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THE MAGNETIC MOMENT OF THE NEUTRINO

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Neutral, spin ½-particles can have a magnetic moment. When neutrino was first postulated in order to account for the missing energy in the β-decay of the nucleus, Pauli[1] endowed it with a magnetic moment μ, and coupled it to the electromagnetic field with, what is now called, "Pauli anomalous magnetic moment term", $\sigma_{\mu\nu}F^{\mu\nu}\psi$, added to the linear Dirac equation:

$$(\gamma^{\mu}p_{\mu} - m + \kappa\sigma^{\mu\nu}F_{\mu\nu})\psi = 0. \tag{1}$$

This equation satisfies all the principles of relativistic quantum theory, with or without the mass m, and contains two conserved currents, $\bar{\psi}\gamma_{\mu}\psi$ (matter density) and $j^{\mu} = (\bar{\psi}\sigma^{\mu\nu}\psi)_{,\nu}$ = electromagnetic current density, and these two currents are no longer proportional as is the case for the electron and minimal coupling. Equation (1) was first used by Carlson and Oppenheimer [2] to derive the ionization loss of "neutrons" (as the neutrinos were then called) in interaction with the electron. To lowest order they find a cross section proportional to lnE. (See later)

Bethe [3] has recalculated the electron-neutrino cross-section, but again in lowest order of perturbation theory. A factor 1/W was later corrected in Bethe's cross-section formula [4], although early estimates of the magnetic moment have used Bethe's formula.

Pauli also envisaged that the neutrino due to its magnetic moment forms actually a bound state with the electron and proton to form what we now call neutron, hence would be a building block of hadrons and nuclei. This is possible if there would be a deep enough well to hold the electron and neutrino down to the nuclear size. Such a possibility could not be realized at that time, and this idea eventually was replaced by the Fermi notion of "creation of electron and neutrino" in the β-decay [5]. We shall come back to the question of bound states of the neutrino.

For m = 0, Eq. (1) necessarily implies a 4-component "neutrino". With an electromagnetic field present, the equation can no longer be split into two 2-component-Weyl equations. Away from the interaction region, asymptotocally, we can split, as usual, the four component equation into its left and right components in a relativitically invariant way (but of course violating parity).

In order to combine the attractive features of both the bound states properties of the 4-component neutrinos with anomalous, magnetic moment, and the simpler 2-component properties of free neutrinos, we make the hypothesis that there exists 4-component neutrinos with magnetic moment forming strongly interacting quasi-bound states, whereas the neutrinos observed coming from weak decays of such bound-states far away from the nucleus behave almost like two-component neutrinos.

Indeed we experiment on neutrinos which are coming from π; K, .. decays; these are nearly polarized, hence can have only a very small magnetic moment. Theoretical & experimental limits for neutrino magnetic moment have been discussed since 1954 [7]; they all refer to 2-component neutrinos. There are some reasons to believe that decaying neutrinos may be nearly polarized four-component neutrinos.

An exact two-component massless ν_L cannot have any magnetic moment, not even a magnetic form factor. Including all radiative effects the electromagnetic current (under CPT and the condition of 2-componentness) can only exhibit a single electric form factor

$$(\nu,p'|J_\mu^{em}(0)|\nu,p) \sim F_1(q^2)\bar{U}_\nu(p')\gamma_\mu(1-\gamma_5)U_\nu(p), \tag{2}$$

with $F_1(0) = 0$. The term $F_2(q^2)\bar{U}_\nu(p')\sigma_{\mu\nu}q^\nu(1-\gamma_5)U_\nu(p)$ vanishes automatically [6]. This is simply because a magnetic moment interaction must flip the spin, and if the ν has just one spin direction, it cannot be flipped.

Houtermanns and Thirring [7] have estimated the magnetic moment of ν from its virtual dissociation into n, $\bar{p}$, e^-. But if the ν is a 2-component ν, this virtual diagram must then only lead to an electric form factor, for the 4-component ν the diagram might lead to a magnetic form factor. The existence of intermediate bosons implies a simpler virtual diagram for the magnetic moment, namely the virtual dissociation of ν into electron and the intermediate boson W.

ELECTROMAGNETIC PROCESSES FOR A 4-COMPONENT NEUTRINO WITH MAGNETIC MOMENT μ (MASSIVE OR MASSLESS):

Electron-Neutrino Scattering: To lowest order the amplitude is proportional to

$$A \sim e\mu\bar{U}_{e'}\gamma^\mu U_e \bar{U}_{\nu'}\sigma_{\mu\nu}q^\nu U_\nu .$$

In the laboratory frame with $p_\nu = (E_\nu,\vec{p}_\nu)$, $p_e = (m_e, \vec{0})$ and $p_{e'} = (E', \vec{p}_{e'})$, we have $t \equiv (p_{e'}-p_e)^2 = -2m_e(E'-m_e) = -2m_eW$, $s \equiv 2m_e(E_\nu+m_e)$, or, $y \equiv \frac{E'}{E_\nu} = -\frac{t-2m_e^2}{s-2m_e^2} \simeq -\frac{t}{s}$. Hence

$\frac{d\sigma}{dy} = \frac{d\sigma}{dt}\frac{dt}{dy} = -(s-m_e^2)\frac{d\sigma}{dt}$, or $\frac{1}{(s-m_e^2)}\frac{d\sigma}{dy} = -\frac{d\sigma}{dt} = \frac{d\sigma}{d|t|}$. The differential cross-section to lowest order if for $m_\nu \ll E_\nu$

$d\sigma = \mu^2\pi r_o^2(1-\frac{W}{E_\nu})\frac{dW}{W}$, or

$$\frac{d\sigma}{d|t|} = \frac{1}{(s-2m_e^2)}\frac{d\sigma}{dy} = \mu^2\pi r_o^2\frac{1}{-t}\left(1+\frac{t}{s-2m_e^2}\right),\ t<0.$$

$$= \mu^2\pi r_o^2\frac{\left(1-y+\frac{2m_e^2}{s-m_e^2}\right)}{ys-m_e^2(2+y)}, \tag{3}$$

where r_o is the characteristic magnetic radius $r_o = \frac{\alpha}{m}$ (see below). For $s \gg m_e^2$, we have the simpler expression

$$\frac{1}{s}\frac{d\sigma}{dy} \simeq \frac{d\sigma}{d|t|} \simeq \mu^2\pi r_o^2\frac{(1-y)}{ys}. \tag{4}$$

The diffrential cross section in lowest order diverges at the lower limit $y_{min} = \frac{2m_e^2}{s-2m_e^2}$. ($t_{min} = 0$). Integrating from t_o to $t_{max} = -\frac{(s-2m_e^2)^2}{s-m_e^2}$ (corresponds to $W_{max} = \frac{2E_\nu^2}{m_e+2E_\nu} \simeq E_\nu$) we obtain for the total cross-section

$$\sigma = \mu^2\pi r_o^2\left[\log\frac{t_{max}}{t_o} + \frac{t_{max}-t_o}{s-2m_e^2}\right] \tag{5}$$

$$= \mu^2\pi r_o^2\left[\log\frac{(s-2m_e^2)^2}{|t_o|(s-m_e^2)} - \frac{s-2m_e^2}{s-m_e^2} + \frac{|t_o|}{s-m_e^2}\right] \xrightarrow[m_e\ll 1]{}$$

$$= \mu^2\pi r_o^2\left[\log s - 1 - \log|t_o| + \frac{|t_o|}{s}\right]$$

The logarithmic increase of the total (elastic) cross-section with energy due to magnetic interactions has recently also been obtained in two other ways [8]. (see later).

ELECTRON-POSITRON ANNIHILATION INTO ν-$\bar{\nu}$ PAIR PRODUCTION:

So far there is no experimental information on the $\nu\bar{\nu}$-pair production. For a 4-component neutrino the magnetic interaction gives to lowest order [4]

$$\sigma \equiv \mu_o^2\pi r_o^2\frac{s+2m_e^2}{6\sqrt{s^2-4sm_e^2}} \tag{6}$$

Other processes due to the magnetic moment of the neutrino that would have observable effects are spin precession in strong magnetic fields, plasmon dissociation into ν and $\bar{\nu}$, etc.

UPPER LIMIT ON THE NEUTRINO MAGNETIC MOMENT FROM e-ν SCATTERING:

Assuming that all the e-ν scattering is due to the magnetic moment of the ν we obtain, comparing eq. (3) with the Fermi and Salam-Weinberg theories, including form factors [9], an upper limit

$$\mu_\nu \text{ betw. } 1.4\text{x}10^{-8} \text{ and } 3.5\text{x}10^{-9}\mu_B.$$

It is remarkable that such a small magnetic moment would give in Born approximation the same cross-sections as the whole of weak interactions.

We will surely be able to say more about the relevance of magnetic interactions when angular distribution and energy-distribution of (e-ν) differential cross-section become available experimentally. For example in the Salam Weinberg theory the angular distribution is of the form.[10]

$$\frac{1}{s}\frac{d\sigma}{dy} = a + b(1-y)^2 + cy/s.$$

The magnetic interactions have another more striking effect beyond the Born approximation which we discuss in the next section.

EXACT SOLUTION OF THE DIRAC EQUATION FOR THE e-ν SYSTEM:

Even if we work with the very small magnetic moment of an approximately two-component neutrino, $\mu \sim 10^{-9}\mu_B$, very large magnetic fields occur near the dipole fields of electrons or protons. Unlike the Coulomb-case, the charge-dipole and dipole-dipole interactions cannot be treated in Born-approximation at short distances (or high energies). There is a new-phenomenon of magnetic pairing, charge-dipole or dipole-dipole quasi bound-states or narrow resonances.

We shall first describe this phenomenon in a simple intuitive nonrelativistic model and then show it in an exact solution in the case of the neutrino.

Simple Model

The Hamiltonian of a charge e in the field of a fixed charged quantum dipole μ can be written as

$$H = \frac{1}{2m}(\vec{p}-e\vec{A})^2 + V + S_{12}, \qquad (8)$$

where A is the vector potential of the dipole moment μ;

$$A = \mu\,\frac{\vec{\sigma}\text{x}\vec{r}}{r^3},$$

V is the Coulomb potential and S_{12} the dipole-dipole term. This Hamiltonian can be written as

$$H = \frac{1}{2m}p^2 + V - \frac{e\mu}{m}\frac{\vec{\sigma}.\vec{L}}{r^3} + \frac{e^2\mu^2}{m}\frac{1}{r^4} + S_{12} \qquad (9)$$

Usually one solves the "unperturbed" problem $H_o = \frac{1}{2m}p^2 + V$, and then treats the remaining terms as perturbations. This procedure is incorrect and overlooks an important phenomenon [11]. Instead of perturbation theory, if we plot the total potential in (9) we obtain a deep potential well $(-\frac{1}{r^3})$ with a repulsive core $(+\frac{1}{r^4})$. This structure at short distances, namely the deep potential well and the barrier is capable of holding very narrow, positive energy resonances.

ELECTRON-NEUTRINO SYSTEM:

In the rest frame of the electron, the 4-component neutrino with a magnetic moment μ is described by eq. (1). We can think of this neutrino as the limit of the electron with $e\to 0$, $m\to 0$, but $e/2m$ = const. = $\mu(r)$, the magnetic moment form factor. The radial components of the wave equation are [12]

$$\frac{df}{dr} = \frac{\kappa-1}{r}f + (m-E)g + V_m f$$

$$\frac{dg}{dr} = -\frac{\kappa+1}{r}g + (m+E)f - V_m f, \qquad (10)$$

where the magnetic potential V_m is given by

$$V_m = e\mu(r)\frac{1}{r^2},$$

we have neglected for simplicity the S_{12}-term which could be easily added. From (10) we obtain the second order equation

$$\left(\frac{d^2}{dr^2} - V_{eff}(r) + \lambda^2\right)U = 0, \qquad (11)$$

with

$$V_{eff} = \frac{v_2}{r^2} + \varepsilon\frac{v_3}{r^3} + \frac{v_4}{r^4} \qquad (12)$$

Potentials of this type are exactly soluble. [13] For example, for the e-ν case without S_{12}, we obtain a zero-energy resonance, i.e. $m_{Resonance} \approx m_e + m_\nu$, is definite parity states. The normalizable bound state solutions for $\varepsilon = -1$ (opposite signs of charge and dipole moment), for example, are of the form $\begin{Bmatrix} ig_\kappa = +1 \\ 0 \end{Bmatrix}$, i.e. of positive parity and having only large components. It is being presently investigated to find out what sort of wave functions of the neutrino tunnels through the barrier when such a state decays and what polarization it will have.

We conclude that the perturbative treatment of magnetic interactions overlooks some important effects at high energies, even for small magnetic moments. These effects have been used elsewhere [10,8] for hadronic processes. For the neutrino, even a small magnetic moment could lead to its capture by other charged particles and to deviations from lowest order scattering cross sections at very high energies.

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NEUTRINO OSCILLATIONS OF THE SECOND CLASS

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I - INTRODUCTION

In this talk I would like to discuss a new class of oscillations which occurs when Majorana and Dirac mass mixings are present in the Lagrangian of the charge zero leptons. The talk will be very elementary in nature and I refer the interested reader to the references for the general formalism and the proofs of the various statements. I have tried to define the concepts of Majorana mass mixings and Majorana neutrinos in the simplest possible way in Section II. Section III considers the standard model of weak interactions and the possible mass terms for the neutrinos. First and second class oscillations are defined in Section IV and the phenomenology of these oscillations is reviewed in Section V. Conclusions and heresies are presented in the last section. For the convenience of the reader a brief appendix contains the various properties of Majorana neutrinos. The literature on second class oscillations is growing fast. I apologize to any author whose work I may have omitted through ignorance.

II - MAJORANA MASS MIXINGS AND MAJORANA NEUTRINOS

All theoretical predictions are obtained from a perturbation expansion, the only thing we can do at present. The full Hamiltonian is as usual split into a "free" Hamiltonian H_o and an interaction term H_{int},

$$H = H_o + H_{int}.$$

To obtain any answer the conventional procedure must be followed: one diagonalizes H_o (finds the eigenstates) and then one expands in powers of H_{int} in the eigenbasis which diagonalizes H_o. H_o usually contains two independent pieces corresponding to kinetic energy terms and mass terms. The kinetic energy terms will be neglected in the sequel: they take care of themselves[1]. We shall look in detail at the mass terms. Note that the eigenstates of the free Hamiltonian, i.e., the mass eigenstates, will be the asymptotic particle states. Let us look at the simplest example of a mass term. The Dirac equation for the electron comes from the Lagrangian

$$\mathcal{L}_D = \bar{\psi}(i\not\partial)\psi + m_e\bar{\psi}\psi \qquad (1)$$

The mass term is $m_e \bar{\psi}\psi$. Its interpretation is clear: m_e is the