## DOUBLE BETA DECAY AND LTMITS on the mass of majorana neutrinos ${ }^{*} \dagger$

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## ABSTRACT

The basic theory of no-neutrino double beta decay is reviewed, and the measured lifetimes are used to set limits on the masses of Majorana neutrinos. When two neutrons are responsible for the decay, the neutrino mass must either be less than 200 ev , or greater than 4 Gev. When $N^{*}$ resonances are involved, the lower ifmit on heavy neutrinos can be raised by several orders of magnitude.

## INTRODUCTION

I am very glad that in her talk on double beta decay experiments, Professor $\mathrm{Wu}^{1}$ referred to the pre-1957 era when parity nonconservation had not been discovered. In those "good old days", the relationship between double beta decay and the neutrino was thought to be very simple: either the neutrino was a "Majorana" particle and double beta decay was dominantly a no-neutrino process,

$$
\begin{equation*}
(A, Z) \rightarrow(A, Z+2)+2 e^{-} \tag{1}
\end{equation*}
$$

or the neutrino was a "Dirac" particle and only the much slower process of two-neutrino double beta decay,

$$
\begin{equation*}
(A, Z) \rightarrow(A, Z+2)+2 e^{-}+2 \bar{v}_{e} \tag{2}
\end{equation*}
$$

could occur. With the discovery of maximal parity nonconservation in $\beta$-decay, it became apparent that, irrespective of the nature of the neutrino, the amplitude for the no-neutrino decay would alvays be strongly inhibited by the perfect, or near perfect chirality with which the neutrino field appears in the charged weak current. Consequently all double beta decays are likely to proceed at the slower rates typical of the two-neutrino process, and it is now a much more difficult problen to determine whether or not the no-neutrino process of eqs. (1) really does occur.

In this talk I would like to review the fundamental questions associated with double beta decay, and to use the present data to set limits on the parameters which control the no-neutrino amplitude. Amongst these parameters is the mass of the neutrino, and we shall derive upper limits on the mass of light neutrinos, together with lower limits on the masses of heavy ones.
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## 1. DIPAC AND MAJORANA NEUTRINOS

Let us begin by considering the distinction between Dirac and Majorana neutrinos. A neutrino $v$, represented by a field $\psi_{v}(x)$, is said to be a Dirac particle if it is distinct from its charge conjugate partner $v{ }^{c}$.

$$
\begin{equation*}
v \neq v^{c} \quad \text { (Dirac) } \tag{3}
\end{equation*}
$$

where $v^{c}$ is represented by the charge conjugate field:

$$
\begin{equation*}
\varphi_{v c}(x)=c \stackrel{\rightharpoonup}{\psi}_{v}^{*}(x) \tag{4}
\end{equation*}
$$

The word "distinct" is defined operationally": two particles are "distinct" if one of them cannot do all the things that the other can. In a similar way, two particles are "identical" if they both do the same things with the same relative probabilities.

We can apply this test to the neutrino by observing that, by definition, an anti-neutrino $v_{e}^{c}$ is emitted in the $\beta$-decay of the neutron:

$$
\begin{equation*}
n \rightarrow p+e^{-}+v_{e}^{c} \tag{5}
\end{equation*}
$$

By the usual rules of field theory; this then fmplies that a neutrino $v$ can interact with a neutron to produce an electron in the final state:

$$
\begin{equation*}
v_{e}+n \rightarrow p+e^{-} \tag{6}
\end{equation*}
$$

The question of the distinctness of neutrino and anti-neutrino then rests upon whether the $v_{e}^{c}$ produced in eq. (5) can also interact with a neutron to produce the same final state as in eq. (6):

$$
v_{e}^{c}+n^{-} p+e^{-}
$$

If it does so with the same cross-section as $v_{e}$, then it will be identical with $v_{e}$; but if it does not, or if its cross-section is very different from that of $v_{e}$, then $v^{c}$ will be distinct from $v^{c}$,

The search for the sequence of reactions (5) and ( $6^{\prime}$ ) was, in
 which he used reactor anti-neutrinos to stimulate the $C \ell \rightarrow A r^{3}$ reaction. His negative results support the notion that $v_{e}$ and $v_{e}^{c}$ are distinct particles.

A neutrino is said to be a Majorana particle if it is identical with its anti-particle:

$$
\begin{equation*}
v \equiv v^{c} \equiv v_{M} \quad \text { (Majorana) } \tag{7}
\end{equation*}
$$

This means that a Majorana neutrino must be represented by a field which is an eigenstate of charge conjugation:

$$
\begin{equation*}
\phi_{v}(x)=\frac{1}{\sqrt{2}}\left(\psi_{v}(x)+C{\underset{\psi}{v}}^{z}(x)\right) \tag{8}
\end{equation*}
$$

On an historical note, Majorana ${ }^{4}$ was originally much more interested in developing a symmetric theory of the electron and position than he was in the neutrino, and it was Racah ${ }^{5}$ who developed the full
implications of Majorana's work for the neutrino. In fact, he discussed the basic test using the sequence of eqs. (5) and ( $6^{\circ}$ ):

$$
\left.\begin{array}{l}
n \rightarrow p+e^{-}+v_{M} \\
v_{M}+n \rightarrow p+e^{-}
\end{array}\right\} \text {(Racah sequence) }
$$

which is allowed for Majorana neutrinos but not for Dirac ones
Even when the neutrino is a Majorana particle, the probability amplitude for the Racah sequence of eq. (9) can be strongly inhibited by helicity arguments. If the neutrino field always appears in the weak current in the combination $\left(1+\gamma_{5}\right) \phi_{v}$, then the neutron always prefers to create a neutrino $v_{M}$ in a fight-handed helicity state, but to absorb it in a left-handed helicity state. Consequent ly, the neutrino produced in the first stage of the Racah sequence eq. (9) is in the wrong state for maximal re-absorption by the neutron at the second stage. The probability amplitude for eq. (9) is therefore proportional to any parameters which violate the perfect helicity of the neutrino.

We can break the perfect helicity of the neutrino "explicitly" by including a component of the wrong helicity, $\left(1-\gamma_{5}\right) \phi_{V}$, in the weak current The present experimental data certainly do not exclude such a component at the 5 to $10 \%$ level. 6 Alternatively we can break it "inplicitly" by preserving the pure chirality of the neutrino field in the weak current and letting its mass serve a the helicity breaking parameter. 7 In practice, should no-neutrino double beta decay be observed, then it is likely that both of these helicity breaking mechanisms will be at work.

In sumary then, we see that in order to detect effects of Majorana neutrinos we must:
(1) break the "Dirac-ness" of the neutrino field; and
(ii) break the pure helicities of the states in which neutrinos are emitted and absorbed.
We now formulate the weak Hamiltonian with these two points in mind.

## 2. general form of the heak bamiltonian

For our purposes we need only deal with the charged-current part of weak interactions. As pointed out by Pauli ${ }^{8}$ and Pursey ${ }^{9}$ in 1957, the most general form of the weak Hamilitonian for $B$-decay is:

$$
\begin{align*}
H_{W} & =\frac{G_{F}}{\sqrt{2}} \underset{V, A, S, P, T}{ }{ }^{\Sigma}\left(\bar{\psi}_{p} r^{(\lambda)} \psi_{D}\right) \\
& \times\left(C_{\lambda}\left(\bar{\psi}_{e} r^{(\lambda)}\left(1+\delta_{\lambda} r_{S}\right) \psi_{v}\right)+D_{\lambda}\left(\bar{\psi}_{e} r^{(\lambda)}\left(1+n_{\lambda} r_{S}\right) \phi_{V} c\right)\right. \tag{10}
\end{align*}
$$

The presence of the $D_{\lambda}$ terms in addition to the $C_{\text {, }}$ ones allows for the possibility that the neutrino might be, at least in part, a Majorana paricie: and the $\delta_{\text {, and }} n_{\text {, coefficients allow for admix- }}$


It was shown by Enz ${ }^{10}$ that the probability for the Racah sequence eq. (9) depends upon certain combinations of coupling constants which are independent of neutrino mass, namely:

$$
\begin{align*}
& I_{\lambda \mu}=C_{\mu} D_{\lambda}\left(1-\delta_{\mu} n_{\lambda}\right)+C_{\lambda} D_{\mu}\left(1-\delta_{\lambda} n_{\mu}\right)  \tag{11}\\
& J_{\lambda \mu}=C_{\mu} D_{\lambda}\left(\delta_{\mu}-n_{\lambda}\right)-C_{\lambda} D_{\mu}\left(\delta_{\lambda}-n_{\mu}\right)
\end{align*}
$$

and upon others which are proportional to the neutrino mass:

$$
\begin{align*}
& \left.m_{v} I_{\lambda \mu}^{\prime}=m_{v} c_{\mu} D_{\lambda}\left(1+\delta_{\mu} n_{\lambda}\right)+c_{\lambda} D_{\mu}\left(1+\delta_{\lambda} n_{v}\right)\right\}  \tag{12}\\
& m_{v} J_{\lambda \mu}^{\prime}=m_{v}\left\{c_{\mu} D_{\lambda}\left(\delta_{\mu}+n_{\lambda}\right)+c_{\lambda} D_{\mu}\left(\delta_{\lambda}+n_{\mu}\right)\right\}
\end{align*}
$$

We shall consider two simple "Majorana" special cases of these general formulae. In the first, we break helicity explicitly in the weak current, and we neglect the neutrino mass:

Case 1

$$
\left.\begin{array}{rl}
C_{x} & =D_{x}=\frac{1}{\sqrt{2}} F_{x}(1+n) \\
\delta_{x} & =\eta_{x}=\frac{(1-n)}{(1+n)}  \tag{13}\\
m_{V}=0
\end{array}\right\} \quad(x=\nabla, A)
$$

The amplitudes in eq. (12) vanish and those in eq. (11) then become proportional to $\eta$. In the second special case, we break helicity implicitly through a non-zero neutrino mass:

Case 2


The non-zero amplitudes are now those of eq. (12), and they are all proportional to $\mathrm{m}_{v}$.
3. SEARCHES FOR THE RACAH SEQUENCE

We can search for the Racah sequence (eq. 9) using real neutrinos or virtual ones. The search using real neutrinos was based upon two suggestions by Pontecorvo ${ }^{\text {(1) }}$ first that reactors would be a copious source of neutrinos from $B^{-}$decay; and second that the reaction,

$$
\begin{equation*}
\text { "reactor neutrino" }+\mathrm{Cl}^{37} \rightarrow \mathrm{Ar}^{37}+\mathrm{e}^{-} \tag{15}
\end{equation*}
$$

would be a good means for detecting these neutrinos.
Davis ${ }^{3}$ developed radiochemical methods for observing the $\mathrm{Ar}^{37}$ produced by the reaction of eq. (15) in a tank of chlorine, and he carried out the experiment in 1955. His null result suggested that carried out the experiment in $19{ }^{c}$. rather than a Majorana $w_{M}$; howthe reactor neutrino is a Dirac $v_{e}^{c}$ rather than a Majorana w; however, if we assume the reactor neutrino is a Majorana particie, and atribute this null result obtained by Davis to a helicity suppress then the limits set upon the parameter $n$ of tight:

$$
\begin{equation*}
|n| \leq 0.05-0.1 \tag{16}
\end{equation*}
$$

Since measurements of the longitudinal polarization of $\beta$-rays do not field limits on $\eta$ more severe than that of eq. (16), we must turn to virtual neutrinos if we want to do better. The Racah sequence makes it possible for two neutrons inside a nucleus to exchange a virtual neutrino between them, and thereby to undergo a


## Fig. 1: Neutrino Exchange

electrons as in Fig. (1). In the language of noncovariant perturbation theory, the virtual neutrino in Fig. (1) has a much higher average energy ( $(\mathbb{E}\rangle \neq 35 \mathrm{Mev}$ ) than the real neutrino in the reactor experiment ( $(E)=1-2 \mathrm{Mev}$ ), and it is this feature which enables us to set a sharper limit on the parameter $\eta$.

The lifetime for no-neutrino double beta decay was first calcuated by Fury ${ }^{12}$ (1939) and Touschek ${ }^{13}$ (1949), and subsequently by Primakoff ${ }^{14}$ (1952), Konopinski 15 (1955), and Primakoff and Rosen ${ }^{2}, 16$ (1959, 65, and 69). As far as general orders of magnitude are concerned, it turns out to be

$$
\begin{equation*}
T_{1 / 2}(O v)=10^{14}|n|^{-2} \text { years } \tag{17}
\end{equation*}
$$

By contrast, the iffetime for two-neutrino double beta decay (eq. 2), which will always occur no matter whether the neutrino is a Majorana or Dirac particle, is of order

$$
\begin{equation*}
T_{1 / 2}(2 v)=10^{20} \text { years } \tag{18}
\end{equation*}
$$

A comparison of eqs. (17) and (18) Indicates that we should be able to set limits on $|n|$ of order $10^{-3}$. In fact we can do better than this, and, as will be seen below, we can set limits in the range:
$|n| \leq 5 \times 10^{-5}-10^{-4}$
Before examining these limits, however, we must make two notes about the above discussion.

Note (1): the limit in eq. (19) is really a limit on the product $n \xi$ where $\xi \equiv\left(D_{x} / C_{x}\right)$ measures the "amount" of Majorana neutrino in the weak current. We have assumed the maximal case $\xi=1$.
Note (2): the no-neutrino decay has been treated as a secondorder weak process, but as suggested by Pontecorvo ${ }^{17}$ in 1968, it could be engendered by a $\Delta Q=2$ companion of the $\Delta S=2$ superweak interaction invented by Wolfenstein ${ }^{18}$ to describe CP violation.

## 4. LIFETIME ESTIMATES FOR NO-NEUTRINO BB DECAY

From the systematics of nuclear masses and energy levels, the most favorable candidates for double beta decay are to be found amongst even-even nuclei. In many cases, single beta decay to the adjacent odd-odd nuclei is either forbidden by energy conservation, or very strongly inhibited by large spin changes; double beta decay to the ground state of the next even-even nucleus then becomes the most probable transition for the parent nucleus. It follows that the spin and parity selection rules correspond to

$$
\begin{equation*}
0^{+} \rightarrow 0^{+} \tag{20}
\end{equation*}
$$

transitions between parent and daughter nucifi. Among the transitions that have been studied experimentally ${ }^{19}$ are:
$\left.\begin{array}{lll}\mathrm{Te}^{128,130} & \rightarrow & \mathrm{Xe}^{128,130} \\ \mathrm{Se}^{82} & \rightarrow & \mathrm{Kr}^{82} \\ \mathrm{Ge}^{76} & \rightarrow & \mathrm{Se}^{76} \\ \mathrm{Ca}^{48} & \rightarrow & \mathrm{Ti}^{48}\end{array}\right\}$

As observed in preceding sections, two constituents of the parent nucleus must take part in the process. This observation, which follows from the fact that nucleons have isospin $T=1 / 2$ and cannot change their electric charge by two units, is very useful because the average separation ( $\overrightarrow{\mathbf{r}}_{\mathrm{r}}$ ) between the constituents provides a natural cut-off for an mitherwise divergent matrix element. If the nucleon had had an isospin $T=1$, 11ke the pion, then it would have been possible for a single constituent to emit two elecmon (he matrix element for double beta decay would have been highly divergent. Fortunately this is not the case.

It has been general practice ${ }^{2,12-16}$ to calculate the matrix element for no-neutrino double beta decay using non-covariant perturbation theory:

$$
\begin{equation*}
A\left(1+f e_{1}^{-} e_{2}^{-}\right)=\sum_{m, v}\left\{\frac{\left\langle f ; e_{1}^{-} e_{2}^{-}\right| H_{w}\left|m ; e_{1}^{-v}\right\rangle\left\langle m ; e_{1}^{-} v\right| H_{w}|1\rangle}{E_{w}-E_{i}+E_{1}+E_{v}}-(1 \leftrightarrow 2)\right\} \tag{22}
\end{equation*}
$$

where $1, m, f$ denote the initial, intermediate, and final nuclear states respectively, and the summation $o v e r m$ and $v$ covers all the intermediate nuclear states and all energies of the virtual, intermediate neutrino.

To estimate the magnitude of this matrix element, the nuclear energy difference ( $E_{m}-E_{1}$ ) in the denominator of eq. (22) is replaced by an average ${ }^{m}$ value $\left\langle E_{m i}\right\rangle$, and the sum over $m$ is done by
 $\left|\vec{r}_{\ell}-\mathbf{r}_{j}\right|^{-1}$ involving the separation between constituents, and it is replaced by an average value $R^{-1}$, where $R$ is the nuclear radius:

$$
\begin{equation*}
R=1.2 A^{1 / 3} \quad \text { fermi } \tag{23}
\end{equation*}
$$

In the "allowed" approximation, therg are two matrix elements for $0^{+} \rightarrow 0^{+}$double beta decay transitions". One is a "Fermi"-type matrix element

$$
\begin{equation*}
\langle f| F|i\rangle \equiv\left\langle\left. f\right|_{\ell, j} ^{\Sigma_{j}} \tau_{\ell}^{+} \tau_{j}^{+} \mid i\right\rangle \tag{24}
\end{equation*}
$$

and the other a "Gamow-Teller" one,

$$
\begin{equation*}
\langle f| G T|i\rangle \equiv\left\langle\left. f\right|_{\ell, j} \tau_{\ell}^{+} \tau_{j}^{+} \vec{\sigma}_{\ell} \cdot \vec{\sigma}_{j} \mid i\right\rangle \tag{25}
\end{equation*}
$$

Where $\tau^{+}$is the isospin raising operator for the $\ell^{\text {th }}$ nucleon, and $\vec{a}$ is its spin operator. The operator $F$ is actually the square of the total isospin raising operator,

$$
\begin{equation*}
\mathrm{T}^{+}=\Sigma_{\ell} \tau_{\ell}^{+} \tag{26}
\end{equation*}
$$

and it can only connect states with the same isospin. Therefore, in the approximation that isospin is a good quantum number and the initial and final nuclear states have different total isospins,

$$
\begin{equation*}
T_{i} \nLeftarrow T_{f} \tag{27}
\end{equation*}
$$

the matrix element of $F$ must vanish. Consequently we expect the Gamow-Teller matrix element to dominate the decay rate.

Following Primakoff and Rosen ${ }^{16}$ (1969), we can now estimate the 1ifetime of the nucleus $(A, 2)$ in terms of the $n$-parameter:
$\left.T_{1 / 2}=\frac{10^{20}}{|n|^{2} f\left(\varepsilon_{0}\right)}\left[\frac{1-e^{-2 \pi a Z}}{2 \pi a Z}\right]^{2} \quad\left(\frac{A}{130}\right)^{2 / 3}|\langle f| G T| i\right\rangle\left.\right|^{-2}$ years
where $\varepsilon_{0}$ is the energy release in units of $m_{e} c^{2}$, and

$$
\begin{equation*}
f\left(\varepsilon_{0}\right)=\varepsilon_{0}^{4}\left(\varepsilon_{0}^{3}+13 \varepsilon_{0}^{2}+77 \varepsilon_{0}+70\right) \tag{29}
\end{equation*}
$$

We now use this formula to obtain limits on $n$.

## 5. LIMITS ON $n$

 measured by geochemical means and is found to be 10 . cannot tell directly whether the two electrons take up all of the energy released $\left(\varepsilon_{0}=5\right)$, or whether they share it with two neutrinos. We can set an upper bound on the product of the parameter $\eta$ and the nuclear matrix element by assuming that the entire lifetime is due to no-neutrino double beta decay; we then find that

$$
n \mid\langle f| G T|1|\rangle \leq 10^{-4}
$$

To extract $n$ from this equation, we need the value of the nuclear matrix element. In their orginal work, Primakoff and Rosen ${ }^{16}$ (1969) did not attempt to calculate $\langle f| G T|i\rangle$, but rather made an "educated guess" that it would fall somewhere between an allowed matrix element and a first forbidden one. For single beta decay, the corresponding range for a Gamow-Teller matrix element <m $\tau_{\ell} \tau_{q}|1\rangle$ runs from 0 to 1 to for double beta decay, the appropriate range for $\langle f| G T|1\rangle$, which is roughly the square of a single beta decay matrix element, is 0.01 to 1 . Accordingly, Primakoff and Rosen took a central value of 0.1 , but made generous allowance for errors on either side:

$$
\begin{equation*}
\langle f| G T|i\rangle=0.1 \quad\left\{\times 10^{ \pm 1}\right\} \tag{31}
\end{equation*}
$$

Inserting the central value in eq. (30), we have

$$
\begin{equation*}
|n| \leq 10^{-3} \tag{32}
\end{equation*}
$$

Stephenson and Haxton ${ }^{21}$ have recently done a full-fledged calculation of the nuclear matrix element, and they find that Henry and I were much too conservative in our "educated guess". Their work, which I regard as the most significant theoretical advance in this field for many years, indicates that the matrix element has a value much closer to 1.5 than to 0.1 , and hence it implies that $n$ is an order of magnitude smaller than the limit in eq. (2). (See the talk by Dr. Stephenson in this session for details).

Support for this result comes from Professor Wu's experiment ${ }^{22}$ on the transition $\mathrm{Se}^{82} \rightarrow \mathrm{Kr}^{82}\left(\varepsilon_{0}=5.9\right)$. From her own lower limit of $10^{21.49}$ years on the no-neutiono mode, and the geochemical lifetime ${ }^{23}$ of $10^{20.42 \pm 0.14}$ years, she concludes that:

$$
\begin{equation*}
|n| \leq 3 \times 10^{-4} \tag{33}
\end{equation*}
$$

If, instead of the geochemical lifetime, we use the much shorter Iffetime of $10^{19}$ years recently measured by Moe and Lowenthal ${ }^{24}$ in a
laboratory experiment, we find an even lower limit:

$$
\begin{equation*}
|n| \leq 6 \times 10^{-5} \tag{34}
\end{equation*}
$$

This value for $\eta$ agrees with one obtained by Bryman and
 $\mathrm{Te} \mathrm{e}^{130}$ and $\mathrm{Te}^{128}$ If the nuclear matrix elements for $\mathrm{Te}^{+10}+\mathrm{Xe}^{\mathrm{C}}$
and $\mathrm{Te}^{128}+\mathrm{Xe}^{128}$ are assumed to be equal then the ratio of their and $\mathrm{Te}^{128}+\mathrm{Xe}^{12}$ are assumed is controlled by phase space ${ }^{16,17 \text {. For pure no-neutrino }}$ decay, the phase space behaves roughly like ( $\varepsilon_{0}$ ) ${ }^{5}$ and the $\mathrm{Te}^{128}$ lifetime would be $(5.0 / 1.7)^{5}=200$ times longer than that for $\mathrm{Te}^{130}$; for pure two-neutrino decay, phase space behaves roughly like $\left(\varepsilon_{0}\right)^{8}$, and the $\mathrm{Te}^{128}$ 11fetime would be $(5 / 1.7)^{8}=6 \times 10^{3}$ times that for $\mathrm{Te}^{130}$. In actual fact, the ratio falls between these two extremes, and so Bryman and Picciotto argue that the decay must be a combination of the two-neutrino and no-neutrino modes corresponding to a value for $n$ of ${ }^{19}$.

$$
\begin{equation*}
|n|=(4.3 \pm 0.1) \times 10^{-5} \tag{35}
\end{equation*}
$$

Further discussion of the matrix elements for both tellurium isotopes can be found in the work of Stephenson and Eaxton. 21
6. NEUTRINO MASS AS helicity-vIOLATING Parameter

Let us now consider the case in which the mass of the Majorana neutrino serves as the helicity violating parameter., The noneutrino double beta decay amplitude will then be proportional to the factor:

$$
\begin{equation*}
\mathbf{r}=\frac{m_{v} p_{v}}{\left(m_{v}^{2}+p_{v}^{2}\right)} \tag{36}
\end{equation*}
$$

The mass appearing in the numerator comes directly from the breakdown of the helicity rule, and the denominator comes from the propagator of the neutrino field.

There are two cases in which the parameter $r$ will be small. In one the neutrino mass is very much smaller than its average momentum, and r is given by

$$
\begin{equation*}
r=\frac{m_{v}}{\left\langle P_{v}\right\rangle}\left(m_{v} \ll\left\langle P_{v}\right\rangle\right) \tag{37}
\end{equation*}
$$

In the other, the mass is very much larger than the mean neutrino momentum, and

$$
\begin{equation*}
r=\frac{\left\langle p_{\nu}\right\rangle}{m_{v}} \quad\left(m_{v} \gg\left\langle p_{v}\right\rangle\right) \tag{38}
\end{equation*}
$$

The first case will give rise to an upper limit on the mass of light neutrinos and the second case will set a lower limit on the mass of heavy neutrinos. 25

To determine these limits, we work out the equivalence between the wass parametrization of the no-neutrino double beta decay amplitude and the parametrization in terms of $n$, and then we translate the limits for $n$ into constraints on the neutrino mass. The
equivalence comes from the relation ${ }^{25}$

$$
\begin{equation*}
m_{v}\left\langle\frac{e^{-m_{v} r_{\ell j}}}{r_{\ell j}}\right\rangle=n\left\{\left|\vec{p}_{1}-\vec{p}_{2}\right|\right\}\left\langle\frac{1}{r_{\ell j}}\right\rangle \tag{39}
\end{equation*}
$$

where the Yukawa-like factor on the left-hand side arises from the exexchange of the massive neutrino between two neutrons in the nucleus. For light neutrinos whose masses are consistent with $m_{v}\left\langle r_{\ell j}\right\rangle \ll 1$, we readily find from eq. (39), that:

$$
\begin{equation*}
m_{v}=n\left\{\left|\vec{p}_{1}-\vec{p}_{2}\right|\right\}=2 n \times 10^{6} \mathrm{ev} \tag{40}
\end{equation*}
$$

where we have assumed that the average momentum difference between the two emitted electrons is 2 Mev . For heavy neutrinos, whose masses are such that $m_{j}\left\langle r_{\ell j}\right\rangle \gg 1$, we must remember that the hard-core of the nucleon-nucleon potential always keeps the two neutrons at least a distance $r$ apart; we therefore evaluate the expectation
 minimum value of $r_{c}$, which ve take to be $\left(3 m_{\pi}\right)^{-1}$, and the nuclear diameter $2 R=2.4\left(m_{\pi}\right)^{-1} A^{1 / 3}$. We then obtain an exponential relationship 25 between $m_{v}^{\pi}$ and $\eta$ :

$$
\begin{equation*}
e^{-m_{v} r_{c}}=0.6 n A^{2 / 3} \quad\left(r_{c}=1 / 3 m_{\pi}\right) \tag{41}
\end{equation*}
$$

If $n<10^{-4}$, then either (eq. 40):

$$
\begin{equation*}
m_{v}<200 \mathrm{ev} \tag{42a}
\end{equation*}
$$

or (eq. 41)

$$
\begin{equation*}
m_{v}>4 \mathrm{Gev} \tag{42b}
\end{equation*}
$$

## 7. the $\mathrm{N}^{\star}$ mechanism

The lower limit on the mass of heavy Majorana neutrinos in eq. (42b) can be raised considerably if even a small part of the nuclear wavefunction includes the $N^{*}(1238)$ resonance. 16,25 Because this resonance has isospin $T=3 / 2$, it can undergo no-neutrino double beta decay through the two-quark process

$$
\begin{equation*}
2 d+2 u+2 e^{-} \tag{43}
\end{equation*}
$$

The average separation between these constituents is now a $=0.5$ Fermi $=\left(\frac{1}{2}\right)$ instead of a nuclear radfus; and, more importantly, there is $\frac{1}{2 m}{ }^{2}$ hard-core between them if modern dieas on asymptotic freedom are to be accepted. Consequently the equivalence relation between the mass- and $n$-paramitifations is no longer exponential in form; instead it is given by

$$
\begin{equation*}
P\left(N^{*}\right) m_{v}\left[\frac{2}{a\left(m_{v} a+2\right)^{2}}\right]=n\left\{\left.\right|_{P_{1}}-\vec{P}_{2} \mid\right\} \cdot \frac{m_{\pi}}{70 A^{1 / 3}} \tag{44}
\end{equation*}
$$

where $P\left(N^{*}\right)$ is the probability of finding the $N^{*}$ in the nucleus, and $a=\left(1 / 2 \mathrm{~m}_{\pi}\right)$.

For low mass neutrinos, this relation becomes

$$
\begin{equation*}
\mathrm{m}_{v}=2 \times 10^{5} \frac{\eta}{P\left(N^{*}\right)} \mathrm{ev} \tag{45a}
\end{equation*}
$$

and for high mass neutrinos, it gives

$$
\begin{equation*}
m_{v}=1.5 \times 10^{3} \frac{\mathrm{P}\left(\mathrm{~N}^{\star}\right)}{n} \mathrm{Gev} \tag{45b}
\end{equation*}
$$

Therefore if we assume that $P\left(N^{*}\right)=10^{-2}$ and $|n|<10^{-4}$, we find that either (eq. 45a)

$$
m_{v}<2 \mathrm{kev}
$$

or (eq. 45b)

$$
\begin{equation*}
m_{v}>1.5 \times 10^{5} \mathrm{Gev} \tag{46b}
\end{equation*}
$$

In the low mass regime, the difference between the $N^{*}$-mechanism limit in eq. (46a) and the two-nucleon one in eq. (42a) probably reflects a difference in the details of the two mechanisms. In the high mass regime, however, the difference between the relatively small 1 imit from the two-nucleon mechanism in eq. ( 42 b ) and the very large one in eq. (46b) comes about because the relationship between
m and $n$ changes from an exponential one in eq. (41) to a polynomial one in eqs. ( 44 and 45 b ). This in turn results from the presence of a hard-core in one case, and its presumed absence in the other. While I might be a little cautious in taking the number in eq. (46b) too literally, I think that a lower limit in the region $10^{3}-10^{4}$ Gev is probably quite conservative.

## 8. CONCLUSIONS

The lesson I would like to draw from is discussion is that there are fundamental reasons for continuing the experimental search for double beta decay. The actual observation of the no-neutrino mode would demonstrate unequivocally that lepton number is not conserved, and the magnitude of the appropriate symmetry violating parameter would then impose serious constraints upon the spectrum of neutral lepton states. These constraints, in their turn, would be extremely important in building grand unified models.

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26. In reference 25 , the transition $n \rightarrow N^{+* *}(1238)+2 e^{-}$was studied, and the equivalence relation was proportional to $\mathrm{P}^{1 / 2}\left(\mathbb{N}^{*}\right)$. G. Stephenson (private communication) has emphasized that the $n \rightarrow N^{*}$ transition, involving a nucleon spin change $1 / 2 \rightarrow 3 / 2$, cannot be embedded in an overall $0^{+}+0^{+}$nuclear transition. We must therefore use a transition of the type $\mathrm{N}^{\star \circ} \rightarrow \mathrm{N}^{\star++}+2 \mathrm{e}^{-}$, and the equivalence relation of eq. (26) is therefore proportional to $\mathrm{P}\left(\mathrm{N}^{*}\right)$.
