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NEUTRINO MIXING IN GRAND UNIFIED THEORIES

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ABSTRACT

Possible forms of the neutrino mass matrix that might appear in grand unified theories are discussed at the SU(2) × U(1) level. Two models are discussed, one based on a heavy Majorana singlet derived from SO(10) and one due to Zee based on a charged heavy scalar boson singlet inspired by SU(5). The patterns of neutrino masses and mixings are very different in the two cases.

It has been noted in many papers that it is quite natural for neutrinos to acquire a small mass in grand unified theories (GUT).[1] By fairly general arguments[2] this mass has the order of magnitude

$$m_\nu \sim (\text{light mass})^2/M \tag{1}$$

where the "light mass" is of the order of charged lepton or quark masses and M is a very heavy mass. Because of the small magnitude of m_ν the best hope for observing this mass may be in the phenomenon of neutrino oscillations. For oscillations to occur, however, not only must there be a non-zero neutrino mass but the mass eigenstates must involve significant mixing among the neutrino flavors. In this talk the major emphasis is on the mixing that might occur in grand unified theories.

Our approach is to look at the neutrino mass matrix at the SU(2) × U(1) level. We assume that the only light particles (masses $\leq M_W$) are those that are in the standard model. In particular, we assume that there are no light neutral lepton singlets so that the neutrino mass matrix necessarily has a Majorana form (connecting ν_L and $\bar{\nu}_R$) with $\Delta L=2$ and $\Delta I_3=1$. If SU(2) is broken only by the vacuum expectation values of Higgs doublets, the lowest dimensionality interaction that can lead to the neutrino mass matrix is given by the form[2]

$$f_{ab\alpha\beta}\, \psi^i_{aL}\, C\, \psi^j_{bL}\, \phi^k_\alpha\, \phi^\ell_\beta\, \epsilon_{ik}\, \epsilon_{j\ell} + \text{h.c.} \tag{2}$$

where a,b are lepton family indices; α,β distinguish different Higgs doublets; i,j are SU(2) indices which are summed over; and C is the Dirac charge conjugation matrix. The neutrino mass matrix is obtained by replacing each ϕ by its vacuum expectation value

$$\sum_{\alpha\beta} f_{ab\alpha\beta}\, \psi^\nu_{aL}\, C\, \psi^\nu_{bL} \left\langle \phi_\alpha^{o} \right\rangle \left\langle \phi_\beta^{o} \right\rangle \tag{3}$$

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From the point of view of GUT, (3) arises as an effective interaction from a diagram of the type indicated schematically in Fig.(1a), where the loop includes one or more heavy particles. The order-of-magnitude estimate of Eq.(1) follows because $\langle\phi\rangle$ times the Higgs Yukawa coupling is proportional to a light mass and $f_{ab\alpha\beta}$ is proportional to M^{-1} by dimensional arguments.[2] This result is not confined to GUT but holds in any theory that reduces to the standard SU(2) × U(1) at "low energies". Thus M need not be the unification mass, but rather represents the mass scale of some interaction beyond SU(2) × U(1). If the "light mass" is of the order of 1 Gev, it follows that the search for m_ν around 1 ev probes a mass scale M of order 10^9 Gev.

The simplest realization of Fig.(1a) occurs in the SO(10) theory where the loop contracts to a single heavy neutral lepton line,[3] Fig.(1b). This heavy lepton N is an SU(2) singlet and it is assumed that in some way it acquires a large Majorana mass, which is the origin of the change in lepton number by 2. The neutrino mass matrix $\mathcal{M}$ may be calculated from Fig.(1b) using perturbation theory

$$\mathcal{M} = \tilde{M}^\nu (\tilde{M}^M)^{-1} \tilde{M}^{\nu^+} \quad (4)$$

where $\tilde{M}^\nu$ is the matrix (in generation space) connecting ν_a to N_c and $\tilde{M}^M$ is the Majorana mass matrix of the SU(2) singlet N. The matrices here are in the representation in which the charged leptons are diagonal. In the discussion of the SO(10) theory the charged leptons are described by a mass matrix M^ℓ, which must be diagonalized[4] by a unitary transformation U_e. Therefore, Eq.(4) may be written

$$\mathcal{M} = U_e M^\nu (M^M)^{-1} M^{\nu^+} U_e^+ \quad (5)$$

where M (without the tilde) represents a matrix in the representation before the leptons are diagonalized. If M^ν is diagonalized by a unitary transformation U_ν

$$\mathcal{M} = U_e U_\nu^+ M_d^\nu (\tilde{M}^M)^{-1} M_d^\nu U_\nu U_e^+ \quad (6)$$

where M_d^ν is the diagonal form of M^ν and

$$\tilde{M}^M = U_\nu M^M U_\nu^+ \quad . \quad (7)$$

If $\tilde{M}^M$ were proportional to the unit matrix, then the neutrino mass matrix would be diagonalized by the matrix $U_e^+ U_\nu$. In the SO(10) theory with a single Higgs representation[5]

$$M^\ell \propto M^D \quad , \quad M^\nu \propto M^U \quad (8a)$$

$$U_e = U_D \quad , \quad U_\nu = U_U \quad (8b)$$

$$U_e^+ U_\nu = U_D^+ U_U = U_{KM} \quad (8c)$$

where D and U refer to up and down quarks and U_{KM} is the Kobayashi-Maskawa matrix. If this were the case, the following qualitative consequences would be expected:

A. Neutrino mixing angles are given by the off-diagonal elements of the Kobayashi-Maskawa matrix U_{KM}, which are expected to be fairly small.[6]

B. The ν_e-ν_τ mixing would be expected to be very small.

C. The neutrinos would have a mass hierarchy with their masses proportional to the square of the generation mass; thus $m(\nu_\tau)$ would scale as m_t^2 and $m(\nu_e)$ as m_u^2. If $m(\nu_\tau)$ is less than the cosmological limit[7] of about 50 ev, this scaling gives

$$m(\nu_\mu) < 0.3 \text{ e.v.} \quad (9a)$$

$$m(\nu_e) < 10^{-4} \text{ e.v.} \quad (9b)$$

Since the oscillation length is inversely proportional to the differences of the squares of the masses, oscillations between ν_μ and ν_e would thus have a very long oscillation length, a factor of 10^4 larger than that between ν_μ and ν_τ.

Retaining the single Higgs relations of Eqs.(8), we now ask whether significant additional mixing can occur from the form of the Majorana mass matrix $\tilde{M}^M$. Combining Eqs.(6) and (8) we write

$$\mathcal{M} = U_{KM}^+ \Delta U_{KM} \quad (10a)$$

$$\Delta_{ab} \propto m_a m_b \sum_{c=1}^{3} V_{ac}^+ V_{cb} (M_c)^{-1} \quad (10b)$$

where m_a is the up-quark mass for generation a, V_{cb} is the unitary matrix diagonalizing $\tilde{M}^M$, and M_c are the masses of the heavy Majorana particles. For the case of two generations (e,μ), V_{ac} is expressed in terms of one mixing angle ϕ and

$$\Delta \propto \begin{pmatrix} \left|\frac{\cos^2\phi}{M_1} + \frac{\sin^2\phi}{M_2}\right| m_u^2 & \sin\phi\cos\phi\left(\frac{1}{M_1} - \frac{1}{M_2}\right) m_u m_c \\ \sin\phi\cos\phi\left(\frac{1}{M_1} - \frac{1}{M_2}\right) m_u m_c & \left(\frac{\sin^2\phi}{M_1} + \frac{\cos^2\phi}{M_2}\right) m_c^2 \end{pmatrix} \quad (10c)$$

The neutrino mass matrix $\mathcal{M}$ is diagonalized by the matrix $U_{KM} U_\Delta$ where U_Δ diagonalizes Δ; the neutrino masses are given by the eigenvalues of Δ. From Eqs.(10) it is seen that the mixing angles needed to diagonalize Δ tend to be proportional to the generational mass ratios (such as m_u/m_c) unless the Majorana mass matrix takes a very special form. This means that in general U_Δ is very close to unity. In order to make all the mixing angles entering U_Δ large it is necessary to have the extreme conditions

$$M_a \propto m_a^2 \quad \text{and} \quad V_{ac} \propto m_a/m_c \ . \tag{11}$$

With the conditions (11) neutrino mixing angles would be large and none of the qualitative results (A)-(C) above would need to be true. However, we cannot imagine a theory that would yield these conditions.

A significant departure of U_Δ from unity can be obtained with the less extreme assumptions

$$M_a \propto m_a \quad \text{and} \quad V_{ab} \propto (m_a/m_b)^{1/2} \tag{12}$$

Combining Eq. (12) with Eq.(10b) and diagonalizing Δ we find that

$$(U_\Delta)_{ab} \propto (m_a/m_b)^{1/2} \tag{13}$$

and that neutrino masses scale as the generation masses rather than the square of the masses. For the case of two generations if we substitute explicitly in Eq.(10c)

$$M_1/M_2 = \alpha\, m_u/m_c \ , \quad \phi^2 = \alpha'\, m_u/m_c \tag{14}$$

then we find

$$\theta_\Delta = 2\,(\alpha'\, m_u/m_c)^{1/2}\,(\alpha + \alpha')^{-1} \tag{15a}$$

$$m(\nu_e)/m(\nu_\mu) = (m_u/m_c)\;\alpha\;(\alpha + \alpha')^{-2} \tag{15b}$$

where θ_Δ is the rotation angle (assumed to be small) needed to diagonalize Δ. In this case the neutrino mass matrix $\mathcal{M}$ is diagonalized by a Cabibbo-like matrix in which the usual Cabibbo angle θ_c is increased or decreased by θ_Δ. Since θ_Δ is less than θ_c, this means that the mixing between ν_e and ν_μ remains small although not exactly given by the Cabibbo angle. In the case of three generations there may be a somewhat larger deviation from the KM matrix because the mixing between ν_μ and ν_τ may be modified by a factor $(m_c/m_t)^{1/2}$ as indicated by Eq.(13), but the ν_e-ν_τ mixing would be expected to remain very small.

The assumption of a single Higgs representation giving the light fermion masses in SO(10) yields some incorrect mass relations.[5,8] A scheme involving two different Higgs representations[8,9] has been applied to the neutrino mixing problem by Hama et al.[10] They find that the mixing angle between ν_μ and ν_τ is given approximately by $\tan^{-1}[3(m_c/m_t)^{1/2}]$, a value that is probably much larger than the mixing given by U_{KM}. On the other hand the mixing angle between ν_e and ν_μ becomes even smaller and the mixing angle between ν_e and ν_τ remains extremely small. The neutrino masses have a hierarchical relation here given by

$$m(\nu_e) : m(\nu_\mu) : m(\nu_\tau) = (m_u/9)^2 : (9\, m_c)^2 : m_t^2 \tag{16}$$

These results are all based on the assumption that the Majorana mass matrix M^M is not significantly different from the unit matrix. As for the case discussed above, the results could be modified significantly if M^M were described by either Eqs.(11) or (12).

In our discussion so far we have made no assumptions about the origin of the Majorana mass matrix M^M. Our major purpose has been instead to emphasize the special features that M^M must have in order to seriously change conclusions about neutrino mixing. A particularly interesting theory of M^M has been given by Witten,[11] who assumes the matrix vanishes at tree level but is non-vanishing as a result of radiative corrections. This case has been discussed in a number of recent papers.[12] Because the diagrams contributing to M^M involve the Higgs couplings connecting the SU(2) singlet N to the light fermions, it is found that M^M is closely related to M^U. As a result, the neutrino mixing and masses tend to have a pattern that agrees with our discussion following Eq.(12).

All the different discussions of the SO(10) scheme using Fig.(1b) yield the following common features:

A. Neutrino mixing angles involving ν_e are small and those connecting ν_e and ν_τ are very small.

B. Neutrino masses have a hierarchical structure such that $m(\nu_e) \ll m(\nu_\tau)$. If $m(\nu_\tau) < 50$ ev, the mass of ν_e is less than .01 ev.

C. The shortest oscillation length and probably the largest mixing angle is that connecting ν_μ and ν_τ.

While these features are not so constraining as those listed after Eq.(8), they still rule out many recent suggested patterns for the neutrino mass matrix. To avoid these conclusions, relatively extreme assumptions must be made for the Majorana mass matrix such as those of Eq.(11).

In addition to the graph of Fig.(1b) one might expect there would be complicated graphs within the SO(10) model that could not be reduced to the form of Fig.(1b). In particular, in the Witten model the same general graphs that produce the Majorana mass M^M must make a direct contribution to the neutrino mass matrix. We have assumed here that these other graphs are much smaller in magnitude. This has been demonstrated for the Witten model.[12]

To find a realization of Fig.(1a) without a heavy fermion singlet, we can introduce a heavy boson singlet. If this singlet is to provide a $\Delta L=2$ interaction, it must be singly (or doubly) charged. This is because $\Delta L=2$ involves the change of a lepton to an anti-lepton and therefore a change in weak hypercharge; since $\Delta I_3=0$ for a singlet, this requires a change in leptonic charge. The simplest example we have found is the diagram Fig.(1c) introduced by Zee.[13] The singlet boson h^+ may be considered as the one colorless component of a <u>10</u> of SU(5). The neutrino mass matrix takes the form[14]

$$\mathcal{M}_{ab} = f_{ab}(m_b^2 - m_a^2)/M \tag{17}$$

where f_{ab} is the $\Delta L=2$ Yukawa coupling of h^+, which is necessarily antisymmetric and therefore off-diagonal, and M is a somewhat complicated expression depending on the h^+ mass. To obtain Eq.(17) we assumed that the SU(2) doublet Higgs Yukawa couplings conserve lepton flavor. This gives the interesting result that the diagonal elements of $\mathcal{M}_{ab}$ vanish, guaranteeing a large amount of mixing.

The crucial difference between Eq.(17) and Eqs.(10) lies in the fact that the (light mass)^2 factor enters as the difference of the squares of the masses instead of the product of the masses. Assuming m_e/m_μ and m_μ/m_τ are very small, Eqs.(10) tend to give small mixing angles (beyond the Cabibbo mixing) and a hierarchy of neutrino masses. In contrast, in this limit, Eq.(17) gives two almost degenerate mass eigenstates, each containing 50% of ν_τ and 50% a mixture of a ν_e and ν_μ, and one mass eigenstate close to zero mass containing the other mixture of ν_e and ν_μ.[14] This means there is one relatively short oscillation length that mixes only ν_e and ν_μ with an arbitrary mixing angle and one relatively long oscillation length that thoroughly mixes ν_τ with some combination of ν_e and ν_μ.

A particularly interesting feature of the Zee model is that as a result of the near degeneracy it is possible for a neutrino with a mass m to have an oscillation length much larger than E/m^2. An example I have considered[14] corresponds to $m(\nu_e) \approx m(\nu_\tau) = 20$ ev with the oscillation length between ν_e and ν_τ characterized by $\Delta m^2 \approx 0.5$ ev^2. In this example there would be a much shorter oscillation characterized by $\Delta m^2 \simeq 400$ ev^2 between ν_e and ν_μ, but I have judiciously and arbitrarily chosen the mixing angle to be small.

In Fig. 2 we show the neutrino mass patterns for the two models, assuming for illustration that the heaviest neutrino has a mass between 10 and 100 ev. Some interesting comparisons between these two patterns are listed in Table 1.

I am indebted to K. Tanaka, G. Branco, S. L. Glashow, R. Cahn and M. Chanowitz for discussions. This research was supported in part by the High Energy Physics Division of the U. S. Department of Energy under contracts W-7405-ENG-48 and EY-76-02-3066.

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Table 1

Comparison of the two models

Feature	Zee SU(5)	SO(10)
Heaviest neutrino	ν_τ and (ν_μ, ν_e) mixture	ν_τ
$m(\nu_e)$	mixed heavy and light	very light
Shortest oscillation length	$\nu_\mu \rightleftarrows \nu_e$	$\nu_\mu \rightleftarrows \nu_\tau$
Amount of mixing	Large	Probably small
Mixing of ν_e	Can be large	very small
Ratio of oscillation lengths	m_τ^2/m_μ	m_τ^2/m_μ^2

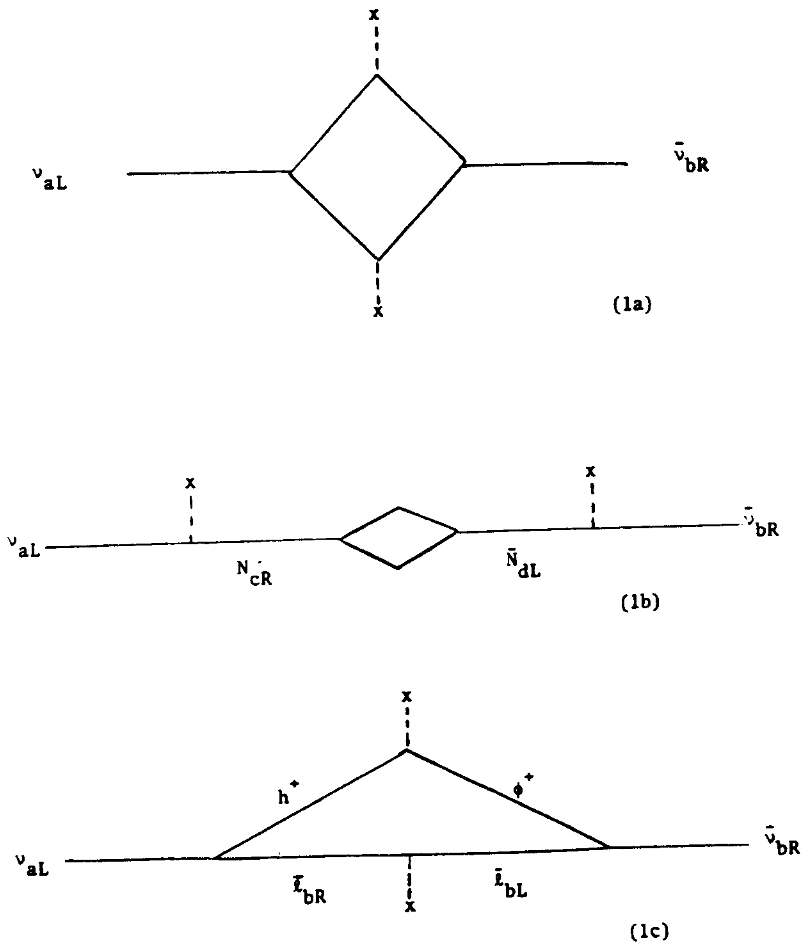

Fig. 1. Neutrino mass matrix diagrams. Dashed lines are Higgs doublets ϕ and x is a vacuum expectation value. (1a) is a general diagram with an undefined loop. In (1b) N is a heavy Majorana particle and the loop represents the Majorana mass insertion. In (1c) $\bar{\ell}$ is a charged anti-lepton and h^+ a heavy scalar boson.

Fig. 2. Comparison of the two models. The absolute mass scale is arbitrary for each model; the main point is the ratio of the masses, which is shown for typical parameters of the two models. Above each level the mixture corresponding to the level is given. c and s are arbitrary parameters such that $c^2 + s^2 = 1$. ε is probably but not necessarily small.

m(ev)	Zee SU(5)	SO(10)
100	$\frac{1}{\sqrt{2}}\{\nu_\tau \pm (c\,\nu_\mu + s\,\nu_e)\}$	$\nu_\tau + \varepsilon\,\nu_\mu$
10		
1		$\nu_\mu - \varepsilon\,\nu_\tau$
10^{-1}	$s\,\nu_\mu - c\,\nu_e$	
10^{-2}		ν_e