 & \text{small}\\ \hline \text{small}^\dagger & (\text{small})^2\end{array}\right),$$

where small means order of (M_2^{-1}). Low mass neutral current neutrino oscillations will not be important in such a case.

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NEUTRINO MASS IN THE SO(10) MODEL*

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ABSTRACT

The question of neutrino mass in the SO(10) grand unified gauge theory is considered. It is pointed out that while the radiative corrections generate the left hand Majorana mass of neutrino, it is smaller than that obtained by diagonalization of the mass matrix consisting of the Dirac mass and a large right hand Majorana mass.

In the standard electroweak theory, there is no particular reason for neutrino not to have mass. A real mystery, however, is apparent smallness of its value, if not zero. Several experiments (though all are merely circumstantial evidences at best) indicates[1] that some of the neutrinos may have masses in the range of $1 \sim 30$ eV. In recent articles,[2,3] a suggestion has been made that a Higgs mechanism in the SO(10) gauge model[4] can generate a large Majorana mass for the right handed (RH) neutrino and then a small mass for the left handed (LH) neutrino results from the diagonalization of the mass matrix.

In this talk, we discuss the problem of the LH neutrino mass by radiative corrections and see whether the above mechanism for the explanation of the observed small neutrino mass is spoiled or not.

Let us start with a general discussion of the Dirac mass and the Majorana mass. A general expression for the mass term for neutrino is given by

$$-\mathcal{L} = \frac{1}{2} M_R(\overline{\nu_R^c}\nu_R + \bar{\nu}_R\nu_R^c) + \frac{1}{2} m_L(\overline{\nu_L^c}\nu_L + \bar{\nu}_L\nu_L^c) + m(\bar{\nu}_L\nu_R + \bar{\nu}_R\nu_L) \tag{1}$$

where M_R (m_L) and m are the Majorana masses for the RH (LH) neutrino ν_R (ν_L) and the Dirac mass respectively. The suffix c stands for the charge conjugated field and is defined by

$$\psi^c = C\bar{\psi}^T, \qquad \bar{\psi}^c = -\psi^T C^{-1} \tag{2}$$

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