19. G. Marx and A. Szalay, Proc. of Int. Neutrino Conf. (1973).

20. J. Gunn, B. Lee, I. Lerche, D.N. Schramm and G. Steigman, Ap. J. 223, 1015 (1978).

21. G. Steigman, Ann. Rev. Nucl. Part. Sci. 29, 313 (1979).

22. Tremaine and J.E. Gunn, Phys. Rev. Lett. 42, 407 (1979).

WHAT LIMITS (IF ANY) DOES BIG BANG NUCLEOSYNTHESIS PLACE ON THE NUMBER OF NEUTRINO FLAVORS?

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ABSTRACT

The mass fraction of ^{4}He synthesized in the big bang, Y_p, depends upon the neutron halflife, $\tau_{1/2}$, the baryon-to-photon ratio, η, and the number of 2-component neutrino species, N_ν. New observational and experimental data have led to a re-examination of the constraints on particle physics and cosmology which follow from primordial nucleosynthesis. If baryons provide most of the mass which binds binary and small groups of galaxies, then N_ν must be $\lesssim 4$. However, if massive neutrinos (or other non-baryonic matter) provide this mass, then at present no firm limit can be placed on N_ν. In addition we find that η must lie in the range $10^{-9.9\pm1}$, implying that baryons alone cannot close the universe; the related baryon-to-specific entropy ratio must lie in the range $10^{-10.8\pm1}$. If the universe is dominated by non-baryonic matter, then there is no contradiction between the predictions of primordial nucleosynthesis and the observations of ^{4}He provided that $Y_p \gtrsim 0.15$.

I. INTRODUCTION

There is an impressive body of evidence which supports the hot big bang theory. This evidence includes: (*i*) the expansion of the universe discovered by Hubble and others, (*ii*) the 3K cosmic microwave background discovered by Penzias and Wilson, (*iii*) the singularity theorems of Hawking and Penrose, (*iv*) the abundance of ^{4}He and several other light elements which were produced $\sim$ 3 min after the big bang, and perhaps, (*v*) the presence of only matter in the universe rather than equal amounts of matter and antimatter. We have reason to believe that this is a result of a slight excess of baryons over antibaryons having evolved during the epoch of baryosynthesis ($t \sim 10^{-35}$ sec), and later when all the antibaryons and most of the baryons annihilated ($t \sim 10^{-6}$ sec) the $\sim$ 1 baryon per 10^{10} photons we see today was left due to this excess.[2] The time-temperature relation in the standard hot big bang model (Friedman-Robertson-Walker cosmology) when the energy density of the universe is dominated by relativistic particles ($t \lesssim 10^{12}$ sec) is

$$t = 2.42 \times 10^{-7}\ \text{sec}\ (100/N)^{1/2}\ T_{GeV}^{-2} \tag{1}$$

where N is the sum of the statistical weights of all the particle species present [$\equiv \sum_{bosons} g_i + (7/8) \sum_{fermions} g_i$] and T is the temperature measured in GeV (1.16×10^{13} K = 1 GeV). From (1) it is clear that at early times particle energies were very high. At the planck time ($t \sim 10^{-43}$ sec) particle energies were as large as

$\sim 10^{19}$ GeV (earlier than the planck time quantum corrections to general relativity may become important). As it has often been said, "The early universe is the ultimate high energy physics laboratory" - unfortunately it ceased operating $\sim$ 15 billion years ago. In order to take advantage of this "marvelous machine" we must search for fossils or relics which remain from the earliest epochs.

Perhaps the most useful relic is the large abundance of ^{4}He ($\sim$ 25% by mass) observed in the universe. Although stars can (and we believe did) produce the heavier elements we see today ($\sim$ 2% by mass), the types of stars present today could not have produced this much ^{4}He. In regions where stars have processed primordial material, there is good evidence that their contribution to the ^{4}He mass fraction is $\lesssim$ 6% (ref. 1). In addition, it is not possible for stars to destroy a substantial amount of ^{4}He without grossly over producing heavier elements. Thus, a strong case can be made that most of the ^{4}He present today is a relic of a much earlier epoch. The very detailed calculations of Peebles[3] and Wagoner, Fowler, and Hoyle[4] showed that this amount of ^{4}He (and smaller amounts of ^{2}He, ^{3}He, ^{6}Li and ^{7}Li) could have been produced during an early epoch ($t \sim$ 0.01 - 200 sec) when the conditions in the universe were just right for nucleosynthesis. The striking agreement between their calculations and the observations makes big bang nucleosynthesis one of the crowning jewels of the big bang theory.

Because of the excellent concordance, the abundance of ^{4}He can be used as a probe of the early universe ($t \sim$ 0.01 - 200 sec, $T \sim$ 10 MeV - 0.1 MeV). Since much smaller amounts of ^{2}H, ^{3}He, ^{6}Li and ^{7}Li were produced, and these elements are more easily produced (with the exception of ^{2}H) and destroyed during the course of stellar and galactic evolution, they are of less use in probing the early universe. Most of you are probably familiar with the use of ^{4}He to place a limit on the number of neutrino flavors.[5] However, the question that particle physicists always have in the back of their minds is, just how reliable are these cosmological limits? I want to bring this question out of the closet and into the open, and answer it by first reviewing big bang nucleosynthesis and the assumptions involved, and then discussing how the limits on N_ν follow. I will pay particular attention to the possibility that neutrinos may have non-zero rest masses.

Let me again emphasize that the nucleosynthesis calculations have been done with great care and in complete detail; however, here I will just briefly discuss the highlights. The two basic underlying assumptions are: (*i*) the validity of the Friedman-Robertson-Walker model, and (*ii*) the lepton numbers-to-photon number, L_i (i = e, μ, τ...), are small ($\lesssim 10^{-2}$). The isotropy of the 3K background and of the Hubble flow, and the distribution of radio sources strongly support the assumption that the universe is isotropic and homogeneous and can be described by the F-R-W metric. Because of the charge neutrality of the universe, any significant lepton numbers must reside in the cosmic background sea of neutrinos - which are undetectable. However, the baryon number-to-photon number is very small ($\lesssim 10^{-9}$), making it very plausible that the L_i are also of this order. Also, it has been argued that if the ideas of grandunification are correct, the L_i must be of order $\lesssim 10^{-9}$ (ref. 6).

The mass fraction of ^{4}He synthesized, Y_p, depends upon three quantities: the neutron halflife, $\tau_{1/2}$, the baryon-to-photon ratio, $\eta = 10^{-10\pm1}$, and N_ν, the number of light ($m \lesssim 1$ MeV), 2-component neutrino species [I will use baryon to mean proton or neutron and not any of the other exotic baryons which have been postulated]. Negligible nucleosynthesis beyond ^{4}He occurs because of the lack of stable nuclei with A = 5 or 8 and the coulomb barriers which become insurmountable as the universe cools. Because of the large binding energy of ^{4}He essentially all of the neutrons present when nucleosynthesis takes place ($T_N \simeq 10^9$ K) end up in ^{4}He. For these reasons nucleosynthesis is most easily described in terms of the n/p ratio. As the temperature of the universe decreases, this ratio "attempts to track" its equilibrium value which is determined by the Saha equation,

$$n/p = \exp(-\Delta mc^2/T) = \exp(-1.29\ \text{MeV}/T). \quad (2)$$

[Note that if L_e were $\gtrsim 10^{-2}$ (2) would have to be modified; for $L_e > 0$ (< 0) n/p would be smaller (larger) for a given temperature].

The reactions which allow neutrons and protons to transform into one another are the usual β-reactions,

$$n + e^+ \leftrightarrow p + \bar{\nu}_e,\ n + \nu_e \leftrightarrow p + e^-, \quad (3a)$$

$$n \leftrightarrow p + e^- + \bar{\nu}_e. \quad (3b)$$

The rate (per nucleon) for reactions (3a) $\Gamma_\beta \propto T^5 \tau_{1/2}^{-1}$. [Note: the same matrix element determines the rates for 3a, b, c]. As long as Γ_β is $\gtrsim \dot{T}/T \sim t^{-1} \sim N^{1/2}T^2$, the n/p ratio can adjust to the changing temperature of the universe and track its equilibrium value [Note: for the times of interest neutron decay is not important, i.e., $t \ll \tau_{1/2}$]. Now let us discuss the evolution of this ratio (summarized in figure 1 below).

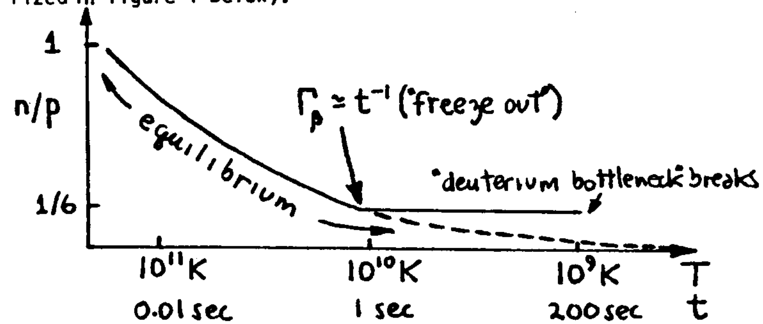


FIGURE 1

Frame 1: $T \simeq 10^{11}$ K $\simeq$ 8.6 MeV, $t \simeq 0.01$ sec. The energy density of the universe is dominated by relativistic particles: photons, $e^{\pm}$ pairs, and $\nu\bar{\nu}$ pairs [$N = 5.5 + (7/4)N_\nu$]. All particle species are interacting rapidly, $\Gamma > \dot{T}/T$, so thermal distributions are maintained for all species. In particular, Γ_β is $\gg \dot{T}/T$, so that n/p is given by (2), and n/p = exp(-1.29/8.6) $\simeq$ 0.86. Nucleosynthesis is not proceeding yet because of the "deuterium bottleneck". The weak binding of the deuteron (2.22 MeV) and the large photon-to-baryon ratio, $\eta^{-1} \sim 10^{10}$, conspire to keep the abundance of deuterium very small. The abundance of ^{2}H is determined by the Saha equation,

$$n_n\, n_p = n_d\, n_\gamma \exp(-2.22\ \text{MeV}/T), \qquad (4a)$$

$$n_d/n_b = \eta \exp(2.22\ \text{MeV}/T), \qquad (4b)$$

where n_n, n_p, n_d, n_b and n_γ are the neutron, proton, deuteron, baryon, and photon number densities and in deriving (4b) I have assumed that $n_n \simeq n_p \simeq n_b$. For $T \simeq 10^{11}$ K, $n_d/n_b \simeq 1.3\eta \simeq 10^{-10}$.

Frame 2: $T \simeq 8.4 \times 10^9$ K $\simeq$ 0.72 MeV, $t \simeq 1$ sec. The energy density is dominated by the same relativistic species; however, the neutrino species have decoupled (rate of interaction < $\dot{T}/T$) and from this time forward they freely expand with their temperature being reshifted $\propto R(t)^{-1}$ (R $\sim$ scale factor of the universe). At this temperature the rate of reactions (3a), Γ_β, is about equal to $\dot{T}/T$; since $\Gamma_\beta \propto T^5 \tau_{1/2}^{-1}$ and $\dot{T}/T \propto N^{1/2} T^2$ from this point forward Γ_β will be < $\dot{T}/T$. The β-reactions effectively cease to occur and the n/p ratio "freezes out" at its current value,

$$n/p = \exp(-1.29\ \text{MeV}/T_f) \simeq \exp(-1.29/0.72) \simeq 1/6, \qquad (5)$$

where T_f is the "freeze out" temperature and

$$T_f \propto N^{1/6}\, \tau_{1/2}^{1/3}. \qquad (6)$$

Nucleosynthesis is still prevented by the "deuterium bottleneck", $n_d/n_b \simeq \eta \exp(2.22/0.72) \simeq 20\eta \simeq 10^{-9}$.

Frame 3: $T \simeq 10^9$ K $\simeq$ 0.086 MeV, $t \simeq 200$ sec. The energy density is dominated by photons and the neutrino pairs; the $e^{\pm}$ pairs annihilated when T was $\sim 3 \times 10^9$ K, heating the photons relative to the neutrinos, so that $T_\nu \simeq (4/11)^{1/3}\, T_\gamma$. The effective value of N is $2 + (7/4) N_\nu\, (4/11)^{4/3}$. At this temperature the "deuterium bottleneck" breaks and essentially all of the neutrons are bound into ^{4}He; the temperature T_N is roughly determined by $n_d/n_b \simeq 1 \simeq \eta \exp(2.22\ \text{MeV}/T_N)$, and

$$T_N \simeq 10^9\ \text{K}\ [1 + 0.04 \ln(\eta/10^{-10})]. \qquad (7)$$

The n/p ratio has decreased slightly due to occasional neutron decays to $\simeq$ 1/7. The mass fraction of ^{4}He synthesized is

$$Y_p \simeq 2(n/p)/[1 + (n/p)], \qquad (8)$$

which for n/p $\simeq$ 1/7 is $\simeq$ 0.25.

How does Y_p depend upon the parameters $\tau_{1/2}$, N_ν, and η? An increase in the value of $\tau_{1/2}$ raises the "freeze out" temperature, $T_f \propto \tau_{1/2}^{1/3} N^{1/6}$, and hence the value of (n/p) at "freeze out". In addition, the number of neutrons which decay between "freeze out" and nucleosynthesis, which is $\propto \exp(t_N/\tau_{1/2})$ (note: $t_N - t_f \simeq t_N$), is smaller. Both effects result in a higher n/p at nucleosynthesis and therefore more ^{4}He. Increasing N_ν increases N, and thereby raises T_f ($\propto \tau_{1/2}^{1/3} N^{1/6}$) and n/p at "freeze out". Increasing N_ν changes the time-temperature relation, equation (1), so that the time between "freeze out" and nucleosynthesis, $t_N - t_f \simeq t_N$, decreases ($t \propto N^{-1/2} T^{-2}$), and fewer neutrons decay between "freeze out" and nucleosynthesis. Again, both effects raise the value of n/p and thereby increase Y_p. Finally, η affects nucleosynthesis by its influence on when the "deuterium bottleneck" breaks, cf. equation (7). A larger value of η (fewer photons per baryon) allows nucleosynthesis to commence earlier, when fewer neutrons have decayed, so that n/p is larger, and more ^{4}He is synthesized. Y_p does not depend sensitively on η, which is fortunate as we shall see. In summary, Y_p increases with increasing $\tau_{1/2}$, N_ν, and η.

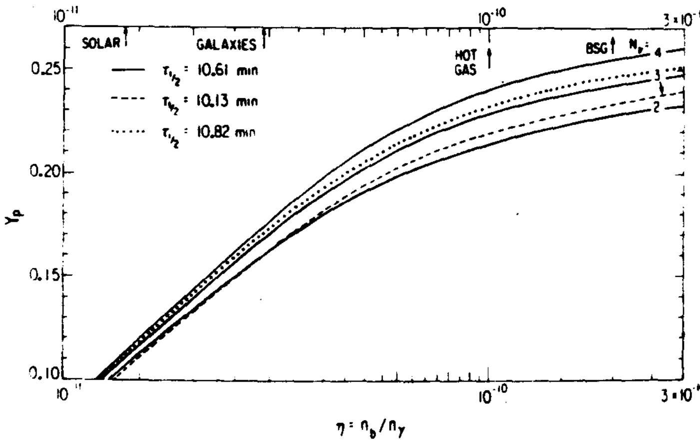


FIGURE 2 - Y_p as a function of η for $\tau_{1/2}$ = 10.13, 10.82 (N_ν = 3) and 10.61 min (N_ν = 2, 3, 4). The various lower bounds on η are indicated by arrows.

In principle, Y_p also depends upon all the other reaction rates which go into the nucleosynthesis code. However, when the uncertainties in all these rates, and the numerical errors associated with the code are taken into account, the uncertainty in Y_p is < 0.004 (ref. 7). Therefore, in practice, $Y_p \equiv Y_p\ (\tau_{1/2}, \eta, N_\nu)$. $Y_p\ (\tau_{1/2}, \eta, N_\nu)$ is shown above in figure 2.

Since Y_p increases with increasing $\tau_{1/2}$, η, and N_ν, lower bounds on η and $\tau_{1/2}$, and an upper limit on Y_p place an upper limit on N_ν. In §II I will discuss our knowledge of the quantities $\tau_{1/2}$, η, and Y_p and in §III the limits which can be placed upon N_ν. §IV is a summary and contains some concluding remarks. The material in §II, III is discussed in much greater detail in reference 1.

II. OUR KNOWLEDGE OF $\tau_{1/2}$, Y_p, and η

(a) The Neutron Halflife

The rate, Γ_β, of reactions (3a, b, c) depends on the measured value of the neutron halflife. Until recently, the accepted value was $\tau_{1/2}$ = 10.61 ± 0.16 min (ref. 8). This re-examination of the limits on N_ν was stimulated, in part, by a report of a new determination by Bondarenko et al.[9] which yielded a significantly different value: $\tau_{1/2}$ = 10.13 ± 0.09 min. As discussed earlier a lower value of $\tau_{1/2}$ implies a lower value of Y_p, and potentially "room" for more neutrino species (recall Y_p increases with both $\tau_{1/2}$ and N_ν).

There is other data which does not support this new, shorter lifetime. Kugler, Paul, and Trinks[10,11] found a value of $\tau_{1/2}$ = 10.62 min. and a lower limit of $\tau_{1/2}$ > 10.5 ± 0.8 min. Very recently, Byrne et al.[12] reported a preliminary value of $\tau_{1/2}$ = 10.82 ± 0.20 min. As techniques for storing neutrons are refined, an accurate (to better than ± 0.1 min) determination of $\tau_{1/2}$ should be forthcoming. For the subsequent analysis I will consider halflives in the range: 10.13 min $\leq \tau_{1/2} \leq$ 10.82 min.

(b) The Primordial Abundance of ^{4}He

Since ^{4}He is produced during stellar and galactic evolution, the abundance derived from objects at the present epoch provides an upper limit to the primordial abundance. The observational and theoretical situation is discussed in great detail in reference 1. The discussion here is a brief summary of the current situation.

There is general agreement among a large number of observers that for the "average", normal metal abundance (∿ 2%), galactic HII region, Y = 0.30 ± 0.02 [Note, an HII region is a region which contains ionized hydrogen gas]. For HII regions formed from material which has undergone less stellar processing (metal abundance < 2%), the observed ^{4}He abundance is lower, Y = 0.23 ± 0.02. Other data also support this result which is good evidence for $Y_p \leq 0.25$.

It is possible to try to extrapolate data from the present epoch to a truly primordial value. It has been suggested, and some of the data support a ΔY vs. ΔZ correlation (Z = abundance of heavy metals), e.g., $Y \simeq Y_p + 3Z$. Such extrapolations lead to a value Y_p = 0.20 - 0.24.

At present the data strongly suggest that $Y_p \lesssim 0.25$ provides a reliable upper limit to the primordial abundance of ^{4}He, and evidence is accumulating (especially from low Z HII regions) that $Y_p \lesssim 0.23$ might provide a better limit. In subsequent discussion I will also consider the possibility that Y_p is as large as 0.27.

(c) The Baryon-to-Photon Ratio

Although the only cosmological parameter that Y_p depends upon is η, η is not directly accessible to measurement, and must be measured through intermediaries. The present number densities of baryons and photons are

$$n_b = 1.13 \times 10^{-5}\ \Omega_N\ h_o^2\ \mathrm{cm}^{-3}, \tag{9}$$

$$n_\gamma = 400.\ (T_o/2.7\mathrm{K})^3\ \mathrm{cm}^{-3}, \tag{10}$$

where $\Omega_N = \rho_N/\rho_c$, ρ_N is the baryon mass density, $\rho_c = 3H_o^2/8\pi G$ is the present critical density, $H_o = 100 h_o$ kms^{-1} Mpc^{-1} is the present value of the Hubble parameter, and T_o is the present temperature of the microwave background. The baryon-to-photon ratio η is therefore given by

$$\eta = 2.83 \times 10^{-8}\ \Omega_N\ h_o^2\ (2.7\mathrm{K}/T_o)^3. \tag{11}$$

All measurements of the microwave background are consistent with a temperature: 2.7K $\lesssim T_o \lesssim$ 3.0K. In recent years there have been several independent determinations of H_o by various groups using different techniques[13-17], and although the internal errors in each determination are small, the vast discrepancy among these results suggests residual systematic errors. The range of probable values at present is: $50 \lesssim H_o \lesssim 100$ kms^{-1} Mpc^{-1}, or $\frac{1}{2} \lesssim h_o \lesssim 1$. Combining these two results ($h_o \gtrsim \frac{1}{2}$, $T_o \lesssim$ 3K) with (11) we obtain

$$\eta \gtrsim 5.16 \times 10^{-9}\ \Omega_N. \tag{12}$$

The standard dynamical approach to determining Ω_N (see, e.g., ref. 18) involves measuring the gravitational mass of a region (e.g., by analyzing rotational curves of galaxies, or by applying the virial theorem to groups of galaxies) and comparing this to the light emitted by that region -- constructing a mass-to-light ratio. Then the mass density of the universe can be found by multiplying the mass-to-light ratio times the luminosity density of the universe. There are at least two inherent problems with this procedure: (*i*) the mass inferred does not distinguish between baryons and other forms of gravitating material (e.g., massive neutrinos) and (*ii*) one is not sure to what extent a given mass-to-light ratio is "typical" of most of the luminous mass in the universe. The mass-to-light ratios and Ω's which are inferred from them are compiled in Table 1 for various scales.

Table 1

Scale	M/L (Solar Units)	Ω
Solar Neighborhood Material	2 ± 1	$(0.0014 \pm 0.0007)/h_0$
Central Region of Galaxies	$(8\text{-}20)\ h_0$	0.006-0.014
Binaries and Small Groups of Galaxies (BSG)	$(60\text{-}180)\ h_0$	0.04-0.13
Rich Clusters	$(300\text{-}1000)\ h_0$	0.2-0.7
Hot Gas in Clusters	---	$\gtrsim 0.007\ h_0^{-3/2}$

The material in the solar neighborhood is most certainly baryons, and from equation (12) we obtain the rather weak lower bound $\eta \gtrsim 0.14 \times 10^{-10}$. There is every reason to believe that the luminous material in the central portions of galaxies is predominantly baryonic; the range in the M/L's does not reflect observational uncertainties but rather real variations from spiral (closer to 8 h_0) to elliptical galaxies (closer to 20 h_0) due to the different stellar populations present. Using $\Omega_N = 0.006$ we obtain the lower bound $\eta \gtrsim 0.29 \times 10^{-10}$.

Most galaxies in the universe find themselves in binaries or small groups, so it seems reasonable that the M/L inferred from BSG is characteristic of the luminous mass in the universe. Using $\Omega_N = 0.04$, we obtain $\eta \gtrsim 2 \times 10^{-10}$. There is however, the uncertainty as to whether or not the mass which is observed is baryonic. Schramm and Steigman (see Schramm's contribution to these proceedings) have pointed out that neutrinos of mass $\gtrsim O(10\text{eV})$ may cluster on these scales and dominate the mass of BSG.

From rich clusters we can deduce a lower bound of $\eta \gtrsim 10^{-9}$; however, since most galaxies are not in rich clusters, there is reason to believe that the M/L ratio inferred for these object is not characteristic of most of the luminous matter in the universe. Finally, the x-ray emitting hot gas found in clusters is most certainly baryons and implies a lower bound on η of $\gtrsim 1 \times 10^{-10}$. Again, there is the uncertainty as to whether or not all galaxies have this much gas associated with them; in BSG or in isolated galaxies this gas, even if it were present, would not be hot enough to emit detectable amounts of x-rays as the gravitational potential wells of these objects are not nearly as deep as those in rich clusters. I should also mention that for $\eta < 1 \times 10^{-10}$ a large abundance of deuterium is produced primordially: $X_D \gtrsim 3 \times 10^{-4}$ (ref. 5), more than an order of magnitude greater than the abundance observed. Unless more than 90% of all baryons have been cycled through stars, this also suggests that $\eta \gtrsim 1 \times 10^{-10}$ (note: deuterium production decreases with increasing η).

In Table 2, I have summarized the lower bounds on η, from most reliable (M/L's for the solar neighborhood) to least reliable (BSG). Because most of the luminous matter in the universe is not in rich clusters I have not included that entry.

Table 2

Method of Determination	Lower Bound
Solar neighborhood material	$\eta \gtrsim 0.14 \times 10^{-10}$
Central regions of galaxies	$\eta \gtrsim 0.29 \times 10^{-10}$
Hot gas/Deuterium	$\eta \gtrsim 1.0 \times 10^{-10}$
BSG	$\eta \gtrsim 2.0 \times 10^{-10}$

III. WHAT ARE THE LIMITS ON N_ν?

At present most theories of elementary particles are quark-lepton symmetrical, i.e., for each quark pair there is a corresponding lepton pair (and hence neutrino), so that by counting the number of neutrino types, one also counts the number of generations. In standard QCD (e.g., without technicolor), there can be no more than 8 generations without spoiling asymptotic freedom ($N_\nu \lesssim 8$). The limits on N_ν derived from big bang nucleosynthesis are potentially much more restrictive.[5] I will now review the present situation paying particular attention to the possibility that neutrinos have non-zero rest masses.

As long as a neutrino species is relativistic during the epoch of nucleosynthesis ($m_\nu < 1$ MeV) it will contribute to the energy density and affect nucleosynthesis just as a massless neutrino would. The e- and μ-neutrinos each satisfy this condition and have been shown to be distinct species. The properties of the τ-neutrino are not well known and it is possible that its mass is as large as 250 MeV or that it is not a separate species, so that it might not contribute to the energy density of the universe during nucleosynthesis.

N_ν is the number of distinct, light 2-component neutrino species. Massive neutrinos can be of two varieties: (*i*) 2-component Majorana neutrinos ($\nu \equiv \bar{\nu}$), in which case each light species contributes one unit to N_ν, or (*ii*) 4-component Dirac neutrinos. In this case, if the right-handed components have interactions strong enough so that they remain in thermal contact with the rest of the universe at least until $T \simeq 10^{12}$K $\simeq$ 100MeV (this is when π's and μ's decoupled heating all those species still in thermal equilibrium), then they will affect nucleosynthesis just as their left-handed counterparts do, and each light species will contribute two units to N_ν. However, if they interact much more weakly than left-handed neutrinos, then they will decouple earlier, will have a lower temperature during nucleosynthesis, and will contribute much less to the energy density than a left-handed neutrino. If the right-handed components couple sufficiently weakly ("G_R" < $G_F/500$), then each light species effectively contributes only ≈ one unit to N_ν. Right-left interactions are not sufficiently strong to keep right-handed neutrinos in thermal contact late enough (i.e., until $T \approx 10^{12}$K) so that they contribute significantly to the energy density during nucleosynthesis. Unless there exist purely right-handed interactions of sufficient strength, right-handed neutrinos will have no significant effect on nucleosynthesis and each new species will change N_ν by only one unit. To summarize, for Majorana neutrinos each species changes N_ν by one unit, and for Dirac neutrinos each species changes N_ν by between one unit ("G_R" << G_F), and two units ("G_R" ≈ G_F).

(a) $m_\nu \lesssim$ few eV (for all species)

In this case neutrinos cannot cluster on scales of BSG[19] (also see Schramm's contribution to these proceedings) and there is no reason to believe the mass which binds BSG is anything but baryons, so that the lower bound on η of $\gtrsim 2 \times 10^{-10}$ is applicable. The number of allowed neutrino species, N_ν, as a function of $\tau_{1/2}$ for $Y_p \lesssim 0.25$ is shown below in figure 3 (the case for $\tau_{1/2} = 10.70$ min and $Y_p \le 0.25$ was discussed in ref. 5), and the limits on N_ν for $\eta \gtrsim 2 \times 10^{-10}$, $\tau_{1/2} \gtrsim 10.13$, 10.61, 10.82 min., and $Y_p \le 0.23$, 0.25, 0.27 are summarized in Table 3 below.

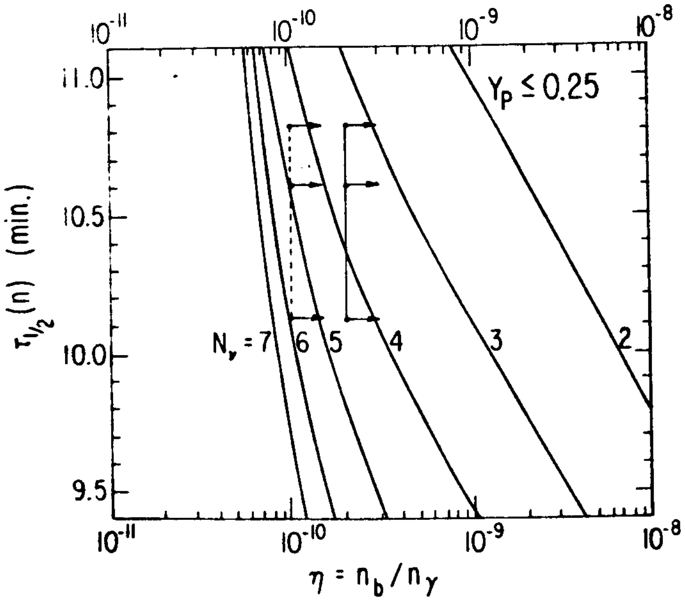


FIGURE 3 - The number of allowed neutrino species as a function of η and $\tau_{1/2}$ for $Y_p \le 0.25$.

Table 3 - Limits on N_ν for $\eta \gtrsim 2 \times 10^{-10}$

	$Y_p \le 0.23$	$Y_p \lesssim 0.25$	$Y_p \lesssim 0.27$
$\tau_{1/2} \ge 10.82$ min	$\lesssim 2$	$\lesssim 3$	$\lesssim 5$
$\tau_{1/2} \gtrsim 10.61$ min	$\lesssim 2$	$\lesssim 3$	$\lesssim 5$
$\tau_{1/2} \gtrsim 10.13$ min	$\lesssim 2$	$\lesssim 4$	$\lesssim 6$

Let me summarize this section by saying that if $m_\nu \lesssim$ few eV, then for $Y_p \le 0.25$ and $\tau_{1/2} \gtrsim 10.13$ min at most 4 two-component neutrino species are allowed (unless another form of non-baryonic matter dominates the masses of BSG, e.g., monopoles or heavy neutral stable leptons).

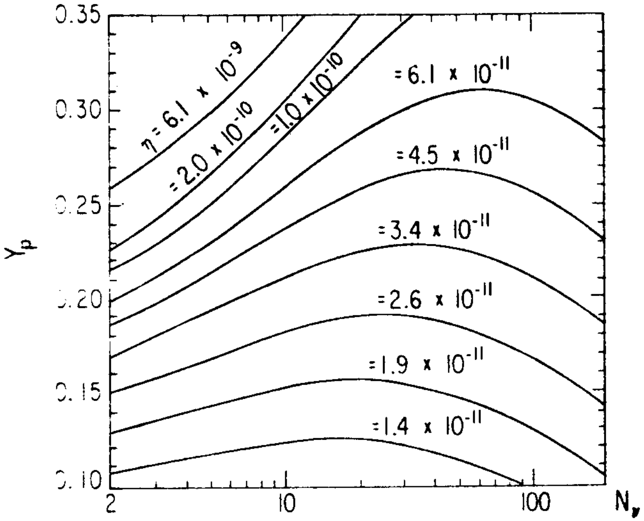
(b) $m_\nu \gtrsim O$(10eV) (for at least one species)

If at least one neutrino species is more massive than O(10eV), then this species may cluster on scales of BSG and dominate the mass of BSG; or other non-baryonic matter (e.g., monopoles, PBHs, heavy neutral stable leptons, etc.) may dominate the mass of BSG, and in either case $\eta \gtrsim 2 \times 10^{-10}$ can not be used as a lower bound on η. Therefore, a more reliable lower bound for η must be used. If the lower limit based on hot gas in clusters and deuterium is used (recall the uncertainties discussed in §II(c), then the number of allowed neutrino species as a function of $\tau_{1/2}$ is shown in figure 3 for $Y_p \le 0.25$, and summarized below in Table 4 for $Y_p \lesssim 0.23$, 0.25, 0.27.

Table 4 - Limits on N_ν for $\eta \ge 1 \times 10^{-10}$

	$Y_p \le 0.23$	$Y_p \lesssim 0.25$	$Y_p \lesssim 0.27$
$\tau_{1/2} \ge 10.82$ min	≤ 2	$\lesssim 4$	$\lesssim 7$
$\tau_{1/2} \gtrsim 10.61$ min	$\lesssim 3$	$\lesssim 5$	$\lesssim 7$
$\tau_{1/2} \gtrsim 10.13$ min	$\lesssim 4$	$\lesssim 6$	$\lesssim 8$

A very reliable lower bound on η is that derived from the inner, luminous parts of galaxies, $\eta \gtrsim 0.29 \times 10^{-10}$. Suprisingly, this bound results in no limit on N_ν, unless Y_p is determined to be $\lesssim 0.21$. Let me briefly explain the new twist here. Increasing N_ν speeds up the expansion, resulting in an earlier "freeze out", a higher n/p ratio, and more ^{4}He. However, if the expansion is sped up too much, the reactions which produce ^{4}He [e.g., ^{3}H(^{2}H, n)^{4}He] are not occuring rapidly on the expansion timescale (i.e., $\Gamma < t^{-1}$), and so there is not enough time to produce ^{4}He, and Y_p begins to decrease with increasing N_ν. This problem is exacerbated for small - since these rates (per baryon) are $\propto \eta$. Because of this, Y_p as a function of N_ν has a maximum value. For $\eta = 0.29 \times 10^{-10}$ and $\tau_{1/2} =$ 10.61 min the maximum value is $Y_p \approx 0.21$. This effect is illustrated in figures 4a and 4b below.



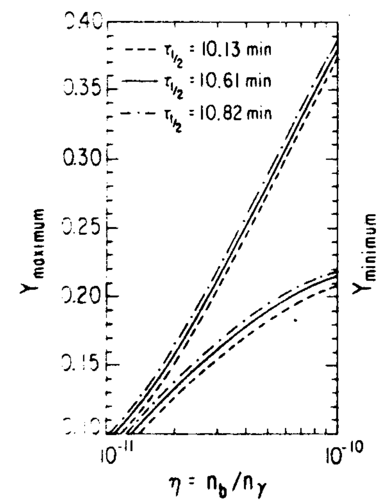


FIGURE 4a - Y_p as a function of N_ν for various values of η.

FIGURE 4b - The maximum amount of ^{4}He that can be produced (by varying N_ν - see fig. 4a), and the minimum amount that can be produced ($N_\nu = 2$) by primordial nucleosynthesis as a function of η; the actual amount produced must lie somewhere in between.

With 0.23 as an upper bound on Y_p there are no limits on N_ν until η is $\gtrsim 0.36 \times 10^{-10}$. The situation could improve. Although, the M/L ratios for the inner parts of galaxies range from (8-20) h_0, the lower bound, $\eta \geq 0.29 \times 10^{-10}$, was derived by assuming the extreme lower limit, M/L = 8 h_0; if, for example, it were shown by further analysis that most of the light in the universe from galaxies was from galaxies with M/L $\approx$ 15 h_0, then the lower bound becomes $\eta \gtrsim 0.55 \times 10^{-10}$. In this case for $Y_p \lesssim 0.25$ and $\tau_{1/2} \geq 10.61$ min at most 9 (or more than $\sim$ 100) neutrino types are permitted.

(c) How Reliable is the Standard Model?

A primordial abundance of $Y_p \lesssim 0.25$ is consistent with the observations of low metal abundant HII regions, although recent data suggest that $Y_p \approx 0.23$ might provide a better upper limit. Some extrapolations from the data suggest that $Y_p \lesssim 0.22$, and such low values lead one to question the consistency of the standard model.[20] In order to address this question the low η and Y_p portion of the Y_p vs. η curve is shown in figure 2, and in Table 5 below Y_p is tabulated for $\tau_{1/2}$ = 10.13, 10.61, 10.82 min, N_ν = 3, and η = 0.14, 0.29, 1.0, 2.0 x 10^{-10}.

Table 5 - Mass Fraction of ^{4}He Synthesized

	$\tau_{1/2}$ = 10.13 min	$\tau_{1/2}$ = 10.61 min	$\tau_{1/2}$ = 10.82 min
$\eta = 0.14 \times 10^{-10}$ (SOLAR)	0.11	0.11	0.12
$\eta = 0.29 \times 10^{-10}$ (GALAXIES)	0.16	0.17	0.17
$\eta = 1.0 \times 10^{-10}$ (HOT GAS)	0.22	0.23	0.23
$\eta = 2.0 \times 10^{-10}$ (BSG)	0.23	0.24	0.24

From Table 5 we see that a primordial ^{4}He abundance as low as $Y_p \approx 0.23$ may still be consistent with a baryon dominated universe. For a neutrino or other non-baryon dominated universe, the constraints on η are less stringent. Using the mass inferred from galactic M/L's ($\eta \gtrsim 0.29 \times 10^{-10}$) the standard model can produce Y_p as low as 0.16; with $\eta \gtrsim 0.14 \times 10^{-10}$ (M/L's for the solar neighborhood) Y_p can be as small as 0.11. In summary, given our present uncertainty with regard to η, the standard model (perhaps neutrino dominated) is not in contradiction with observations unless Y_p is found to be $\lesssim 0.16$ (may be even as low as 0.11).

IV. SUMMARY

The ^{4}He mass fraction synthesized in the big bang, Y_p, depends upon $\tau_{1/2}$, η, and N_ν. In order to set a limit on N_ν, an upper bound on Y_p and lower bounds on $\tau_{1/2}$ and η are needed. Observations strongly suggest that $Y_p \lesssim 0.25$ is a firm upper bound, and the experiments done to date are consistent with $\tau_{1/2} \geq 10.13$ min. The baryon-to-photon ratio η is more elusive. If all the light neutrinos are less massive than a few eV, then they cannot cluster on scales of BSG, and there is no reason to believe that the mass which binds BSG is anything but baryons. The lower bound $\eta \gtrsim 2 \times 10^{-10}$ derived from BSG is applicable, and for $\tau_{1/2} \geq 10.13$ min. and $Y_p \leq 0.25$ N_ν can be at most 4, i.e., 4 Majorana neutrinos, 4 Dirac neutrinos whose right-handed components interact very weakly ("G_R" << G_F), or 2 Dirac neutrinos whose right-handed components interact with full strength ("G_R" = G_F). With Y_p as high as 0.27 and $\tau_{1/2} \geq 10.13$ min N_ν can be as large as 6.

If there exists at least one neutrino with mass $\gtrsim 0(10\text{eV})$, then it may cluster on scales of BSG and dominate the mass of BSG. In this case (or if some other form of non-baryonic matter dominates the mass of BSG), the lower limit on η derived from BSG is not applicable. If the less than certain lower bound derived from hot gas and deuterium is used, $\eta \gtrsim 1 \times 10^{-10}$, then for $Y_p \leq 0.25$ and $\tau_{1/2} \geq 10.13$ min. N_ν can be at most 6. However, if the very firm, but less restrictive lower bound from the M/L ratios of the central regions of galaxies is used, $\eta \gtrsim 0.29 \times 10^{-10}$, then, at present, no limit can be placed on N_ν. This situation could change if further analysis permits a more stringent lower limit on η or if Y_p is found to be $\lesssim 0.21$.

I also mention in summarizing that primordial nucleosynthesis can be used to place an upper bound on η. For $Y_p \leq 0.25$ and $\tau_{1/2} \geq 10.13$ min, η must $\lesssim 10^{-9.0}$. Combining this with the lower bound derived from the solar neighborhood, we find that η must be in the range $10^{-9.9\pm1}$ - which implies that baryons alone cannot close the universe. The related baryon number to-specific entropy ratio, which has remained constant since the epoch of baryosynthesis (assuming the expansion has been isentropic), is constrained to $10^{-10.8\pm1}$. Finally, unless Y_p is found to be ≤ 0.16 (or possibly as low as 0.11) the standard model (perhaps neutrino dominated) is in no serious trouble at the present time.

Two years ago Yang et al.[5] used primordial nucleosynthesis to set the limit $N_\nu \leq 4$. Today the situation is, as I have described, somewhat less certain. To a large degree the uncertainty has to do with changes or possible changes in the microphysics - the neutron halflife and neutrino rest masses. I have tried to clearly state the assumptions upon which primordial nucleosynthesis is based, to explain the events leading to ^{4}He synthesis, and to detail the process involved in setting limits on N_ν. From this I hope that you are able to better understand both the strengths and weaknesses of using primordial nucleosynthesis as a probe of the early universe. The situation with regard to limits on N_ν promises to improve: (*i*) further studies of galactic mass-to-light ratios, of hot gas, and of the abundances of other light elements produced during the epoch of nucleosynthesis should yield a more stringent lower limit on η[21], (*ii*) improved confinement techniques should allow an accurate determination of $\tau_{1/2}$, and (*iii*) laboratory measurements of neutrino rest masses should help settle the question of whether or not they provide the mass which binds BSG. Of course, when the width of the Z^0 is finally measured, N_ν will be directly determined. This will be the moment of truth for those of us who use cosmology to probe particle physics. In any case, a direct determination of N_ν will provide yet another rigorous test for the standard model, and another method of indirectly determining η.

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REFERENCES

1. K.A. Olive, D.N. Schramm, G. Steigman, M.S. Turner, and J. Yang, EFI preprint 80-42, submitted to Ap.J. (1980). Most of the material presented in this talk is contained in this paper.

2. A. Sakharov, JETP Lett. 5, 24 (1967); M. Yoshimura, Phys. Rev. Lett. 41, 281 (1978); A. Ignatiev, N. Krasnikov, V. Kuzmin, and A. Taukhelidze, Phys. Lett. 76B, 436 (1978); D. Toussaint, S.B. Treiman, F. Wilczek, and A. Zee, Phys. Rev. D19, 1036 (1979); S. Dimopoulos and L. Susskind, Phys. Rev. D18, 4500 (1978); J. Ellis, M.K. Gaillard, and D.V. Nanopoulos, Phys. Lett. 80B, 360 (1979); S. Weinberg, Phys. Rev. Lett 42, 850 (1979); a brief review of big bang baryosynthesis is given in M.S. Turner, "Big Bang Baryosynthesis", in the Proceedings of the First Workshop on Grand Unification (1980).

3. P.J.E. Peebles, Ap.J. 146, 542 (1966).

4. R.V. Wagoner, W.A. Fowler, and F. Hoyle, Ap.J. 148, 3 (1967).

5. J. Yang, D.N. Schramm, G. Steigman, and R.T. Rood, Ap.J. 227, 697 (1979).

6. D.N. Schramm and G. Steigman, Phys. Lett. 87B, 141 (1979). M.S. Turner, "On the lepton numbers of the universe", EFI preprint 80-41, submitted to Phys. Lett. B (1980).

7. R.V. Wagoner, Ap.J. Suppl. 18, 247 (1969).

8. C.J. Christensen, A. Nielsen, A. Bahnsen, W.K. Brown, and B.M. Rustad, Phys. Rev. D5, 1628 (1972).

9. L.N. Bondarenko, V.V. Kurguzov, Yu.A. Prokof'ev, E.V. Rogov, and P.E. Spivak, JETP Lett. 28, 303 (1978).

10. K.J. Kugler, W.Paul, and U. Trinks, Phys. Lett. 72B, 422 (1978).

11. K.J. Kugler, W. Paul, and U. Trinks, IEEE 26, 3152 (1979).

12. J. Byrne, J. Morse, K.F. Smith, F. Shaikh, K. Green, and G.L. Greene, Phys. Lett. 92B, 274 (1980).

13. D. Branch, MNRAS 186, 609 (1979).

14. G. deVaucouleurs and G. Bollinger, Ap.J. 233, 433 (1979).

15. M. Aaronson, J. Mould, J. Huchra, W.T. Sullivan, R.A. Schommer, and G.D. Bothun, Ap. J. 239, 12 (1980).

16. R.P. Kirshner and J. Kwan, Ap.J. 193, 27 (1974).

17. A. Sandage and G.A. Tammann, Ap.J. 210, 7 (1976).

18. S.M. Faber and J.S. Gallagher, Ann. Rev. Astron. Astrophys. 17, 135 (1979).

19. S. Tremaine and J.E. Gunn, Phys. Rev. Lett. 42, 407 (1979).

20. F.W. Stecker, Phys. Rev. Lett. 44, 1237 (1980).

21. J. Yang, M.S. Turner, G. Steigman, D.N. Schramm, and K.A. Olive, work in preparation (1980).

DYNAMICAL FRICTION AND MASSIVE NEUTRINOS

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ABSTRACT

The requirement that dynamical friction by massive neutrinos not damp out all peculiar motions of galaxies places a weak upper bound of about 100 eV on the sum of masses of neutrinos which were once in equilibrium with the cosmic microwave radiation background.

INTRODUCTION

There has been much recent interest in neutrino masses in cosmology, as massive neutrinos can reconcile the large mass density required by the virial theorem applied to the dynamics of clusters of galaxies and the smaller upper bound on the baryon density inferred from cosmological helium synthesis.[1] Neutrinos which were in equilibrium early in the universe and which decouple at $kT \sim$ (few) MeV are roughly as abundant as photons today, whereas there is only one baryon per 10^9 photons, so even a small neutrino mass can contribute significantly to the total mass density of the universe. If the neutrinos are light enough to be bound to large clusters but not to individual galaxies, thus providing a "natural" explanation of the increasing mass-to-light ratio seen with increasing length scale, then galaxies are in some sense test particles moving in a sea of neutrinos which define the gravitational potential. That these test particles are in fact moving, that galaxies have peculiar velocities relative to the cosmological expansion, allows us to place an upper limit on m_ν, for in this situation the motions are damped by a process which has become known as dynamical friction.

DYNAMICAL FRICTION

As a massive object moves through a distribution of lighter objects, it deflects gravitationally those bodies which pass near it, with an exchange of momentum $\Delta p_\perp = mv \sin\Theta$, $\Delta p_{||} = mv\,(1 - \cos\Theta)$, where m and v are the mass and asymptotic velocity of the light particle, and Θ is the deflection angle. Summarizing over a large number of collisions, there is no average transverse force, but there is a net transfer of momentum in the logitudinal direction. This is the effect called dynamical friction, first studied by Chandrasekhar[2] in the context of stellar clusters. In our picture, the time scale of dynamical friction is