NEUTRINO MASS AS THE PROBE OF INTERMEDIATE MASS SCALES*

Goran Senjanović
Brookhaven National Laboratory, Upton, N.Y. 11973

ABSTRACT

A discussion of the calculability of neutrino mass is presented. We analyze in detail the possibility of neutrinos being either Dirac or Majorana particles. We offer arguments in favor of the Majorana case, where we succeed to link the smallness of neutrino mass to the maximality of parity violation in weak interactions. It is shown how the measured value of neutrino mass would probe the existence of an intermediate mass scale, presumably in the TeV region, at which parity is supposed to become a good symmetry. Experimental consequences of the proposed scheme are discussed, in particular the neutrino-less double β decay, where observation would provide a crucial test of our model, and rare muon decays such as $\mu \to e\gamma$ and $\mu \to ee\bar{e}$. Finally, we offer an analysis of the embedding of this model in an O(10) grand unified theory, with the emphasis on the implications for intermediate mass scales that it offers. It is concluded that the proposed scheme provides a distinct and testable alternative for understanding the smallness of neutrino mass.

I. INTRODUCTION

The conventional belief, until recently at least, was that neutrinos were massless, the main reasons being the quite small upper limit on the mass of electron neutrino: $m_\nu < 35$ eV and the γ_5 invariance of the two-component Weyl equation. I will not offer any comments on the status of the experimental situation, but rather discuss the theoretical side of the issue. The usual argument is that the absence of ν_R, experimentally suggestive by the V-A nature of weak interactions, automatically guarantees the γ_5 invariance and therefore the vanishing mass for the neutrino. That is false! Namely, even in the two component case one has a left-handed particle ν_L and a right-handed antiparticle $\nu^c_R \equiv C(\bar{\nu}_L)^T$ (where C is the Dirac charge conjugation matrix), which can be combined into a massive particle through a so-called Majorana mass term[1]

$$m\bar{\nu}_L \nu^c_R = m\nu_L^T C \nu_L \qquad (1.1)$$

The characteristic of the above mass term is, of course, the lepton number nonsonservation. It is, actually the lack of experimental indication for the violation of lepton number that prompted the belief that neutrino is strictly massless, similarly as the non-observation of proton decay led to the assumption of baryon number conservation. It is important to stress that in both cases the conservation law is not tied up to any fundamental physical principle.

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In this talk I would like to describe the attempts to construct the models with the calculable violations of the lepton number. Now, if neutrino is massive then the question poses itself as to why its mass is much smaller than the mass of the accompanying SU(2) partner, the electron. This question seems to be intimately connected with the nature of weak interactions, and so we will try to make the maximal use of that fact.

Let us start first by reviewing the situation with respect to the neutrino mass in the $SU(2)_L \times U(1)$ theory, which provides a description of low energy properties of weak interactions. In the standard scheme,[2] neutrino is kept massless by the choice of the Higgs sector and the assumed absence of ν_R from the theory. Namely, since we have only ν_L at our disposal it can neither develop a Dirac mass nor does the Higgs doublet couple to $\nu_L^T C \nu_L$.

What are then the possibilities of having nonvanishing neutrino mass in the context of the $SU(2)_L \times U(1)$ theory? There are two:

a) Add ν_R, in which case neutrino becomes a massive Dirac particle. The mystery then remains as to why $m_\nu \ll m_e$.

b) Add a Higgs triplet Δ, which can couple to $\nu_L^T \nu_L$. Again, since $m_\nu \propto \langle\Delta\rangle$ and the neutral current phenomena imply only $\langle\Delta\rangle \leq \langle\phi\rangle/10$ (ϕ is the SU(2) doublet), why is $m_\nu/m_e \leq 10^{-5}$? We find both possibilities rather arbitrary. But then, in order to understand the smallness of the ratio m_ν/m_e, we are forced to extend the gauge theory of weak interactions beyond the standard scheme.

Let us be more precise. Our hope of calculating m_ν would be achieved if we could construct an effective triplet Δ out of the doublet, i.e. $\Delta \sim \phi\phi$. In that case, we would obtain a Majorana mass term

$$m_\nu \sim \frac{\langle\phi\rangle^2}{M} \qquad (1.2)$$

where M is some large, SU(2) invariant mass scale, needed for dimensional reasons. Such a term is, for the reasons of renormalizability forbidden in the basic Lagrangian. Therefore, one could expect $m_\nu/m_e \sim \langle\phi\rangle/M \ll 1$ naturally. The question is then what physical energy scale does M correspond to?

In this talk I will discuss the possibility that M is the scale at which parity violation disappears. This is naturally incorporated in the left-right symmetric[3] theories devised a few years ago by Pati, Salam, Mohapatra and this author. The basic idea behind this approach is to start with the theory which is originally invariant under space reflection, by the analogy with electromagnetic and strong interactions. It turns out that the noninvariance of the vacuum under parity transformation, leads to a naturally heavier right-handed gauge meson and therefore, to a predominent V-A character of low energy weak interactions. The subsequent phenomenological analysis leads to a requirement $M^2_{W_R} \geq 10\, M^2_{W_L}$, so that in the TeV region, one may be able to observe the gradual restoration of parity as a good symmetry.

In these models, since left-right symmetry requires the presence of both ν_L and ν_R, neutrino is naturally a massive particle. We will discuss in detail attempts to explain the smallness of neutrino mass, both in Dirac and Majorana cases. We will argue on several grounds that the Majorana case is more realistic. First, in this case one can link the small mass of neutrino to the maximality of parity violation in low-energy weak interactions. Also, since the Dirac spinor is just the combination of two degenerate (in mass) Majorana spinors, Majorana case is more general and therefore more likely to occur. As we shall see, the price for understanding the smallness of m_ν in the Dirac case is quite large and one would be led to a very complicated and aesthetically unappealing theory.

In the preferred, Majorana case, we will be able to obtain the following, approximate relation between the neutrino and electron mass

$$m_\nu = \text{const.}\ \frac{m_e^2}{M_{W_R}} \tag{1.3}$$

where the constant in (1.3) is expected to be of order 1-10 and M_{W_R} is the mass of the right-handed charged gauge meson. In the V-A limit of the theory ($M_{W_R} \to \infty$), m_ν vanishes, which provides the mentioned link between parity violation and m_ν.

It turns out that the experimental consequences of such a scheme are quite intriguing. For sufficiently low value of M_{W_R}, one is bound to expect the neutrino-less double β decay with the life-time of order of 10^{24} years (likely to be observed by the next generation of experiments) and also lepton flavor changing processes such as $\mu \to e\gamma$, $\mu \to ee\bar{e}$ etc. with $B(\mu \to e\gamma) \approx 10^{-13}$-$10^{-9}$. Observation of m_{ν_e} in the eV region and (or) neutrino-less double β decay with a right life-time would be a strong indication of the existence of an intermediate mass scale (M_{W_R}) in the TeV region, contrary to the expectations based on the minimal SU(5) grand unified theory, which predicts the desert in energies between M_{W_L} and the unification scale ($\sim 10^{15}$GeV).

The rest of this paper is then organized as follows: In section II we discuss the general properties of left-right symmetric theories, with the emphasis on the minimality of the scheme. There we lay the groundwork for the discussion of neutrino masses. In section III, a Dirac case is analyzed in detail and it is shown that the only reasonable scheme with calculable m_ν requires $m_\nu = 0$. We discuss such a model and end up arguing against it, due to its complexity, especially in the Higgs sector of the theory. Section IV is where we reach the central part of our work: the connection between parity nonconservation and (Majorana) neutrino mass. We pay special attention to the experimental predictions in section V, in particular the neutrino-less double β decay. In section VI an analysis of embedding our model into grand unified theories is presented. We show how such considerations lead to rather stringent constraints on the intermediate mass scale (M_{W_R}). Finally, in section VII we summarize the main results of our work.

II. LEFT-RIGHT SYMMETRIC THEORIES: GENERAL FEATURES

Since these models were discusses extensively in the literature, we cover here only their basic properties. What we are after is a theory in which lepton number plays an active role, presumably in the form of one of the generators of the gauge group. The nice feature of this scheme is that that happens automatically. The gauge group is $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, where the U(1) generator[6] is B-L. That is seen from the formula for the electric charge generator

$$Q = T_{3L} + T_{3R} + \frac{B-L}{2} \tag{2.1}$$

together with the fact that the fermionic assignment, dictated by left-right symmetry, is (for one generation)

$$\psi_L = \begin{pmatrix}\nu\\ e\end{pmatrix}_L \quad \left(\frac{1}{2},\ 0,\ -1\right); \qquad \psi_R = \begin{pmatrix}\nu\\ e\end{pmatrix}_R \quad \left(0,\ \frac{1}{2},\ -1\right)$$

$$Q_L = \begin{pmatrix}u\\ d\end{pmatrix}_L \quad \left(\frac{1}{2},\ 0,\ \frac{1}{3}\right); \qquad Q_R = \begin{pmatrix}u\\ d\end{pmatrix}_R \quad \left(0,\ \frac{1}{2},\ \frac{1}{3}\right) \tag{2.2}$$

The numbers in the brackets correspond to $SU(2)_L$, $SU(2)_R$ and U(1) representation content.

At low energies, $SU(2)_L \times U(1)$ is a good symmetry, so that at energies of order 100 GeV $\Delta T_{3L} = 0$, $\Delta Q = 0$ and so

$$\Delta(B-L) = -2\Delta T_{3R} \tag{2.3}$$

Above relation is encouraging, since we can expect naturally the breakdown of lepton number conservation, or in other words the nonvanishing (presumably Majorana) neutrino mass. Also, it provides the connection between the lepton number violation and the helicity structure of weak interactions.

To implement the physical ideas discussed above, we need to specify the Higgs sector of the theory. The minimal set required to produce a realistic theory is

$$\phi = \left(\frac{1}{2},\ \frac{1}{2},\ 0\right)$$

$$\rho_L = (T_{3L},\ 0,\ B-L); \quad \rho_R = (0,\ T_{3R},\ B-L) \tag{2.4}$$

where we purposely do not specify the nature of ρ multiplets. Led by notions of simplicity, we will choose ρ's to be doublets or triplets. We shall use experiment and theory to discriminate between the two options. We should add that ϕ is needed to provide the fermionic masses and ρ's serve the purpose of breaking the parity. One can show that, consistent with the minimization of the potential,[7] the pattern of symmetry breaking is

$$\langle\rho_L\rangle = \gamma\frac{\langle\phi\rangle^2}{\langle\rho_R\rangle}, \quad \langle\rho_R\rangle \gg \langle\theta\rangle \tag{2.5}$$

where constant γ in (2.5) may vanish, depending on the particular scheme chosen. In any case, since $M^2_{W_R} \approx g^2\langle\rho_R\rangle^2$, $M^2_{W_L} \approx g^2[\langle\phi\rangle^2+\langle\rho_L\rangle^2]$, we have the following scenario for the symmetry breaking of the original gauge group

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \xrightarrow{\langle\rho_R\rangle \neq 0} SU(2)_L \times U(1)$$
$$\xrightarrow{\langle\phi\rangle \neq 0,\ \langle\rho_L\rangle \neq 0(?)} U(1)_{em}$$

Since $\langle\rho_L\rangle/\langle\phi\rangle \simeq \langle\phi\rangle/\langle\rho_R\rangle \simeq M_{W_L}/M_{W_R} << 1$, so that $\langle\phi\rangle$ gives most of its mass to W_L, one cannot rule out the possibility of ρ's being triplets. Namely, the ϕ multiplet consists of two $SU(2)_L$ doublets so that the success of the celebrated relation $M^2_W = M^2_Z\cos^2\theta_W$, predicted by the standard model is preserved, up to the factors $M^2_{W_L}/M^2_{W_R}$. We shall need further theoretical hints to be able to decide on the nature of ρ's. That is the topic of the next two sections, where we discuss the cases of doublets and triplets, respectively, paying the special attention to the properties of neutrinos.

III. DIRAC NEUTRINOS AND CALCULABILITY OF NEUTRINO MASS

Let us choose ρ multiplets to be SU(2) doublets

$$\rho_L = \left(\frac{1}{2},\ 0,\ 1\right); \qquad \rho_R = \left(0,\ \frac{1}{2},\ 1\right) \tag{3.1}$$

In this case only ϕ can couple to fermions, so that we get for the most general Yukawa couplings[8]

$$\mathscr{L}_Y = h\ \bar{\psi}_L\phi\psi_R + \text{h.c.} \tag{3.2}$$

where we concentrate on leptons only. Since, in general

$$\langle\phi\rangle = \begin{pmatrix}\kappa' & 0\\ 0 & \kappa\end{pmatrix} \tag{3.3}$$

One gets the Dirac mass for neutrino and electron

$$m_\nu = h\kappa'$$
$$m_e = h\kappa \tag{3.4}$$

The question of the smallness of neutrino mass becomes

$$\frac{m_\nu}{m_e} = \frac{\kappa'}{\kappa} \lesssim 10^{-5} \tag{3.5}$$

Why? Furthermore, one could question the Dirac character of the neutrino and the lepton number conservation that it leads to on the basis of our relation $\Delta(B-L) = -2\Delta T_{3R}$, which suggests the breaking of B-L, or L for that matter. The answer to the second problem is much simpler, so we give it first. What happens is that there is an extra global symmetry which can be identified with B-L. In some sense the lepton number conservation is accidental in this scheme, since it does not follow from any general principle.

Let us be more precise about it. By a discrete symmetry one can forbid the trilinear couplings. It is easy to show that the Higgs potential is invariant under two separate U(1) symmetries

$$U(1):\quad \rho_L \to \rho_L,\ \phi \to \phi,\ \rho_R \to e^{i\alpha}\rho_R,\ \psi_{L,R} \to e^{i\beta}\psi_{L,R}$$
$$U'(1):\quad \rho_L \to e^{i\alpha'}\rho_L,\ \phi \to \phi,\ \rho_R \to \rho_R,\ \psi_{L,R} \to e^{i\beta}\psi_{L,R} \tag{3.6}$$

The elimination of the trilinear couplings leads to the minimum of the Higgs potential

$$\langle\rho_L\rangle = 0, \qquad \langle\rho_R\rangle >> \langle\phi\rangle \tag{3.7}$$

But then U'(1) is the unbroken global symmetry, preserved even after symmetry breaking. It can and will be identified as B-L. Hence, the conservation of lepton number is linked to the Dirac character of neutrino.

We are still faced with the mystery as to why $m_\nu/m_e = \kappa'/\kappa < 10^{-5}$. I will describe here an attempt[9] by Branco and the author to resolve the difficulty. We have tried the most simple approach: arrange at the tree level $\kappa' = 0$, i.e. $m_\nu = 0$ and have either calculable and finite or vanishing neutrino mass in higher orders in perturbation theory, depending whether γ_5 symmetry is broken or not. At the first glance, the approach seems remarkably successful, since neutrino mass gets induced at the one-loop level through W_L-W_R mixing.

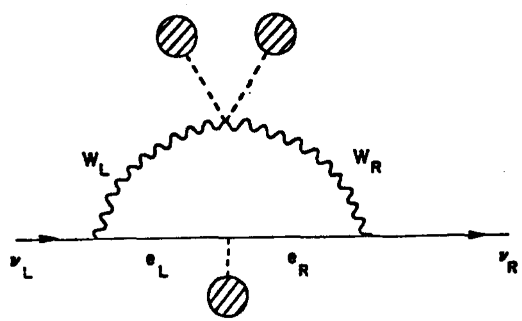

Fig. 1. Induced neutrino mass through W_L-W_R mixing.

It is a simple exercise to calculate neutrino mass from the diagram in Fig. 1

$$m_\nu = m_e\frac{\alpha}{\pi}\sin 2\xi_{LR}\ \ln\frac{M^2_{W_R}}{M^2_{W_L}} \tag{3.8}$$

which for expected small left-right mixing $\xi_{LR} \sim$ few% would lead to m_ν in the 10-100 eV region. Has one been able to calculate neutrino mass and link it to the parity violation? Alas, what appears to be a triumph of the theory is actually its disease. Namely, if one closes the neutrino line in Fig. 1 by coupling it to ϕ as in Fig. 2

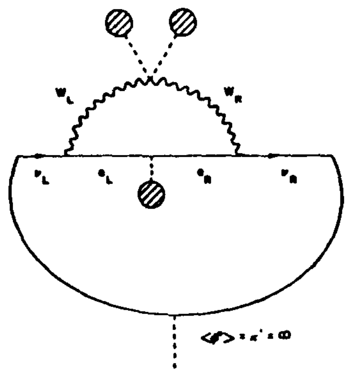

Fig. 2. Induced $\kappa' = \infty$ at the two loop level.

one clearly obtains an infinite value for κ' at the two loop level[10]. However, there is no counterterm to absorb the infinity, since we assumed $\kappa'_{tree} = 0$. Clearly then, we have to forbid $W_L - W_R$ mixing at the tree level, which makes both graphs in Figs. 1 and 2 vanish and so neutrino remains massless.

To see that above condition has nontrivial implications for the structure of charged weak currents, we display a graph in Fig. 3 which leads to induced infinite $W_L - W_R$ mixing at the one loop level.

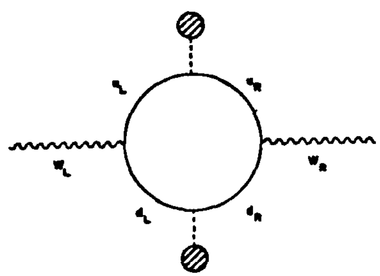

Fig. 3. Infinite $W_L - W_R$ mixing induced through the nonvanishing quark masses.

Again, simple dimensional analysis shows that $\xi_{LR}^{1\text{-loop}} = \infty$ which cannot be absorbed due to the nonexistence of the counterterm at the tree level. But then, Fig. 3 has to be forbidden. We have been able to achieve by choosing, what we called "orthogonal" structure of left and right handed charged currents

$$\begin{pmatrix} u \\ d(\theta) \end{pmatrix}_L, \quad \begin{pmatrix} c \\ s(\theta) \end{pmatrix}_L, \quad \begin{pmatrix} t_1 \\ b_1(\phi) \end{pmatrix}_L, \quad \begin{pmatrix} t_2 \\ b_2(\phi) \end{pmatrix}_L$$

$$\begin{pmatrix} u \\ b_1(\alpha) \end{pmatrix}_R, \quad \begin{pmatrix} c \\ b_2(\alpha) \end{pmatrix}_R, \quad \begin{pmatrix} t_1 \\ d(\beta) \end{pmatrix}_R, \quad \begin{pmatrix} t_2 \\ s(\beta) \end{pmatrix}_R \qquad (3.9)$$

with θ, ϕ, α, β being various Cabbibo-like mixing angles. The pattern of (3.9) is obvious: we have coupled the right handed light quarks to the heavy quarks only, so that the diagram in Fig. 3 vanishes. That, of course, leads to the requirement of the minimum of eight quark flavors.

I will not go into the details of the technical aspects, i.e. the choice of discrete (or global) symmetries which lead naturally to the above described picture. It is sufficient to say that the required Higgs sector is rather complicated. Above analysis was included more for pedagogical reasons: to show the constraints which appear in the issues of the calcuability of fermionic masses and, more importantly, to argue against the Dirac neutrinos on the basis of complexity.

Now, the case of two nondegenerate Majorana neutrinos, to be described in the next section, is more general than the Dirac case. Namely, Dirac particle is equivalent to two degenerate Majorana spinors, since it can be described by $\psi_1 = 1/\sqrt{2}\,(\psi + \psi^c)$, $\psi_2 = 1/\sqrt{2}\,(\psi - \psi^c)$ (where $\psi_1{}^c = \psi_1$, $\psi_2{}^c = \psi_2$) and the mass term can be written as

$$m\bar{\psi}\psi = m(\psi_1{}^T C\psi_1 + \psi_2{}^T C\psi_2) \qquad (3.10)$$

IV. MAJORANA NEUTRINOS AND SPONTANEOUS PARITY VIOLATION

In this section I will discuss the work done in collaboration with Rabindra Mohapatra. In terms of technical aspects of left-right symmetric models, Majorana case, as we shall see, corresponds to the ρ multiplets being triplets. The Higgs sector is then

$$\phi(\tfrac{1}{2}, \tfrac{1}{2}, 0)$$

$$\Delta_L(1, 0, 2)\ ; \quad \Delta_R(0, 1, 2) \qquad (4.1)$$

where we use symbol Δ instead of ρ.

The main change is that now quantum numbers of Δ's allow further Yukawa couplings, so that we have

$$\mathcal{L}_Y = h_1\bar{\psi}_L\phi\psi_R + h_2(\psi_L{}^T C\,\tau_2\Delta_L\psi_L + \psi_R{}^T C\tau_2\Delta_R\psi_R) + \text{h.c.} \qquad (4.2)$$

In a sense, this set of Higgs multiplets fully utilize their purpose, since (4.2) are the most general Yukawa couplings. Namely, the Higgs fields transform as fermion-fermion bilinears $\phi = \bar{\psi}_L\psi_R$ and $\Delta_{L,i} = \psi_L{}^T C\tau_2\tau_i\psi_L$, $\Delta_{R,i} = \psi_R{}^T C\tau_2\tau_i\psi_R$, where $\Delta_{L,R} = 1/\sqrt{2}\tau_i\Delta_{L,Ri}$. This is an important point, since it is exactly a situation one envisions in the case of dynamical symmetry breaking. Therefore, our results should hold true, at least qualitatively, even if there are no fundamental Higgs scalars.

The minimization of the Higgs potential leads to the following pattern of symmetry breaking

$$\langle\Delta_R\rangle \gg \langle\phi\rangle$$

$$\langle\Delta_L\rangle = \gamma \frac{\langle\phi\rangle^2}{\langle\Delta_R\rangle} \tag{4.3}$$

where γ is a particular combination[11] of triplic self-couplings in the Higgs potential. It turns out that γ is necassarily non-vanishing,[12] if one is to allow the Yukawa coupling h_2 in (4.2). We shall come to it later.

To see what (4.3) implies physically, we display the gauge meson eigenstates and their masses. First, in the charged sector we have

$$W_L^{\pm}:\quad M_{W_L}^2 = \frac{g^2}{2}\left(2\langle\Delta_L\rangle^2 + \langle\phi\rangle^2\right)$$

$$W_R^{\pm}:\quad M_{W_R}^2 = \frac{g^2}{2}\left(2\langle\Delta_R\rangle^2 + \langle\phi\rangle^2\right) \tag{4.4}$$

where we ignore the tiny mixing between W_L and W_R. The condition $\langle\Delta_R\rangle \gg \langle\phi\rangle \gg \langle\Delta_L\rangle$ implies then $M_{W_R}^2 \gg M_{W_L}^2$.

In the neutral sector, the physical states (to the leading order in M_{W_L}/M_{W_R}) are

$$A:\quad M_A^2 = 0$$

$$Z_L:\quad M_{Z_L}^2 \simeq \frac{M_{W_L}^2}{\cos^2\theta_W}$$

$$Z_R:\quad M_{Z_R}^2 \simeq 2\,\frac{\cos^2\theta_W}{\cos 2\theta_W}\, M_{W_R}^2 \tag{4.5}$$

where $\tan\theta_W \equiv g'/\sqrt{g^2+g'^2}$. Besides the photon A, we have a light boson Z_L which corresponds to the Z boson of $SU(2)_L \times U(1)$ and a heavy boson Z_R with a mass of order M_{W_R}. At low energies, then, the $SU(2)_L \times U(1)$ theory emerges, up to the $M_{W_L}^2/M_{W_R}^2$ terms. The correction terms can be used to set the lower limits on the heavy gauge meson masses. The result turns out to be[13]

$$M_{Z_R} > 300 \text{ GeV}$$

$$M_{W_R} > 180 \text{ GeV} \tag{4.6}$$

We turn next to the main topic of our discussion, that is lepton masses, in particular the ratio m_ν/m_e. Substituting the vacuum expectation values (4.3) into the Yukawa couplings (4.2), we get the following expression for lepton masses

$$m_e = h_1 \langle\phi\rangle$$

$$\mathcal{L}_\nu^{mass} = h_2[\langle\Delta_L\rangle \nu_L^T C\nu_L + \langle\Delta_R\rangle \nu_R^T C\nu_R] + h_1\langle\phi\rangle\overline{\nu}_L\nu_R + \text{h.c.} \tag{4.7}$$

where we for simplicity assume[14] $\kappa' = \kappa$. The more complete analysis is presented in ref. 11. The complicated mixture of Dirac and Majorana mass terms in (4.7) can be easily simplified by the change of variables

$$\nu \equiv \nu_L, \quad N \equiv C(\bar{\nu}_R)^T \tag{4.8}$$

which leads to

$$\mathcal{L}_\nu^{mass} = h_2[\langle\Delta_L\rangle \nu^T C\nu + \langle\Delta_R\rangle N^T CN] + h_1\langle\phi\rangle\nu^T CN + \text{h.c.} \tag{4.9}$$

Since $\langle\Delta_R\rangle \gg \langle\phi\rangle \gg \langle\Delta_L\rangle$, the approximate eigenstates of the mass matrix[15] for ν and N

$$\begin{matrix} \nu \\ N \end{matrix}\begin{pmatrix} h_2\langle\Delta_L\rangle & h_1\langle\phi\rangle \\ h_1\langle\phi\rangle & h_2\langle\Delta_R\rangle \end{pmatrix} \tag{4.10}$$

are

$$m_N \simeq h_2\langle\Delta_R\rangle$$

$$m_\nu \simeq \frac{h_2^2\langle\Delta_L\rangle\langle\Delta_R\rangle - h_1^2\langle\phi\rangle^2}{h_2\langle\Delta_R\rangle} \tag{4.11}$$

We shall furthermore assume, for simplicity and naturalness that $\gamma h_2^2 \sim h_1^2$. Using (4.3) and (4.7) we obtain an approximate relation

$$m_N = \frac{h_2}{g} M_{W_R}$$

$$m_\nu = \frac{g}{h_2}\frac{m_e^2}{M_{W_R}} \tag{4.12}$$

Eq. (4.12) has several noteworthy features

(a) It lifts the degeneracy between two equal mass Majorana spinors (Dirac case). It is a more general possibility and, we believe, more likely to occur.

(b) It achieves the link between parity violation and neutrino mass. In the V-A limit of the theory, i.e. when $M_{W_R} \to \infty$, $m_\nu \to 0$ and so the smallness of neutrino is naturally connected to the maximality of observed parity nonconservation in weak interactions.

(c) $m_\nu \ll m_e$ is naturally achieved. If we assume $g/h_2 = 1\text{-}10$ and take $M_{W_R} \simeq 200$ GeV, we get

$$m_N = 100 \text{ GeV}$$

$$m_\nu = (1\text{-}10) \text{ ev} \tag{4.13}$$

Higher Lepton Generations

Up to now, we have dealt only with the first family of fermions. The situation in the realistic, multigeneration case is pretty much the same. One easily obtains[11] the general qualitative result:

$m_{\nu_i} \propto 1/M_{W_R}$, for any generation, so that the link between the m_{ν_i} and V-A structure of charged currents is preserved. Previous relation between m_ν and m_e generalizes, under some assumptions, to[11]

$$m_{\nu_i} = (1\text{-}10)\,\frac{m^2_{\ell_i}}{M_{W_R}} \qquad (4.14)$$

where m_{ℓ_i} stands for the masses of charged leptons (i = 1,2,3). Order of magnitude estimates are then ($M_{W_R} \simeq 200$ GeV)

$$m_{\nu_e} \simeq 10 \text{ eV}, \quad m_{\nu_\mu} \simeq 100 \text{ KeV}, \quad m_{\nu_\tau} \simeq 100 \text{ MeV} \qquad (4.15)$$

We must see whether these values are in accord with experiment and observation. They easily past the test of laboratory limits:[16] $m_{\nu_e} < 35$ eV; $m_{\nu_\mu} < 500$ KeV; $m_{\nu_\tau} < 250$ MeV. The situation is somewhat different in regard to cosmological limit on relatively stable neutrinos[17]

$$\sum m_{\nu_i} < 100 \text{ eV} \qquad (4.16)$$

assuming $\tau_{\nu_i} < 10^{18}$ sec (the age of universe). It seems that ν_μ and ν_τ violate (4.16). However, ν_τ, due to its large mass can decay sufficiently fast into a $e\bar{e}\,\nu_e$ final state (it's a tree level process analogous to $\mu \to \bar{\nu}_e \nu_\mu e$ decay). One can easily estimate $\tau_{\nu_\tau} \simeq \tau_\mu \theta^{-2}$, where τ_μ is the muon lifetime and θ is the mixing between ν_τ and e in the gauge current. Since $\tau_\mu \simeq 10^{-6}$ sec, we conclude $\tau_{\nu_\tau} \ll 10^{18}$ sec. The limit (4.16) does not apply to such a heavy neutrino. The case of ν_μ is more problematic, due to its small mass. Its dominant decay modes are $\nu_\mu \to \nu_e \gamma$ and $\nu_\mu \to \nu_e \nu_e \bar{\nu}_e$. From the constraints on $\mu \to e\gamma$ and $\mu \to ee\bar{e}$ processes, one expects roughly

$$\tau_{\nu_\mu} \simeq \tau_{\mu\to e\gamma}\left(\frac{m_\mu}{m_{\nu_\mu}}\right)^5 > 10^{15} \times 10^3 \text{ sec} = 10^{18} \text{ sec} \qquad (4.17)$$

which would then imply $m_{\nu_\mu} < 100$ eV. The possible ways out of this impasse are being analyzed.[18] One alternative seems to be the existence of a heavy charged lepton ($m_e \simeq m_W$) which reduces the GIM suppression. Further analysis is clearly needed.

V. TESTS OF THE THEORY: LEPTON NUMBER VIOLATION

The characteristic feature of the model described above is the existence of massive light neutrinos ($m \sim 1$ eV-100 MeV) and heavy neutral leptons ($m \sim 100$ GeV), all Majorana particles. As we emphasized in the introduction, that leads to lepton number non-conservation. The main test of the theory is actually provided by such a process: neutrino-less double β decay $(\beta\beta)_o$.

(a) Neutrino-less double β decay

It is well-known[19] that a massive neutral Majorana lepton can induce $(\beta\beta)_o$ process, as shown in Fig. 4.

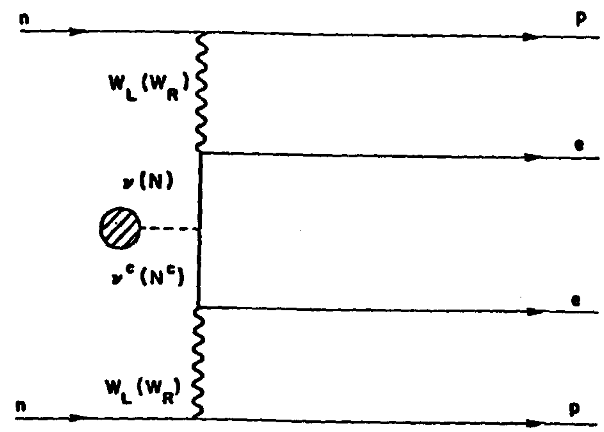

Fig. 4. Neutrino-less double β decay mediated by internal Majorana neutral leptons $\nu(N)$.

It is useful to use the parametrization[20] in which the intermediate lepton mass is traded for a dimensionless parameter η, defined through

$$J_\mu = \bar{e}\gamma_\mu\left[\frac{1+\gamma_5}{2} + \eta\,\frac{1-\gamma_5}{2}\right]\nu \qquad (5.1)$$

The present limits on η, coming from decays $Ca^{48} \to Ti^{48}$, $Ge^{76} \to Se^{76}$ and $Se^{82} \to Kr^{82}$ is approximately

$$\eta < (10^{-4} - 10^{-5}) \qquad (5.2)$$

That would correspond to a neutrino mass of order keV. The exchange of W_L leads then to the following bound on the products of masses and mixing angles

$$O^2_{L11} m_{\nu_e} + O^2_{L12} m_{\nu_\mu} + O^2_{L13} m_{\nu_\tau} < 1 \text{ keV} \qquad (5.3)$$

where O_L is the analog of the Cabbibo notation in leptonic current. From $m_{\nu_\mu} \lesssim 100$ keV, $m_{\nu_\tau} \lesssim 100$ MeV, (5.3) leads to the constraints

$$O_{L12} < 1/10, \quad O_{L13} < 3/100 \qquad (5.4)$$

which is expected from other considerations,[21] too. To make a definite prediction, we need to know the mixing angles.

The situation is more interesting in the case of right-handed currents. The equivalent η in this case is easily found to be[22]

$$\eta_R \simeq \frac{M^4_{W_L}}{M^4_{W_R}}\frac{m_p}{m_N} \qquad (5.5)$$

where $m_p \sim 1$ GeV and $m_N \sim 100$ GeV. Assuming $M^2_{W_L}/M^2_{W_R} \simeq 1/10$, we get

$$\eta_R < 4 \times 10^{-5} \qquad (5.6)$$

The above value of η implies a half-life for $(\beta\beta)_o$ decay

$$T_{1/2}^{(\beta\beta)_o} \simeq 10^{23\pm 2} \text{ years} \tag{5.7}$$

Therefore, a measurement of $T_{1/2}^{(\beta\beta)_o}$ up to 10^{24} years would have an important impact on the validity of the ideas presented here. Hoepfully, a next generation of experiments will achieve it.

(b) Muon number changing processes

These are processes which conserve the total lepton number, but not leptonic flavors. The examples are

(i) rare muon decays: $\mu \to e\gamma$, $\mu \to ee\bar{e}$

(ii) rare $\Delta S = 1$ decay: $K_L^o \to \mu^\pm e^\mp$

(iii) muon capture by nucleus: $\mu + p \to e + p$.

Let us cover in some detail $\mu \to e\gamma$ decay and only briefly comment on the other processes. We have separated the neutrino and heavy neutral lepton contribution, arising from the exchanges of W_L and W_R, respectively. First, the left-handed contribution is estimated to be

$$B_\nu(\mu\to e\gamma) \simeq \frac{\alpha}{\pi}\left(\sum_i O_{L_{i1}} O_{L_{i2}} \frac{m_{\nu_i}^2}{M_{W_L}^2}\right)^2 \tag{5.8}$$

where $B(\mu\to e\gamma) \equiv \Gamma(\mu\to e\gamma)/\Gamma_{total}$ and the expression in brackets is the well known[23] GIM suppression. Clearly, B_ν is hopelessly small: $B_\nu \leq \frac{\alpha}{\pi} O_{L_{13}} (m_{\nu_\tau}/M_{W_L})^4 \leq 10^{-18}$, for $m_{\nu_\tau} \leq 100$ MeV, $O_{L_{13}} \leq 10^{-3}$.

Similarly, the exchange of N's gives

$$B_N(\mu\to e\gamma) \simeq \frac{\alpha}{\pi}\frac{M_{W_L}^4}{M_{W_R}^4}\sum_i \left(O_{R_{i1}} O_{R_{i2}} \frac{m_{N_i}^2}{M_{W_R}^2}\right)^2 \tag{5.9}$$

where O_R is the Cabbibo-like rotation in the right-handed currents. For $M_{W_L}^2/M_{W_R}^2 = 1/10$ and specifying, for simplicity, to the case of two generations only, we get

$$B_N(\mu\to e\gamma) \simeq 10^{-5}(\sin\theta_R \cos\theta_R)^2 \frac{m_{N_2}^2 - m_{N_1}^2}{M_{W_R}^2} \tag{5.10}$$

With additional estimates $\Delta m_N^2/M_{W_R}^2 = 1/10$, $\theta_R \simeq 10^{-2}$ eq. (5.10) becomes

$$B(\mu\to e\gamma) \simeq B_N(\mu\to e\gamma) \simeq 10^{-10} \tag{5.11}$$

Due to uncertainties, I would say that $B(\mu\to e\gamma)$ can be expected in the range $10^{-13} \leq B(\mu\to e\gamma) \leq 10^{-9}$. Therefore, $\mu\to e\gamma$ process could serve as an important distinction between left-right symmetric theories and $SU(2)_L \times U(1)$ or grand unified models, which predict hopelessly small $\Gamma(\mu\to e\gamma)$.

Admittedly, our predictions are somewhat plagued by the lack of knowledge of M_{W_R} and (or) mixing angles. There is, however, an uncertainty free prediction of the model[11]

$$\frac{\Gamma(\mu\to ee\bar{e})}{\Gamma(\mu\to e\gamma)} \simeq \frac{\alpha}{\sin^2\theta_W} = \text{a few \%} \tag{5.12}$$

which is characteristic of models with neutral heavy leptons.[29] Eq. (5.12) could serve as a clear test of the theory, once $B(\mu\to e\gamma)$ and $B(\mu\to ee\bar{e})$ are precisely calcualted. Precise theoretical calculations and more sensitive experiments[25] are called for. All that we have said about rare muon decays, can be easily translated for other muon number changing processes (ii) and (iii).

VI. LEFT-RIGHT SYMMETRY AND GRAND UNIFICATION

In previous chapters we have discussed in detail the possibility that parity violation is only a low energy phenomenon which might disappear at energies above 1000 GeV or so. Since the unification suggested was only partial, we were not able to predict the corresponding mass scale. It is fair, then to ask whether such a situation can be made compatible with the idea of grand unification.[26] It is well known that the minimal grand unified theory, SU(5) of Georgi and Glashow,[26] predicts the desert in energies between a 100 and 10^{15} GeV. What is the price that would have to be paid to allow for the low intermediate mass scale, such as M_{W_R}?

The simplest, and rather popular, candidate for left-right symmetric grand unified theory is based on O(10) gauge group[27]. This group is large enough to embedd the $SU(2)_L \times SU(2)_R \times SU(4)_c$ group of Pati and Salam.[26]

Let us first review some basic properties of O(10) theory. The fermions are placed in 16 dimensional spinorial representations, They contain a right-handed neutrino as in left-right symmetric theories.

There are forty-five gauge mesons (adjoint representation of O(10) is 45 dimensional two index antisymmetric representation). Twenty-one of these are associated with the $SU(2)_L \times SU(2)_R \times SU(4)_c$ and the remaining twenty-four are superheavy bosons which mediate baryon number violating processes and therefore lead to nucleon decay. Besides the unification mass scale M_u, we can imagine the existence of intermediate mass scales M_c associated with the breaking of SU(4) color theory, M_{W_R} at which parity gets broken and M_{W_L} at which the symmetry breaking is completed down to $SU(3)_c \times U(1)_{em}$. In other words, we imagine the following scenario of symmetry breaking

$$O(10) \xrightarrow{M_u} SU(2)_L \times SU(2)_R \times SU(4)_c \xrightarrow{M_c} SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c$$
$$\xrightarrow{M_R} SU(2)_L \times U(1) \times SU(3)_c \xrightarrow{M_W} U(1)_{em} \times SU(3)_c$$

I should mention, that O(10) could be first broken down to SU(5), which then breaks in the usual manner. However, in this case $M_{W_R} \geq M_x \simeq 10^{15}$ GeV and so it is not of interest to us.

In what follows, we shall present the analysis of symmetry breaking and derive the constraints on M_c and M_{W_R} that follow from the value of $\sin^2\theta_W$ at low energies. Of course, we are after M_{W_R}, but we shall keep our discussion general, for the sake of completeness and pedagogy. The idea is, following Georgi, Quinn and Weinberg,[28] to look at the momentum dependence of $SU(2)_L$, $U(1)$ and $SU(3)_c$ coupling constants (called g, $g_1 = \sqrt{5/3}g'$ and g_s, respectively) in order to utilize the constraints provided by unification at high energies. The method used is based on renormalization group.

Now, since the $SU(2)_L$ group is kept intact all the way up to M_u, we get a simple dependence

$$\frac{1}{g^2(M_W)} = \frac{1}{g^2(M_u)} + 2b_2 \ln\frac{M_u}{M_W} \tag{6.1}$$

where b_2 is the well-known coefficient of β function, given for the gauge group SU(N) by[29]

$$b_N = -\frac{1}{8\pi^2}\left[\frac{11}{3}N - \frac{4}{3}\sum_f T_f(R) - \frac{1}{6}\sum_s T_s(R)\right] \tag{6.2}$$

The first term in (6.2) is the well-known gauge meson contribution and the second and third terms denote the fermionic and Higgs scalar contribution; $T(R)$ is defined by $\mathrm{Tr}T_aT_b \equiv T(R)\delta ab$, and $T_f(T_s)$ are group generators for fermions (scalars).

In the case of $U(1)$ and $SU(3)_c$ coupling constant we have to keep in mind that these groups become part of larger groups at intermediate energies. Following their descending, we get[30]

$$\frac{1}{g_1{}^2(M_W)} = \frac{1}{g_1{}^2(M_u)} + 2b_1 \ln\frac{M_R}{M_W} + 2\left(\frac{2}{5}b_1 + \frac{3}{5}b_2\right)\ln\frac{M_c}{M_R} + 2\left(\frac{3}{5}b_2 + \frac{2}{5}b_4\right)\ln\frac{M_u}{M_c}$$

$$\frac{1}{g_s{}^2(M_W)} = \frac{1}{g_s{}^2(M_u)} + 2b_3 \ln\frac{M_c}{M_W} + 2b_4 \ln\frac{M_u}{M_c} \tag{6.3}$$

Using $g_1(M_u) = g(M_u) = g_s(M_u)$ we can obtain relations which do not depend on the value of the unification coupling constant. In the usual manner,[28] we obtain

$$\sin^2\theta_W(M_W) = \frac{3}{8} - \frac{11}{3}\frac{\alpha(M_W)}{\pi}\left[\frac{5}{8}\ln\frac{M_u}{M_W} - \frac{3}{8}\ln\frac{M_u}{M_R} - \frac{1}{2}\ln\frac{M_u}{M_c}\right]$$

$$1 - \frac{8}{3}\frac{\alpha(M_W)}{\alpha_s(M_W)} = \frac{11}{3}\frac{\alpha(M_W)}{\pi}\left[3\ln\frac{M_u}{M_W} - \ln\frac{M_u}{M_R}\right] \tag{6.4}$$

where we have ignored the Higgs boson contribution, which is small [see (6.2)]. It is worth mentioning that the fermionic contributions to (6.4) cancel, so that the above is the prediction of the theory, independent of the number of generations, as it should be. The first term in above equations corresponds to the case $M_c = M_R = M_u$, i.e. no intermediate mass scales, as in SU(5). On the basis of (6.4) we can set the lower limits on M_R and M_c[31]

$$M_R \geqslant 10^7 - 10^9 \text{ GeV}$$

$$M_c \geqslant 10^{10} - 10^{12} \text{ GeV} \tag{6.5}$$

These bounds come about simply by requiring the unification scale to be below the Planck mass: $M_u < 10^{19}$ GeV. The experimental constraint $\sin^2\theta_W < .25$ leads then to (6.5).

Evidently, O(10) grand unified theory has no way of accommodating a light W_R (in the TeV region). That serves as an important distinction from left-right symmetric theories and it is also reflected in the much smaller predictions for neutrino masses: $m_{\nu_e} \ll m_{\nu_\mu} \ll m_{\nu_\tau} < 1$ eV. Certainly, all the interesting lepton number violating processes are hopelessly small in O(10) theory. Of course, a new class of phenomena is predicted, such as proton decay. The question is how general is our result, i.e. whether it is true for any grand unified theory. It may be, if the unifying group is sufficiently large and (or) its group properties are different, that it can still incorporate low M_{W_R}. One such scheme is based on $[SU(6)]^4$ gauge group[32], which I will not discuss here, but rather present some general attempts in reaching a desired model. Our ideas will be demonstrated on a simple toy model.

We have to understand first why the existence of intermediate steps of symmetry breaking increases the value of $\sin^2\theta_W$, since that is how we obtained the bound on M_{W_R}. The reason can be found from (6.13): making U(1) a part of nonabelian gauge group in some regions of energy scale clearly increases the value of $g_1(M_W)$. Since $\sin^2\theta_W \propto g_1{}^2/g^2$, that in turn increases $\sin^2\theta_W$ over the successful prediction of SU(5) (no intermediate mass scale). That suggest the possible ways out of the problem:

(a) construct theories in such a way that g changes faster with energy, as to compensate the increase in g_1 and (or)

(b) do not embed U(1) into a nonabelian group at all. The requirement (a) means that $SU(2)_L$ should be embedded into a larger group, at least in a region of energy scale.

We give now a toy model[33] that incorporates both features (a) and (b). Let us imagine a gauge group G with the following pattern of symmetry breaking

$$G \xrightarrow{M_u} SU(m)_L \times U(1) \times SU(n)_c \xrightarrow{M_I} SU(2)_L \times U(1) \times SU(3)_c \xrightarrow{M_W} U(1) \times SU(3)_c$$

where U(1) is kept intact at all momenta (up to M_u) and $n > 2$, $m > 3$. We can obtain equations that determine g, g_1 and g_s at M_W

$$\frac{1}{g^2(M_W)} = \frac{1}{g_u{}^2} + 2b_2 \ln\frac{M_I}{M_W} + 2b_m \ln\frac{M_u}{M_I}$$

$$\frac{1}{g_1{}^2(M_W)} = \frac{1}{g_u{}^2} + 2b_1 \ln\frac{M_u}{M_W}$$

$$\frac{1}{g_s{}^2(M_W)} = \frac{1}{g_u{}^2} + 2b_3 \ln\frac{M_I}{M_W} + 2b_n \ln\frac{M_u}{M_I} \tag{6.6}$$

Now, in general $g_1 = Cg'$, where $g'^2 = g^2\tan^2\theta_W$ (in SU(5) and O(10) $C = \sqrt{5/3}$). One then gets

$$\sin^2\theta_W(M_W) = \frac{1}{1+C^2} - \frac{C^2}{1+C^2}\frac{11}{3}\frac{\alpha(M_W)}{\pi}\left[\ln\frac{M_u}{M_W} + \frac{1}{2}(m-2)\ln\frac{M_u}{M_I}\right] \tag{6.7}$$

Abviously, for a well chosen C and m, the right value for $\sin^2\theta_W(M_W)$ will emerge even if $M_I \simeq 10\, M_W$ and $M_u \simeq 10^{15}$ GeV. This simplified model is not realistic, but we believe that an investigation along these lines has a promise of constructing a realistic grand unified theory with a low intermediate scale.[33] At the moment, the question is still very much open.

For the sake of completeness, I should mention that it was shown by Witten[34] that in the minimal O(10) theory, an intermediate mass scale $m_N \sim 10^8$ GeV gets induced in higher orders in perturbation theory. In such case, one obtains a spectrum of neutrino masses $m_{\nu_e} \ll m_{\nu_\mu} \ll m_{\nu_\tau} \simeq 10$ eV. Clearly, this scheme differs substantially from the one proposed by us. We feel, therefore, that the question of neutrino mass will have an important impact on detecting the nature of interactions beyond the ones described by the standard model. An observation of $m_\nu \sim 100$ MeV and the substantial amount of lepton number violation in a $(\beta\beta)_0$ process, would strongly indicate an existence of new phenomena and new mass thresholds in the TeV region.

VII. COMMENTS AND CONCLUSION

The main aspect of this talk was the analysis of neutrino mass in the theories with left-right symmetry. Most of our discussion was in terms of minimal such model based on $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge group, although in the last section we have made some comments regarding the embedding of that scheme into a grand unified theory, such as O(10).

If neutrino is massive, then it is either a Dirac or Majorana particle. We have argued against the Dirac case on various grounds. One of the strong arguments in favor of Majorana character of neutrinos, is that it follows naturally from a rather simple Higgs sector which would be also expected in the case of dynamical symmetric breaking (all the Higgs fields behave as bilinears in fermionic fields). Also, it fully utilizes one of the basic aspects of theory, i.e. the connection between parity violation and B-L nonconservation. The main triumph of the theory is that the smallness of neutrino mass is naturally explained: $m_\nu \ll m_e$ is tied up to the maximality of parity violation. Similarly, the model leads to the predictable amount of lepton number violation through a Majorana character of neutrinos. We have discussed at length the interesting experimental predictions of the theory; neutrino-less double β decay and rare muon decays. It turned out that for a light W_R ($M_{W_R} \simeq 300$ GeV), the theory leads to an appreciable amount of such amplitudes, hopefully observable by a next generation of experiments. Particularly important would be an observation of neutrino-less double β decay which could be a crucial test of the theory.

An important and distinguishing feature of our theory are appreciable neutrino masses: $m_{\nu_e} \simeq (1\text{-}10)$eV, $m_{\nu_\mu} \simeq (10\text{-}100)$keV, $m_{\nu_\tau} \simeq 100$ MeV which emerge if the lower limit on M_{W_R} is saturated. Is it something one expects to happen in grand unified theories? As we have shown, the answer is no. Working within 9(10) grand unified theory, we have derived the lower limit: $M_{W_R} > 10^9$ GeV. The situation in general is not clear and we have offered possible remedies. However, in simple cases there is a clear distinction between our model and grand unified theories, which should prove useful in our search for a true theory at very high energies.

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THE MAGNETIC MOMENT OF THE NEUTRINO

A. O. Barut
Department of Physics
The University of Colorado, Boulder, Colo. 80309

Neutral, spin ½-particles can have a magnetic moment. When neutrino was first postulated in order to account for the missing energy in the β-decay of the nucleus, Pauli[1] endowed it with a magnetic moment μ, and coupled it to the electromagnetic field with, what is now called, "Pauli anomalous magnetic moment term", $\sigma_{\mu\nu}F^{\mu\nu}\psi$, added to the linear Dirac equation:

$$(\gamma^\mu p_\mu - m + \kappa\sigma^{\mu\nu}F_{\mu\nu})\psi = 0. \qquad (1)$$

This equation satisfies all the principles of relativistic quantum theory, with or without the mass m, and contains two conserved currents, $\bar{\psi}\gamma_\mu\psi$ (matter density) and $j^\mu = (\bar{\psi}\sigma^{\mu\nu}\psi)_{,\nu}$ = electromagnetic current density, and these two currents are no longer proportional as is the case for the electron and minimal coupling. Equation (1) was first used by Carlson and Oppenheimer [2] to derive the ionization loss of "neutrons" (as the neutrinos were then called) in interaction with the electron. To lowest order they find a cross section proportional to lnE. (See later)

Bethe [3] has recalculated the electron-neutrino cross-section, but again in lowest order of perturbation theory. A factor 1/W was later corrected in Bethe's cross-section formula [4], although early estimates of the magnetic moment have used Bethe's formula.

Pauli also envisaged that the neutrino due to its magnetic moment forms actually a bound state with the electron and proton to form what we now call neutron, hence would be a building block of hadrons and nuclei. This is possible if there would be a deep enough well to hold the electron and neutrino down to the nuclear size. Such a possibility could not be realized at that time, and this idea eventually was replaced by the Fermi notion of "creation of electron and neutrino" in the β-decay [5]. We shall come back to the question of bound states of the neutrino.

For m = 0, Eq. (1) necessarily implies a 4-component "neutrino". With an electromagnetic field present, the equation can no longer be split into two 2-component-Weyl equations. Away from the interaction region, asymptotocally, we can split, as usual, the four component equation into its left and right components in a relativitically invariant way (but of course violating parity).

In order to combine the attractive features of both the bound states properties of the 4-component neutrinos with anomalous, magnetic moment, and the simpler 2-component properties of free neutrinos, we make the hypothesis that there exists 4-component neutrinos with magnetic moment forming strongly interacting quasi-bound states, whereas the neutrinos observed coming from weak decays