Nelly Halzen are such great cooks it is hard to thank them enough. Marty Olsson, Fred Halzen and the lad will be remembered for their continuing friendship and support, and for teaching me the differences between cows and horses.

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7. See e.g. the talk of V. Barger in these proceedings.
8. This section is essentially cripted from Ref. 1, which contains appropriate references.
9. "L'ours et les deux compagnons", La Fontaine.

APPENDIX

For convenience I will summarize the basic properties of Majorana fermions in this appendix. Rather than study Majorana particles in the Majorana basis, I will use the conventions of Bjorken and Drell (Relativistic Quantum Mechanics, Mc-Graw Hill, 1964, Appendix A). Since we are dealing with self-conjugate fermions in gauge theories, this latter choice is actually better-suited for calculations.

A Majorana fermion ψ is a self conjugate fermion, i.e.

$$\psi^c \equiv C\gamma^0\psi^* = \psi \tag{A1}$$

The charge conjugation matrix $C = i\gamma^2\gamma^0$ satisfies $C = -C^{-1} = -C^\dagger = -\tilde{C}$, $C\,\gamma^\mu\,C^{-1} = -\widetilde{\gamma^\mu}$. Charge conjugation is an involution, i.e.

$$(\psi^c)^c = C\gamma^0(\psi^c)^* = C\gamma^0 C^*\gamma^0\psi = \psi \tag{A2}$$

Note that the definition (A1) is not the most general. We could have introduced an arbitrary phase in the definition, i.e.: $\psi^c = e^{i\eta}\,\psi$. The phase choice $\eta=0$ is usually made for convenience. See Refs. 1,2 for details.

Assume now that Ψ is an arbitrary four-component spinor. Then Ψ^c is also a four-component spinor. Defining

$$\psi_R = \frac{\Psi + \Psi^c}{\sqrt{2}} \qquad \psi_I = -i\,\frac{\Psi - \Psi^c}{\sqrt{2}} \tag{A3}$$

one finds (use A2)

$$\Psi = \frac{\psi_R + i\,\psi_I}{\sqrt{2}}, \quad \psi_R = \psi_R{}^c, \quad \psi_I = \psi_I{}^c \tag{A4}$$

If Ψ describes a particle of mass m, then

$$\begin{aligned} \overline{\Psi} i\partial\!\!\!/ \Psi &= \frac{1}{2}\,\overline{\psi}_R i\partial\!\!\!/\,\psi_R + \frac{1}{2}\,\overline{\psi}_I\,i\partial\!\!\!/\,\psi_I \\ m\overline{\Psi}\Psi &= \frac{1}{2}m\,\overline{\psi}_R\psi_R + \frac{1}{2}\,m\,\overline{\psi}_I\psi_I \end{aligned} \tag{A5}$$

where we have used Eq. A6 below. Hence a four-component Dirac field is equivalent to a pair of degenerate Majorana fields. The Majorana fields ψ_R and ψ_I are two-component objects, as is obvious from (A1). It is clear from (A3, A4) that the decomposition into Majorana fermions is like going from complex to real objects, with complex conjugation replaced by charge conjugation. The factors of 1/2 in A5 are necessary to have the "real" fields normalized correctly.

The following identities hold for any two Majorana anti-commuting fermions ψ and χ, i.e. $\psi^c = \psi$ and $\chi^c = \chi$.

(i) $\bar{\psi}\chi = \bar{\chi}\psi$

(ii) $\bar{\psi}\gamma^{\mu}\chi = -\bar{\chi}\gamma^{\mu}\psi$

(iii) $\bar{\psi}\gamma^{\mu}\gamma^{5}\chi = \bar{\chi}\gamma^{\mu}\gamma^{5}\psi$

(iv) $\bar{\psi}\not{\partial}\chi = \bar{\chi}\not{\partial}\psi$ (A6)

(v) $\bar{\psi}\gamma^{5}\not{\partial}\chi = -\bar{\chi}\gamma^{5}\not{\partial}\psi$

(vi) $\bar{\psi}\sigma^{\mu\nu}\chi = -\bar{\chi}\sigma^{\mu\nu}\psi$

In deriving (A6, iv-v) integration by parts was used and the surface terms were neglected. Furthermore, it must be remembered that ψ and χ are anticommuting c-numbers, i.e. $\widetilde{(\bar{\chi}\psi)} = -\bar{\psi}\tilde{\chi}$. Eq. (A6) summarizes all properties of Majorana fermions. In particular A6(ii) guarantees that they cannot couple to the electromagnetic field, A6(vi) that a Majorana fermion has no magnetic moment. When writing down Feynman rules one must remember that Majorana fermions are real. The symmetry numbers are the same as for a real spinless boson.

THE NEUTRINO MASS IN UNIFIED THEORIES

Dan-di Wu*

Lyman Laboratory of Physics, Harvard University, Cambridge, MA 02138

ABSTRACT

Neutrino mass patterns for one family with both left- and right-handed neutrinos are discussed.

Talking about neutrino masses is very much like guessing the solution to a puzzle without enough information. Different people may guess different things. What I can do is to say something based on my tastes.

If there are both neutrinos in the doublet and singlet of the Weinberg-Salam Model,[1] the most general mass Lagrangian[2,3,4] for one family of neutrinos is:

$$\mathcal{L} = -\tfrac{1}{2}[a\ \bar{\nu}_R\nu_L + a\overline{(\nu^c)}_R(\nu^c)_L + b\overline{(\nu^c)}_R\nu_L + c\ \bar{\nu}_R(\nu^c)_L] + \text{h.c.} \quad (1)$$

where ν_L is the doublet neutrino which appears in the weak charged current

$$\bar{e}_L\gamma_\mu\nu_L \quad (2)$$

and ν_R is the singlet partner of ν_L with the same lepton number; ν^c is

$$\nu^c = \mathcal{C}\ \nu\ \mathcal{C}^{-1} = C\ \bar{\nu}^T,\ C = i\gamma_2\gamma_0,\ C^T = -C \quad (3)$$

the chirality eigenstates are defined as

$$\nu_{\substack{L\\R}} = \frac{1\mp\gamma_5}{2}\ \nu_{\substack{L,\\R}}\ \bar{\nu}_{\substack{L\\R}} = (\nu_{\substack{L\\R}})^+\ \gamma_0$$

$$\frac{1\pm\gamma_5}{2}\nu^{L}_{R} = 0\ . \quad (4)$$

We have

$$\nu^c_{\substack{L\\R}} \equiv (\nu_{\substack{L\\R}})^c = (\nu^c)_{\substack{L\\R}}\ . \quad (5)$$

We notice that because of fermi statistics, we have

$$\overline{(\nu^c)}_R\nu_L = \overline{(\nu_L)}^c\nu_R = -\nu_L^T\ C\ \nu_L \neq 0\ . \quad (6)$$

*Research is supported in part by the National Science Foundation under Grant No. PHY77-22864.

ν_L and ν_R are also called the interaction eigenstates. Upon diagonalizing Eq. (1), we get the mass eigenstates. It has been proved[2] that the mass eigenstates are always Majorana spinors χ_+ and χ_- which are charge conjugation eigenstates

$$C\, \chi_\pm C^{-1} = \pm\chi_\pm \tag{7}$$

and

$$\chi_+ = \cos\theta(\nu_L+\nu_L^c) + \sin\theta(\nu_R+\nu_R^c)$$

$$\chi_- = \sin\theta(\nu_L-\nu_L^c) + \cos\theta(\nu_R-\nu_R^c)\ . \tag{8}$$

By a suitable choice of the phases of ν_L and ν_R and the angle θ, Eq. (1) becomes

$$\mathcal{L} = -\tfrac{1}{2}(m_+\, \bar{\chi}_+\chi_+ + m_-\bar{\chi}_-\chi_-)\ . \tag{9}$$

The Dirac spinors are the eigenstates of parity

$$\mathbb{P}\, \psi\mathbb{P}^{-1} = \psi$$

$$\mathbb{P}\, \psi^c\mathbb{P}^{-1} = -\psi^c \tag{10}$$

where

$$\psi^c = \mathbb{C}\, \psi\mathbb{C}^{-1}\ . \tag{11}$$

Equation (10) is an image of Eq. (7).

The angle θ in Eq. (8) can be measured in "neutrino-antineutrino oscillation" experiments[4] in which we observe the number of neutral current events which varies like

$$\sin^2 2\theta \cos\left(\frac{m_+^2-m_-^2}{2E}\, L\right)\ . \tag{12}$$

The transformation from ν to χ of Eq. (8) is a Pauli transformation[5] which does not change the kinematic part of the neutrino Lagrangian. Thus the effective Lagrangian of the free neutrinos expressed in terms of Majorana spinors is

$$\tfrac{1}{2}\bar{\chi}_+(\not{\partial}-m_+)\chi_+ + \tfrac{1}{2}\bar{\chi}_-(\not{\partial}-m_-)\chi_- \tag{13}$$

and we have the second kind of propagator for the Majorana spinors

$$\overline{\chi_{\pm L}\, \chi_{\pm L}^T} = \frac{1-\gamma_5}{2}\,(\pm)\, \overline{\chi_\pm\bar{\chi}_\pm}\, C^T\, \frac{1-\gamma_5}{2} = \frac{\pm\, im_\pm}{p^2+m_\pm^2}\, C\, \frac{1-\gamma_5}{2} \tag{14}$$

and using the inverse of Eq. (8) we have

$$\overline{\nu_L\nu_L^T} = \cos^2\theta\, \frac{im_+}{p^2+m_+^2}\, C\, \frac{1-\gamma_5}{2} - \sin^2\theta\, \frac{im_-}{p^2+m_-^2}\, C\, \frac{1-\gamma_5}{2}\ . \tag{15}$$

Equation (15) should be the propagator appearing in double β decay[6] if the charged current is exactly Eq. (2).

Now a question is, whether ν_L and ν_R are Dirac spinors. The only information we have gotten from experiments so far is that the charged current may have the form Eq. (2). Suppose the charged current interaction eigenstates are exactly left-handed spinors, we may still have the most general left-handed neutrinos

$$\nu_L = \cos\theta'\, \psi_L + \sin\theta'\, \psi_R^c \tag{16}$$

where ψ is the Dirac spinor which satisfies Eqs. (10) and (11). It is very unlikely that this angle θ' (in general, a unitary transformation from ν to ψ) can be specified experimentally. If we choose $\theta' = 0$, we have

$$P\, \chi_+\, P^{-1} = \chi\ . \tag{17}$$

Equation (16) gives an ambiguity. There is no reason to say that $\theta' = 0$. However, this ambiguity may be resolved if the neutrinos have substructure[7] like the neutron. In this case, I would favor the neutrino mass pattern in Fig. 1a with double fine structures. It is also very likely that $\theta' = 0$ in Eq. (16), especially if the subspinor particles are all charged.

In some unification models, e.g. in the original Weinberg-Salam model[1] and the SU(5) model[8] of Georgi and Glashow, there is no position for the right-handed neutrinos. Then the eigenstate of mass must be χ_+ in Eq. (8) with $\theta = 0$.

Let us stick to the complicated case with both left- and right-handed neutrinos. We put ν_R and e_R in a doublet of $SU(2)_R$, formally

$$\begin{pmatrix} \nu_R \\ e_R \end{pmatrix}\ . \tag{18}$$

The properties of the three terms in Eq. (1) under the group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ are listed below. (Here B is the baryon number.) For convenience in going to the quark sector, we keep track of B though we will not discuss quarks here.

	$SU(2)_L \times SU(2)_R$	B-L	(T_{3L}, T_{3R})	
$\bar{\nu}_R\nu_L$	(2,2)	0	$(\tfrac{1}{2},-\tfrac{1}{2})$	
$\overline{\nu_L^c}\nu_L$	(3,1)	-2	(1,0)	
$\bar{\nu}_R\nu_R^c$	(1,3)	2	(0,1)	(19)

$$Q = T_{3L} + T_{3R} + \frac{B-L}{2} . \tag{20}$$

$$T_{3R} + \frac{B-L}{2} = y . \tag{21}$$

If our Lagrangian has originally $SU(2)_L \times SU(2)_R \times U(1)_B$ symmetry (B is for B-L), then to get corresponding mass terms, we have to introduce corresponding symmetry breaking in the Higgs sector directly or indirectly. The effective low dimension Higgs[9] which may develop useful vacuum expectation values (VEV) and have integer electric charge are in the following list. The quantum number (T_{3L}, T_{3R}) is for the electrically neutral components.

		$SU(2)_L \times SU(2)_R \times U(1)_B$	(T_{3L}, T_{3R})	Survival Symmetry
(I)	(1)	(2,2,0)	$(\frac{1}{2},-\frac{1}{2})$	$(SU(2)_L+SU(2)_R)\times B$
	(2)	(1,2,1)	$(0,-\frac{1}{2})$	$SU(2)_L \times y$
	(3)	(2,1,1)	$(-\frac{1}{2},0)$	$(T_{3L}+\frac{B}{2}) \times SU(2)_R$
	(4)	(1,3,2)	(0,1)	$SU(2)_L \times y$
	(5)	(3,1,2)	(1,0)	$(T_{3L}+\frac{B}{2}) \times SU(2)_R$
(II)	(6)	(1,3,0)	(0,0)	$SU(2)_L \times T_{3R} \times B$
	(7)	(3,1,0)	(0,0)	$SU(2)_R \times T_{3L} \times B$

(22)

Before going ahead, let us discuss the idea of naturalness. This idea has been discussed in detail by Georgi and Pais.[10] Recently, 't Hooft[11] gave a general and useful version of naturalness:

"A physical parameter $\alpha(\mu)$ is allowed to be very small only if the replacement $\alpha(\mu)$ would increase the symmetry of the system."

When $\alpha(\mu) = 0$ implies a symmetry, its radiative corrections must be under control and small, thus it is consistent to put it small at the beginning. This idea can also be used for assigning vacuum expectation values to the Higgs components: a component which breaks the symmetry more seriously should get a smaller VEV. 't Hooft uses this idea to explain why electron mass is small: when electron mass (or any fermion mass) is zero, we get a $U(1)_A$ symmetry (or chiral symmetry). We can explain why the masses of ν_e and ν_μ must be small at the same time in the W-S model[1] too. Suppose ν_e is very small, then ν_μ must be small because when $m_{\nu_e} = m_{\nu_\mu} = 0$, we get electron lepton number and muon lepton number conservations separately. This cannot happen if $m_{\nu_e} = m_\mu = 0$ nor $m_{\nu_\mu} = m_e = 0$. Thus we get

$$m_{\nu_e}, m_{\nu_\mu} \ll m_e, m_\mu . \tag{23}$$

One may argue that if $m_e = m_\mu = 0$, we can get the separate lepton number conservations too, why we do not have, instead of Eq. (23), $m_{\nu_e}, m_{\nu_\mu} \gg m_e, m_\mu$? This is a problem 't Hooft idea cannot solve. There must be some arguments to prove that charged particles cannot be massless. This statement is true for spin zero[12] and spin larger than a half[13] particles. But there is no proof for $s = \frac{1}{2}$ particles.

From the mass of the leptons, e, μ, and τ we find

$$m_e \ll m_\mu \ll m_\tau . \tag{24}$$

This means there may exist several scales in the lepton-quark level. Thus the so-called "one scale grand unification models"[14] may not be the best description of nature, though these models are simple and beautiful. Models with several scales which have less desert than models with one scale should also be in consideration.

Now let us return to the list (22). The Higgs in Category I may break the symmetry more seriously than that in Category II. Also, we notice that we can only break the linear combinations of the three Cartan operators T_{3L}, T_{3R} and B-L which are perpendicular to the operator Eq. (20). Depending on the scale of breaking the Cartan operators, we may get different models with different neutrino mass patterns. For example, we may have two kinds of SO(10) models:[15]

(1) The "one scale" SO(10) models:

In the Gell-Mann-Fritzsch[15] model, Higgs No. 4 (which is in the 126-plet of SO(10)) gets a very big VEV, but No. 5 gets zero VEV which is in the same 126-plet of SO(10). Thus they implicitly used an ansatz

$$m_{\nu_L} = 0 \quad \text{(at the tree level)} . \tag{25}$$

This ansatz has been avoided by Witten[16] in his minimal SO(10) model without Higgs 126-plet. The effective Higgs No. 2 (which is in 16-plet of SO(10)) gets a big VEV - 10^{14} GeV. He gets the mass of the right-handed neutrino to be (Fig. 2)

$$m_{\nu_R} \sim \frac{m_d}{m_L} M\, g\, \alpha^2 \sim 10^7 \text{ GeV} \tag{26}$$

where the factor $(m_d/m_L)g$ is the Yukawa coupling constant and $M \sim 10^{14}$ GeV. $\alpha = g^2/4\pi$ is the gauge coupling constant. All "one scale" SO(10) models give neutrino mass pattern as that in Fig. 1b with very heavy right-handed neutrinos. The low energy phenomenology of such models has nothing not already in the SU(5) model.[8] In the E_6, E_7 and E_8 models, the same pattern can be achieved.

(2) The multi-scale SO(10) models:

If the Higgs No. 6 (which is in the 45-plet of SO(10)) gets a VEV at a scale larger than 10^5 GeV and the Cartan operators are broken by the first effective Higgs in the List (22) at lower energies, then the model will be very different from the "one scale" models. In this case we need, instead of Eq. (25), the ansatz

$$m_\nu = 0 \quad \text{(at the tree level)} \tag{27}$$

which means the neutrino mass, no matter whether it is a Dirac one or a Majorana one, vanishes at the tree level. Owing to Eq. (26) the effective Higgs Nos. 4 and 5 cannot get VEV. The effective Higgs No. 1 must get VEV to give the lepton and quarks masses. No. 1 Higgs appears both in 10-plet and 126-plet of SO(10). By suitable adjustment of the Yukawa couplings, we can obtain Eq. (26). This fine tuning is ugly but accessible (i.e., high order corrections are small) because Eq. (26) gives extra $U(1)_A$ symmetry and, if there are several families, it also gives the family lepton number conservation to a high extent (violation is less than $(m_F/M)^8$, where m_F is a typical fermion mass and $M \sim 10^{14}$ GeV). In such a model, the Dirac mass of the neutrino is given by a one-loop diagram (Fig. 3a)

$$m_\nu \sim m_\ell \frac{m_L}{m_R} \alpha \sim 10^{-5}\, m_\ell \tag{28}$$

where m_L and m_R are the masses of the left- and right-handed W bosons.

The Majorana mass of the neutrino can be much smaller, if instead of Higgs Nos. 2 and 3 in Witten's model,[16] we introduce an effective Higgs (2,2,2) under $SU(2)_L \times SU(2)_R \times U(1)_B$ (which is in the 210-plet of SO(10)) and give its neutral components VEV. In this model, the Majorana mass of the neutrino is given by a two-loop diagram (Fig. 3b)

$$m_{\nu_L} \sim m_{\nu_R} \sim \frac{m_u}{m_W} \frac{V^2}{M} g\alpha^2 \sim 10^{-6} - 10^{-11}\ \text{eV} . \tag{29}$$

The consequences of the multiscale SO(10) model are: (1) We have two kinds of neutrino oscillations: $\nu_e - \nu_\mu$ type[3] and $\nu - \bar{\nu}$ type;[4] (2) The low energy physics is not the $SU(2) \times U(1)$ model but the $SU(2) \times [U(1)]^2$ model.[17] That means we may get two low-lying neutral gauge bosons and the mass of the lighter one is smaller than the mass of the z boson expected by the standard W-S model.

In the extended $SU(2) \times U(1)$ models with right-handed neutrino and nondoublet Higgs, a neutrino may get a Dirac mass and a Majorana mass, too. These have been discussed by Zee, Cheng, and Li.[18] In these models the lepton number violation does not imply baryon number violation. There are also grand unification models which maintain baryon number as a global conservation law.[19]

Acknowledgement

The author thanks I. Affleck for reading the manuscript.

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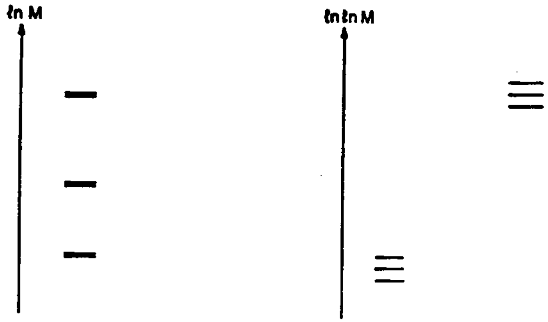
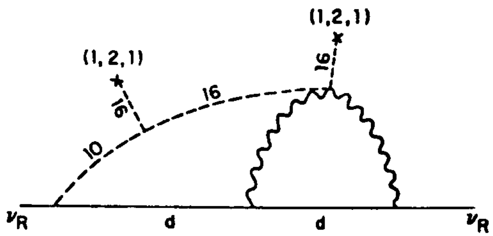
Fig. 1 Neutrino mass pattern for three families of neutrinos (six mass eigenvalues)



(a)
Double line fine structure.

(b)
Masses of left- and right-handed neutrinos are well-separated.

Fig. 2



Witten's diagram for neutrino mass[15]

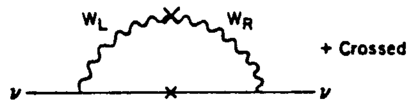


Fig. 3.a

Neutrino mass diagram in the multiscale model

Dirac mass

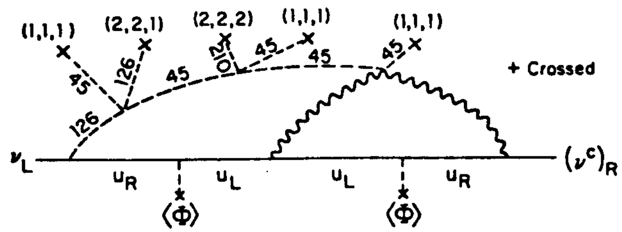


Fig. 3.b

Neutrino mass diagram in the multiscale model

Majorana mass