moment), for example, are of the form $\begin{Bmatrix} ig_{\kappa} = +1 \\ 0 \end{Bmatrix}$, i.e. of positive parity and having only large components. It is being presently investigated to find out what sort of wave functions of the neutrino tunnels through the barrier when such a state decays and what polarization it will have.

We conclude that the perturbative treatment of magnetic interactions overlooks some important effects at high energies, even for small magnetic moments. These effects have been used elsewhere [10,8] for hadronic processes. For the neutrino, even a small magnetic moment could lead to its capture by other charged particles and to deviations from lowest order scattering cross sections at very high energies.

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NEUTRINO OSCILLATIONS OF THE SECOND CLASS

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I - INTRODUCTION

In this talk I would like to discuss a new class of oscillations which occurs when Majorana and Dirac mass mixings are present in the Lagrangian of the charge zero leptons. The talk will be very elementary in nature and I refer the interested reader to the references for the general formalism and the proofs of the various statements. I have tried to define the concepts of Majorana mass mixings and Majorana neutrinos in the simplest possible way in Section II. Section III considers the standard model of weak interactions and the possible mass terms for the neutrinos. First and second class oscillations are defined in Section IV and the phenomenology of these oscillations is reviewed in Section V. Conclusions and heresies are presented in the last section. For the convenience of the reader a brief appendix contains the various properties of Majorana neutrinos. The literature on second class oscillations is growing fast. I apologize to any author whose work I may have omitted through ignorance.

II - MAJORANA MASS MIXINGS AND MAJORANA NEUTRINOS

All theoretical predictions are obtained from a perturbation expansion, the only thing we can do at present. The full Hamiltonian is as usual split into a "free" Hamiltonian H_o and an interaction term H_{int},

$$H = H_o + H_{int}.$$

To obtain any answer the conventional procedure must be followed: one diagonalizes H_o (finds the eigenstates) and then one expands in powers of H_{int} in the eigenbasis which diagonalizes H_o. H_o usually contains two independent pieces corresponding to kinetic energy terms and mass terms. The kinetic energy terms will be neglected in the sequel: they take care of themselves[1]. We shall look in detail at the mass terms. Note that the eigenstates of the free Hamiltonian, i.e., the mass eigenstates, will be the asymptotic particle states. Let us look at the simplest example of a mass term. The Dirac equation for the electron comes from the Lagrangian

$$\mathcal{L}_D = \bar{\psi}(i\not{\partial})\psi + m_e\bar{\psi}\psi \tag{1}$$

The mass term is $m_e\,\bar{\psi}\psi$. Its interpretation is clear: m_e is the mass of the electron and $\bar{\psi}\psi$ the probability (number) density, so the product is the mass energy. We may wonder why no other mass terms appear in Eq. (1). Indeed we know that the positron is described by the wave function (field)

$$\psi^c \equiv C\gamma^0\psi^* = i\,\gamma^2\psi^* \qquad (2)$$

ψ^c is basically the complex conjugate of ψ, the matrices guarantee that it transforms as a Lorentz spinor. We can immediately think of two other bilinear mass terms:

$$\text{(i)}\quad m_c\,\overline{\psi^c}\,\psi^c \qquad (3)$$

$$\text{(ii)}\quad m_1\,\overline{\psi^c}\psi + m_1^*\,\bar{\psi}\,\psi^c$$

These terms are perfectly Lorentz invariant. What is wrong with them? It is a simple exercise to prove that $\overline{\psi^c}\,\psi^c = \bar{\psi}\psi$. Hence (i) is nothing more than the Dirac mass term and does not represent anything new. On the other hand, we can easily see that (ii) is different. $\overline{\psi^c}\psi$ represents the probability that an electron becomes a positron! But this process is ludicrous, since it does not conserve electric charge! Hence if electric charge is to be conserved we cannot allow a term like (ii) in our Hamiltonian. It is instructive and will later be useful to derive this in another way. The concept of electric charge conservation implies a symmetry in the theory: $\psi \to e^{ie\theta}\psi$, $\psi^c \to e^{-ie\theta}\psi^c$ leaves the Hamiltonian invariant. We see indeed that

$$m_1\,\overline{\psi^c}\psi + m_1^*\,\bar{\psi}\psi^c \to m_1\,e^{2ie\theta}\,\overline{\psi^c}\psi + m_1^*\,e^{-2ie\theta}\,\bar{\psi}\psi^c \qquad (4)$$

So (ii) is clearly non-invariant, a statement which is equivalent to saying that it does not conserve electric charge.

Suppose however that we were describing a massive neutrino, which is electrically neutral. The most general mass Hamiltonian we could write down would then be:

$$H_{mass} = d(\bar{\psi}\psi + \overline{\psi^c}\psi^c) + M(\overline{\psi^c}\psi + \bar{\psi}\psi^c) \qquad (5)$$

where we have used $\bar{\psi}\psi = \overline{\psi^c}\,\psi^c$ and assumed real parameters for simplicity. The first two terms are called Dirac type mass terms, the last one (same as 3(ii)) is a Majorana type mass term. The Majorana mass terms connect a field with a conjugate field, hence they violate any kind of conservation law associated with the field.[3] We easily see that they violate lepton number conservation. If ψ carries lepton number +1, ψ^c carries lepton number -1, and the Majorana terms violate lepton number of two units! Hence no useful (additive) quantum number[3] can be defined when Majorana mass terms are present. What are the mass eigenstates associated with (5), i.e., what are the elementary particles states? We can diagonalize Eq. (5) by defining:

$$\chi = \frac{\psi+\psi^c}{\sqrt{2}}, \qquad \phi = i\,\frac{\psi-\psi^c}{\sqrt{2}} \qquad (6)$$

Then:

$$H_{mass} = (M+d)\bar{\chi}\chi + (d-M)\bar{\phi}\phi \qquad (7)$$

The transformation (6) also diagonalizes the kinetic energy term and we see therefore that the Hamiltonian (5) describes two free fermions χ and ϕ with masses 2(M+d), 2(d-M) respectively. Let us investigate these fermions more carefully. Clearly charge conjugating twice leads one back to where one started: $(\psi^c)^c = \psi$. It easily follows that

$$\chi^c = \chi, \qquad \phi^c = \phi \qquad (8)$$

i.e. the fermions χ and ϕ are their own antiparticles (like the π° but fermions). Self-conjugate fermions are called Majorana fermions. A Majorana fermion has by its very definition no charge or other quantum number since the anti-particle would carry the opposite quantum number.[3] Hence only neutrinos can be Majorana fermions. Since the fields χ and ϕ represent fermions, they have four components. The relation (8) guarantees that only two components are independent. A Majorana neutrino is therefore a two-component object.

Lepton number cannot be conserved anymore. This is easily seen since if ψ was a lepton, ψ^c is an antilepton, but the mass eigenstates ϕ and χ are superpositions of leptons and antileptons. We summarize what we have learned until now: the description of neutrinos naturally leads one to consider Majorana mass terms in the Hamiltonian. The mass eigenstates are then two-component Majorana neutrinos. Lepton number cannot be defined anymore.[3]

Are we saying therefore that any theory describing neutrinos will lead to Majorana mass eigenstates? Of course not. We can choose to eliminate the Majorana mass term by simply demanding that lepton number be conserved! In the case of one neutrino ψ, if we require that the transformation

$$\psi \to e^{i\theta}\psi, \qquad \psi^c \to e^{-i\theta}\psi^c \qquad (9)$$

be an invariance of the Hamiltonian, them M=0 in Eq. (5). The mass eigenstate is then the four-component Dirac neutrino ψ.

We conclude this section by stating without proof that the above continues to hold if we are describing many neutrinos. Indeed let ψ_i, i=1,2...N be the fields in the Hamiltonian.

(1) If the mass terms are only of the Dirac type, i.e. $\bar{\psi}_i \psi_j$, then the mass eigenstates will be four-component fermions. A lepton number can be defined and is conserved by the mass Hamiltonian.
(2) If both Majorana and Dirac mass mixings are present, the eigenstates will be 2N two-component Majorana neutrinos. Lepton number cannot be conserved anymore.

III - REALISTIC MODEL

We have only focused our attention so far on the mass term in H_o. We have not worried about the fact that neutrinos are left-handed, etc.... We now study a realistic model, which is a simple extension of the Standard Weak Gauge Interaction Model[4] (SW-GIM). The weak interactions are postulated to be invariant under local transformation of the group SU(2)×U(1). Because the charged weak current is known to be left-handed, the particles are put into the following representation:

$$\left.\begin{array}{llll} \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L & \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L & \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \cdots & \text{weak SU(2) doublets} \\ e^-_R & \mu^-_R & \tau^-_R & \text{charged SU(2) singlets} \\ \omega_{eR} & \omega_{\mu R} & \omega_{\tau R} & \text{neutral SU(2) singlets} \end{array}\right\} \quad (10)$$

Several technical comments must be made before continuing. The subscripts L,R correspond to left-and right-handed chiral projections $(1\mp\gamma^5)/2$ respectively. In writing (10) we have also assumed that the fields e^-, μ^-, τ^-... describe the electron, muon and tau. Hence we have assumed that the mass matrix has already been diagonalized in the charged sector. Since we know that the charged leptons are massive, we need both a left-and right-handed part for them. Since on the other hand the charged weak current does not involve the right-handed electron, we must put e^-_R in an SU(2) singlet (similarly for the other families μ^-_R, τ^-_R...). The addition of the singlets ω_{eR},... is more subtle. If the neutrinos are massless, no right-handed partner is needed. If the neutrinos are massive, then a right-handed singlet will be necessary. At this stage we will just add the singlets in by hand to preserve the symmetry between charged and neutral fermions and also between quarks and leptons.

Because the charged current raises or lowers the isospin only, the doublet members may get involved. Concentrating on the electron family only, we find

$$J_\mu^{\text{charged}} \sim \bar{e}_L \gamma^\mu \nu_{eL} + \ldots$$

the normal V-A current. A ν_{eL} would therefore be created when an electron is absorbed (more on this later).[5] ν_e, ν_μ, ν_τ... are called weak interaction eigenstates. The charged current is simple in terms of them.

The neutral singlets ω_{eR}, ... have no charged current interactions. More generally, they cannot have SU(2) currents. Furthermore, they cannot couple to the photon since they are neutral. Hence the singlets do not interact at all except for the possibility of very weak interactions with Higgs bosons.

What kinds of electrically neutral objects are described by Eq. (10)? Clearly we cannot answer this until we have diagonalized the mass Hamiltonian. As is well-known, mass terms are of the form $\bar{\psi}_L \psi_R$, namely they mix left-and right-handed chiralities. For the sake of generality we will consider the mass Hamiltonian without specifying the Higgs scalars which give rise to these terms. We can distinguish three cases.

Case (i): We impose singlet number conservation by demanding that the transformation

$$\omega_{eR} \to e^{i\alpha_e} \omega_{eR}; \qquad \omega_{\mu R} \to e^{i\alpha_\mu} \omega_{\mu R} \cdots \qquad (11)$$

leave the Hamiltonian invariant. Then the singlets ω_{iR} cannot couple to the doublet members ν_{jL} i,j = e,μ,τ... even via Higgses: the singlets completely decouple. If only a Higgs doublet is present, then the theory will also have another invariance equivalent to lepton number conservation:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \to e^{i\alpha} \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \qquad e^-_R \to e^{i\alpha} e^-_R \qquad (12)$$

+ same for other families

leaves the Hamiltonian invariant. As a consequence, the neutrinos ν_e,... remain massless. This is the standard weak model[4] where neutrinos are both weak interaction eigenstates and mass eigenstates with vanishing mass. Since the singlet members never mix with the rest of the world, one may as well omit them, as was done in the original version of the theory.

Case (ii): Suppose that instead of the constraints (11) and (12) we simply demand that the transformation:

$$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L \to e^{i\alpha}\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L; \quad \begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}_L \to e^{i\alpha}\begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}_L, \ldots$$

$$e^-_R \to e^{i\alpha} e^-_R; \quad \mu^-_R \to e^{i\alpha}\mu^-_R, \ldots \tag{12}$$

$$\omega_{eR} \to e^{i\alpha}\omega_{eR}, \quad \omega_{\mu R} \to e^{i\alpha}\omega_{\mu R}, \ldots$$

leave the Hamiltonian invariant. Clearly we are demanding that lepton number be conserved. Note, however, that we are <u>not</u> demanding separate conservation of electron number, muon number, etc... We cannot have Majorana mixing in the Lagrangian since as emphasized before they would violate lepton number conservation, e.g.

$$\overline{(\omega_{eR})^c}\ \omega_{\mu R} \to e^{2i\alpha}\ \overline{(\omega_{eR})^c}\ \omega_{\mu R} \tag{13}$$

These terms are not invariant, hence not allowed. [Note: $(\omega_{eR})^c$ represents a left-handed field]. Only Dirac mass terms are allowed. Indeed the most general mass terms for the electrically neutral sector is of the form:

$$H_{mass} = \sum_{i=e,\mu,\tau} m_{ij}\ \overline{\nu_{iL}}\ \omega_{jR} + \text{hermitian conjugate} \tag{14}$$

One can easily diagonalize Eq. (14) by a unitary transformation of the left-and right-handed fields. One learns that there are three mass eigenstates ν_1, ν_2, and ν_3. The doublets and singlets are linear superpositions of these mass eigenstates:

$$\nu_{eL} = U_{e1}\ \nu_{1L} + U_{e2}\ \nu_{LL} + U_{e3}\ \nu_{3L}; \ \cdots$$
$$\omega_{eR} = \hat{U}_{e1}\ \nu_{1R} + \hat{U}_{e2}\ \nu_{2R} + \hat{U}_{e3}\ \nu_{eR}; \ \cdots \tag{15}$$

The mass eigenstates carry the same lepton number as the singlet members ω_{eR}... and doublet numbers ν_{eL}... In fact if we only had one family, then ω_{eR} would just be the right-handed neutrino.

<u>Case (iii)</u>: Let us now impose nothing. Then Majorana mixings will occur naturally. In fact "bare" Majorana mass terms arise for the singlets without the need for additional Higgs. $\overline{(\omega_{eR})^c}\ \omega_{eR}$ etc...

The most general mass Hamiltonian is now given by:

$$H_{mass} = \sum_{ij=e,\mu,\tau} M_{ij}\ \overline{(\nu_{iL})}\ (\nu_{jL})^c + S_{ij}\ \overline{(\omega_{iR})^c}\ \omega_{jR} \tag{16}$$
$$+ D_{ij}\ (\overline{\nu_{iL}}\ \omega_{jR} + \overline{(\omega_{jR})^c}(\nu_{iL})^c) + \text{hermitian conjugate.}$$

The first two terms are Majorana mixings, the last one a Dirac mass term. The diagonalization proceeds as usual by making unitary transformations on the left-and right-handed fields. One now finds that there are 6 Majorana mass eigenstates ϕ_i, $i = 1,6$ with masses $m_i, i = 1,6$. As before

$$\nu_{eL} = \sum_{i=1,6} U_{ei}\ \phi_{iL}, \ldots$$
$$\omega_{eR} = \sum_{i=1,6} \hat{U}_{ei}\ \phi_{iR}, \ldots \tag{17}$$

There is no lepton number associated with the ϕ_i's anymore.

Let us look at the case of one family in detail. For simplicity we drop the subscript e. Then Eq. (16) reduces to:

$$H_{mass} = m\ \bar{\nu}_L(\nu_L)^c + M\ \overline{(\omega_R)^c}\ \omega_R + d(\bar{\nu}_L\ \omega_R + \overline{(\omega_R)^c}(\nu_L)^c)$$
$$+ \text{hermitian conjugate} \tag{18}$$

where for simplicity we have chosen the parameters to be real. We can diagonalize Eq. (18) simply by making a rotation, i.e.

$$\nu_L = \cos\theta\ \phi_{1L} + \sin\theta\ \phi_{2L}$$
$$\omega_R = -\sin\theta\ (\phi_{1L})^c + \cos\theta\ (\phi_{2L})^c \tag{19}$$

Choosing $\tan 2\theta = 2d/(M-m)$ brings (18) to the form:

$$H_{mass} = m_1\ \bar{\phi}_1\ \phi_1 + m_2\ \bar{\phi}_2\ \phi_2 \tag{20}$$

with

$$\phi_1 = \phi_{1L} + (\phi_{1L})^c = \phi_1^c, \quad \phi_2 = \phi_{2L} + (\phi_{2L})^c = \phi_2^c \tag{21}$$

and

$$m_1 = \cos^2\theta\ m + \sin^2\theta\ M - 2d\ \sin\theta\ \cos\theta$$
$$m_2 = \cos^2\theta\ M + \sin^2\theta\ m - 2d\ \sin\theta\ \cos\theta \tag{22}$$

Hence the mass eigenstates ϕ_1, ϕ_2 are Majorana fermions as advertised. Two interesting limits can be and have been considered.

(a) $m=0$, $d \ll M$: Clearly $\theta \simeq d/M \ll 1$. It follows from (19) that ν_L is almost purely made up of ϕ_1, while ω_R is almost purely ϕ_2. The masses also reduce to

$$m_1 \simeq d^2/M \simeq 0; \quad m_2 = M + 0\ (d/M).$$

In this limit the left-handed doublet member is a "light" neutrino, while the right-handed singlet is heavy and can never be produced in the laboratory. Some authors have argued that this is what happens in real life.[6]

(b) $m \sim d \sim M$: In this case θ is arbitrary and $m_1 \sim m_2$. This is the case we shall consider in the sequel.

Before concluding this section, we wish to change our notation a little to make Eq. (19) more obvious. Let us define

$$\eta_{iL} = (\omega_{iR})^c \qquad i = e,\mu,\tau \tag{23}$$

We could have done everything in terms of the η_{iL} rather than the ω_{iR}. In case (ii) the η_{iL} would have played the role of left-handed antineutrinos! Eq. (19) can now be written:

$$\begin{pmatrix}\nu_{eL}\\ \eta_{eL}\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\phi_{1L}\\ \phi_{2L}\end{pmatrix} \tag{25}$$

i.e. a rotation in the left-handed field space.

For further use let us quote the expansions of $\eta_{eL}, \ldots$ in terms of the mass eigenstates for both case (ii) and case (iii):

$$\eta_{eL} = \sum_{\alpha=1}^{3} \hat{U}^*_{e\alpha}\,(\nu^c_\alpha)_L; \quad \nu_{eL} = \sum_{\alpha=1}^{3} U_{e\alpha}\,\nu_{\alpha L} \quad \text{case (ii)} \tag{26}$$

$$\eta_{eL} = \sum_{\alpha=1}^{6} \hat{U}^*_{e\alpha}\,(\phi_\alpha)_L; \quad \nu_{eL} = \sum_{\alpha=1}^{6} U_{e\alpha}\,(\phi_\alpha)_L \quad \text{case (iii)} \tag{27}$$

where we have used $\phi_i^c = \phi_i$ in Eq. (27).

Equations (26-27) will be the starting points of the next section leading to two different classes of neutrino oscillations, so it is worth our while to point out the obvious one more time. In case (ii) the left-handed singlets are antineutrinos (left-handed!); they are superpositions of the mass eigenstates ν^c_α, $\alpha = 1,3$ while the electron neutrino (left-handed) is a superposition of the conjugate fields ν_α. Since lepton number is conserved, we cannot possibly have a transition from a left-handed singlet, η_{eL} say, to a left-handed doublet member ν_{eL}. Eq. (27) summarizing the situation in case (iii) reveals otherwise. Because the mass eigenstates are Majorana fermions, $\phi_\alpha = \phi^c_\alpha$, the singlets and doublets are linear superpositions of the same objects. Hence a transition $\eta_{eL} \leftrightarrow \nu_{eL}$ is now at least theoretically possible. We shall see in the next section that these transitions do occur: they are the second class of neutrino oscillations.

IV - NEUTRINO OSCILLATIONS

Let us briefly review the phenomenon of neutrino oscillations.[7] For definiteness let us consider the decay $\pi^+ \to \mu^+\nu_\mu$. ν_μ is a superposition of mass eigenstates ϕ_α. For generality we will not specify how many mass eigenstates there are. (It is also irrelevant to the present discussion whether they are Majorana or Dirac neutrinos)

$$|\nu_\mu\rangle = \sum_{\alpha=1}^{N} a_{\mu\alpha}|\phi_\alpha\rangle \tag{28}$$

The masses of the ϕ_α are denoted by m_α. Let us assume first that the mass differences are large compared with the energy resolution of the apparatus one is using. Then $\pi^+ \to \mu^+\nu_\mu$ would mean $\pi \to \mu^+\phi_1 + \mu^+\phi_2 + \mu^+\phi_3 \ldots$ and the decay rate would be given by the sum of incoherent decay rates:

$$\Gamma(\pi \to \mu^+\nu_\mu) \to \sum_\alpha a^2_{\mu\alpha}\,\Gamma(\pi^+ \to \mu^+\phi_\alpha) \tag{29}$$

Clearly the concept of a ν_μ is not very useful in this case. Assume next that $m_\alpha - m_\beta$ is much less than the energy resolution of the experiment and that the m_α's are small compared to the momenta involved. Then one produces a coherent superposition of the ϕ_α's when the pion decays, i.e. a ν_μ:

$$\Gamma(\pi \to \mu\nu) \sim |\sum_\alpha a_{\mu\alpha}\,\langle\pi|\mu^+\phi_\alpha\rangle|^2 \tag{30}$$

Neutrino oscillations are a statement about the evolution of a ν_μ beam. Before delving into the details, recall that there is a one-to-one correspondence between the weak eigenstate basis ($\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}, \eta_{eL}, \eta_{\mu L}, \eta_{\tau L}, \ldots$) and the mass eigenstates ($\phi_1, \phi_2, \phi_3 \ldots$). Consider a beam of ν_μ created at the time $t=0$ by the decay of pions. At a time t later (or a distance $L \simeq ct$ away) the beam will be made of $\nu_\mu(t)$:

$$|\nu_\mu(t)\rangle = e^{-iHt}|\nu_\mu(0)\rangle = \sum_\alpha a_{\mu\alpha}\,e^{-iE_\alpha t}|\phi_\alpha\rangle \tag{31}$$

We assume that the ϕ_α all have the same momentum $\vec{p}$, so that $E_\alpha = (\vec{p}^2 + m_\alpha^2)^{1/2}$. Expanding the right-hand side of (31) in terms of $|\nu_\mu(0)\rangle$, $|\nu_e(0)\rangle$ etc... we find:

$$|\nu_\mu(t)\rangle = K_{\mu\mu}|\nu_\mu(0)\rangle + K_{\mu e}|\nu_e(0)\rangle + \ldots \qquad (32)$$

Hence the probability that a ν_μ remains a ν_μ is given by

$$\langle\nu_\mu(t)|\nu_\mu(0)\rangle = |K_{\mu\mu}|^2 \neq 1 \qquad (33)$$

A ν_μ beam will contain some ν_e, ν_τ etc... contamination varying with the distance from the source. Note that we mean oscillations of a left-handed object into another left-handed object. Oscillations which flip chirality are strongly suppressed by powers of $(m/E)^2$ with m a typical neutrino mass, and are not considered here.

We can distinguish two classes of oscillations depending upon which model describes the real world, case (ii) or (iii), of the previous section. (Case (i) leads to zero mass neutrinos, hence no oscillations, as is obvious from Eq. (31)).

Case (ii): Recall that there are three mass eigenstates ν_1, ν_2, ν_3 which carry lepton number +1 and are Dirac neutrinos $\nu_1^c \neq \nu_1$ etc... The expansion of the singlets and doublets in terms of the mass eigenstates is given by Eq. (26). Clearly, in the evolution of a neutrino beam only the following oscillations are possible:

$$\eta_{eL} \leftrightarrow \eta_{\mu L} \leftrightarrow \eta_{\tau L} \qquad (34)$$

$$\nu_{eL} \leftrightarrow \nu_{\mu L} \leftrightarrow \nu_{\tau L} \qquad (35)$$

These oscillations are oscillations of the first class. The oscillations (34) will never be seen experimentally since the singlets η_{iL} are never produced by charged current reactions. The flavor oscillations (35) amongst left-handed neutrino species (hence also amongst the "normal" right-handed antineutrinos $\bar{\nu}_{eR}$, $\bar{\nu}_{\mu R}$, $\bar{\nu}_{\tau R}$) have become popular again, although the experimental evidence is dubious at best.

Because of lepton number conservation, singlet-doublet transitions $\eta_{eL} \leftrightarrow \nu_{eL}$ etc... cannot take place here.

Case (iii): The presence of Majorana mixings in the Lagrangian forbids the existence of a conserved lepton number. There are now six Majorana mass eigenstates ϕ_α, $\alpha = 1,6$, $\phi_\alpha^c = \phi_\alpha$. The expansions of the weak eigenstates in terms of mass eigenstates are given in Eq. (27). Since the left-handed singlets are "made of the same constituents" as the neutral members of the left-handed doublets, we can now have two classes of oscillations. As in the previous case we can have:

$$\left.\begin{array}{l}\nu_{eL} \leftrightarrow \nu_{\mu L} \leftrightarrow \nu_{\tau L} \\ \eta_{eL} \leftrightarrow \eta_{\mu L} \leftrightarrow \eta_{\tau L}\end{array}\right\} 1^{st} \text{ class oscillations.}$$

However, we can also have:

$$\left.\begin{array}{l}\nu_{eL} \leftrightarrow \eta_{eL} \\ \nu_{eL} \leftrightarrow \eta_{\mu L} \\ \nu_{eL} \leftrightarrow \eta_{\tau L} \\ \vdots \\ \nu_{\tau L} \leftrightarrow \eta_{\tau L}\end{array}\right\} 2^{nd} \text{ class oscillations.} \qquad (36)$$

The second class oscillations (36) mix singlets with doublet members of the same chirality. Care must be exercised here as the concept of lepton versus antilepton is not defined. Although statements are sometimes made that 2nd class oscillations are neutrino-antineutrino oscillations, these statements are loose parlance at best.

To conclude this section, we emphasize that the more natural case (iii), with no artificial constraints placed on the model, leads naturally to two classes of oscillations. In the next section we take a brief glance at the phenomenology of these oscillations.

V - PHENOMENOLOGICAL IMPLICATIONS OF SECOND CLASS OSCILLATIONS

A detailed analysis of the phenomenological consequences of second class oscillations is extremely difficult. In the simplest realistic case of three families there are six mass eigenstates (a 6 x 6 unitary mixing matrix) and five mass differences. Clearly, both first and second class oscillations occur concurrently. Disentangling their effects will not be an easy task experimentally. The key point in isolating second class oscillations is that the left-handed singlets η_{eL}, $\eta_{\mu L}$, $\eta_{\tau L}$ do not interact (we neglect Higgs interactions). When an incoming neutrino, say ν_{eL}, oscillates into a singlet, it cannot interact until the singlet oscillates back into a ν_{eL}, $\nu_{\mu L}$ or $\nu_{\tau L}$. The net effect is an oscillation of the absolute cross-sections as a function of distance. Although possible in principle, a precise measurement of absolute (charged and neutral) cross-sections appears very difficult in practice. It is unfortunate that this effect is the only possible way to confirm or rule out the existence of second class oscillations.

We now turn to possible phenomenological implications of second class oscillations for current experiment.[8]

Solar: Lepton number violating oscillations have the capability of explaining the deficiency in the ratio of observed to expected solar neutrinos. With first and second class oscillations among three families, the minimum probability for $\nu_e \to \nu_e$ transitions is 1/6, as opposed to 1/3 for first class oscillations only.

Reactor: The cross sections for an initial antineutrino beam scattering on proton and deuteron targets indicate depletions in $\sigma_{CC}(p)$, $\sigma_{CC}(d)$ and $\sigma_{CC}(d)/\sigma_{NC}(d)$ but not (at the ≈ 20% uncertainty level) in $\sigma_{NC}(d)$. To explain both the σ_{CC} and σ_{CC}/σ_{NC} results, first class oscillations are required with $\delta m^2 \approx 1\ eV^2$.

Beam dump: Charged and neutral current events are produced by prompt neutrinos created in the dump. Since the prompt neutrinos originate from decays of charmed particles, identical ν_e and ν_μ spectra and numbers are generated. The charged and neutral current interactions of the prompt neutrinos are measured in bubble chamber and counter experiments at CERN at a distance L ≈ 800-900 m downstream.

In the bubble chamber experiment, the measured e/μ ratio is $R(e/\mu) = 0.48^{+0.24}_{-0.16}$. Such deviations of the e/μ ratio from unity may indicate a $P(\nu_e \to \nu_e)$ depletion arising from oscillations. For the CERN beam dump L/E ≈ 0.01 m/MeV, so the mass scale of the oscillations would be $\delta m^2 \approx 100\ eV^2$. To discuss such oscillations we assume a prompt neutrino beam with equal parts of ν_{eL} and $\nu_{\mu L}$, neglecting any antineutrino contributions for simplicity.

For second class oscillations of the ν_e family alone, the e/μ ratio is given by

$$R(e/\mu) = \langle P(\nu_e \to \nu_e)\sigma_{CC}\rangle / \langle\sigma_{CC}\rangle \tag{37}$$

where σ_{CC} is the inclusive production cross section for e or μ and ⟨ ⟩ denotes a spectrum average. For first class oscillations $\nu_e \to \nu_e$, $\nu_e \to \nu_\tau$ (stringent experimental limits exist on $\nu_\mu \to \nu_e$ and $\nu_\mu \to \nu_\tau$ oscillations in this L/E range), the corresponding prediction is

$$R(e/\mu) = \frac{\langle P(\nu_e \to \nu_e)\sigma_{CC}\rangle + 0.17\langle P(\nu_e \to \nu_\tau)\sigma^\tau_{CC}\rangle}{\langle\sigma_{CC}\rangle + 0.17\langle P(\nu_e \to \nu_\tau)\sigma^\tau_{CC}\rangle} \tag{38}$$

where σ^τ_{CC} is the inclusive τ cross section. For comparable mixing in the two classes, the predictions in Eqs. (37) and (38) are similar. One can discriminate experimentally between the classes of oscillations by ascertaining whether τ is produced.

The beam dump counter experiments measure the ratio N(0μ)/N(1μ) of muonless to single muon events. With second class oscillations of the ν_e family, the prediction is

$$N(0\mu)/N(1\mu) = [\langle(1 + P(\nu_e \to \nu_e))\sigma_{NC}\rangle + \langle P(\nu_e \to \nu_e)\sigma_{CC}\rangle]/\langle\sigma_{CC}\rangle \tag{39}$$

in the limit of perfect acceptance. The corresponding prediction for first class oscillations is

$$\frac{N(0\mu)}{N(1\mu)} = \frac{2\langle\sigma_{NC}\rangle + \langle P(\nu_e \to \nu_e)\sigma_{CC}\rangle + 0.83\langle P(\nu_e \to \nu_\tau)\sigma^\tau_{CC}\rangle}{\langle\sigma_{CC}\rangle + 0.17\langle P(\nu_e \to \nu_\tau)\sigma^\tau_{CC}\rangle}\ . \tag{40}$$

Taking comparable mixing in the two classes (and hence similar R(e/μ) predictions), the value of N(0μ)/N(1μ) is significantly lower for second class oscillations. A detailed analysis with experimental cuts could thereby differentiate between first and second class oscillations in this L/E range on the basis of measured R(e/μ) and N(0μ)/N(1μ) values. Still other alternatives are simultaneous first and second class oscillations or first class oscillations involving additional families.

We shall not discuss other possible experiments to detect oscillations. Of these only modulations of scattering seem feasible. We refer the interested reader to the talk of B. Kayser in these proceedings.

VI - CONCLUSIONS

I have tried to convince the reader that if neutrinos are massive, the mass eigenstates will naturally be Majorana neutrinos unless extraneous symmetries are imposed. The presence of Majorana mass mixings enriches the phenomenon of neutrino oscillations: left-handed neutrinos can now oscillate into left-handed, non-interacting singlets, the second class oscillations. Quite frankly, these oscillations will make the interpretation of experimental data extremely difficult.

To conclude, let me advertise a heretical point of view. In the original SW-GIM model neutrinos remained massless in quite an artificial way. Is it possible that there are good theoretical reasons for neutrinos to remain massless? It should be clear from the above that one must look outside the simple model for such reasons, possibly supersymmetry or more exotic theories. That such an enterprise may not be without merit is summarized in an old French proverb, particularly suited to a meeting in the Northern Woods of Wisconsin: *"Il m'a dit qu'il ne faut jamais vendre la peau de l'ours qu'on ne l'ait mis par terre."*[9]

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6. M. Gell-Mann, P. Ramond, R. Slansky, Rev. Mod. Phys. 50, 721, (1978).
7. See e.g. the talk of V. Barger in these proceedings.
8. This section is essentially cripted from Ref. 1, which contains appropriate references.
9. "L'ours et les deux compagnons", La Fontaine.

APPENDIX

For convenience I will summarize the basic properties of Majorana fermions in this appendix. Rather than study Majorana particles in the Majorana basis, I will use the conventions of Bjorken and Drell (Relativistic Quantum Mechanics, Mc-Graw Hill, 1964, Appendix A). Since we are dealing with self-conjugate fermions in gauge theories, this latter choice is actually better-suited for calculations.

A Majorana fermion ψ is a self conjugate fermion, i.e.

$$\psi^c \equiv C\gamma^0\psi^* = \psi \qquad \text{(A1)}$$

The charge conjugation matrix $C = i\gamma^2\gamma^0$ satisfies $C = -C^{-1} = -C^\dagger = -\tilde{C}$, $C\,\gamma^\mu\,C^{-1} = -\widetilde{\gamma^\mu}$. Charge conjugation is an involution, i.e.

$$(\psi^c)^c = C\gamma^0(\psi^c)^* = C\gamma^0C^*\gamma^0\psi = \psi \qquad \text{(A2)}$$

Note that the definition (A1) is not the most general. We could have introduced an arbitrary phase in the definition, i.e.: $\psi^c = e^{i\eta}\,\psi$. The phase choice $\eta=0$ is usually made for convenience. See Refs. 1,2 for details.

Assume now that Ψ is an arbitrary four-component spinor. Then Ψ^c is also a four-component spinor. Defining

$$\psi_R = \frac{\Psi + \Psi^c}{\sqrt{2}} \qquad \psi_I = -i\,\frac{\Psi - \Psi^c}{\sqrt{2}} \qquad \text{(A3)}$$

one finds (use A2)

$$\Psi = \frac{\psi_R + i\,\psi_I}{\sqrt{2}}, \quad \psi_R = \psi_R{}^c, \quad \psi_I = \psi_I{}^c \qquad \text{(A4)}$$

If Ψ describes a particle of mass m, then

$$\begin{aligned} \bar{\Psi} i\not{\partial}\Psi &= \frac{1}{2}\bar{\psi}_R i\not{\partial}\,\psi_R + \frac{1}{2}\,\bar{\psi}_I\,i\not{\partial}\,\psi_I \\ m\bar{\Psi}\Psi &= \frac{1}{2}m\,\bar{\psi}_R\psi_R + \frac{1}{2}\,m\,\bar{\psi}_I\psi_I \end{aligned} \qquad \text{(A5)}$$

where we have used Eq. A6 below. Hence a four-component Dirac field is equivalent to a pair of degenerate Majorana fields. The Majorana fields ψ_R and ψ_I are two-component objects, as is obvious from (A1). It is clear from (A3, A4) that the decomposition into Majorana fermions is like going from complex to real objects, with complex conjugation replaced by charge conjugation. The factors of 1/2 in A5 are necessary to have the "real" fields normalized correctly.

The following identities hold for any two Majorana anti-commuting fermions ψ and χ, i.e. $\psi^c = \psi$ and $\chi^c = \chi$.

(i) $$\bar\psi\chi = \bar\chi\psi$$

(ii) $$\bar\psi\gamma^\mu\chi = -\bar\chi\gamma^\mu\psi$$

(iii) $$\bar\psi\gamma^\mu\gamma^5\chi = \bar\chi\gamma^\mu\gamma^5\psi$$

(iv) $$\bar\psi\partial\!\!\!/\chi = \bar\chi\partial\!\!\!/\psi \qquad \text{(A6)}$$

(v) $$\bar\psi\gamma^5\partial\!\!\!/\chi = -\bar\chi\gamma^5\partial\!\!\!/\psi$$

(vi) $$\bar\psi\sigma^{\mu\nu}\chi = -\bar\chi\sigma^{\mu\nu}\psi$$

In deriving (A6, iv-v) integration by parts was used and the surface terms were neglected. Furthermore, it must be remembered that ψ and χ are anticommuting c-numbers, i.e. $\widetilde{(\bar\chi\psi)} = -\bar\psi\tilde\chi$. Eq. (A6) summarizes all properties of Majorana fermions. In particular A6(ii) guarantees that they cannot couple to the electromagnetic field, A6(vi) that a Majorana fermion has no magnetic moment. When writing down Feynman rules one must remember that Majorana fermions are <u>real</u>. The symmetry numbers are the same as for a real spinless boson.

THE NEUTRINO MASS IN UNIFIED THEORIES

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ABSTRACT

Neutrino mass patterns for one family with both left- and right-handed neutrinos are discussed.

Talking about neutrino masses is very much like guessing the solution to a puzzle without enough information. Different people may guess different things. What I can do is to say something based on my tastes.

If there are both neutrinos in the doublet and singlet of the Weinberg-Salam Model,[1] the most general mass Lagrangian[2,3,4] for one family of neutrinos is:

$$\mathcal{L} = -\tfrac{1}{2}[a\,\bar\nu_R\nu_L + a\overline{(\nu^c)}_R(\nu^c)_L + b\overline{(\nu^c)}_R\nu_L + c\,\bar\nu_R(\nu^c)_L] + \text{h.c.} \qquad (1)$$

where ν_L is the doublet neutrino which appears in the weak charged current

$$\bar e_L\gamma_\mu\nu_L \qquad (2)$$

and ν_R is the singlet partner of ν_L with the same lepton number; ν^c is

$$\nu^c = \mathcal{C}\,\nu\,\mathcal{C}^{-1} = C\,\bar\nu^T,\; C = i\gamma_2\gamma_0,\; C^T = -C \qquad (3)$$

the chirality eigenstates are defined as

$$\nu_{\substack{L\\R}} = \frac{1\mp\gamma_5}{2}\,\nu_{\substack{L\\R}},\quad \bar\nu_{\substack{L\\R}} = (\nu_{\substack{L\\R}})^+\,\gamma_0$$

$$\frac{1\pm\gamma_5}{2}\,\nu^L_R = 0\,. \qquad (4)$$

We have

$$\nu^c_{\substack{L\\R}} \equiv (\nu_{\substack{L\\R}})^c = (\nu^c)_{\substack{L\\R}}\,. \qquad (5)$$

We notice that because of fermi statistics, we have

$$\overline{(\nu^c)}_R\nu_L = \overline{(\nu_L)}^c\nu_R = -\nu_L^T\,C\,\nu_L \neq 0\,. \qquad (6)$$

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