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NEUTRINOS AND THE BIG BANG

D.N. Schramm
Astronomy and Astrophysics Center
University of Chicago
Chicago, Illinois 60637

G. Steigman
Bartol Research Foundation
of The Franklin Institute
University of Delaware
Newark, Delaware 19711

ABSTRACT

It is shown that the cosmological density implied from the dynamics of clusters of galaxies is greater than the upper limit on the density of matter in baryons from big bang nucleosynthesis if the primordial helium abundance, Y , is ≤ 0.25 . If Y is ≤ 0.23 then even the density implied from the dynamics of binaries and small groups of galaxies cannot be in baryons. The solution to these problems comes if neutrinos have a small rest mass. For $3 \text{ eV} \leq m_\nu \leq 10 \text{ eV}$, the neutrinos will be trapped on the scale of large clusters. For $10 \text{ eV} \leq m_\nu \leq 20 \text{ eV}$, they will be trapped on the scale of binaries and small groups. If neutrinos have a rest mass $\geq 10 \text{ eV}$, then the limits on numbers of neutrino types from big bang nucleosynthesis may be relaxed if it is shown that the density of baryon matter is much less than the density implied by binaries and small groups. If neutrinos have rest mass there is no serious conflict with big bang nucleosynthesis as long as $Y \geq 0.15$.

INTRODUCTION

In this paper we will review the arguments on the density of matter in the universe and show that the density implied from large clusters of galaxies may be too large to be consistent with the upper limit on the density of baryons implied from nucleosynthesis. We will use this point to argue that this probably implies that neutrinos have a small rest mass which enables them to be gravitationally bound in large clusters. We will also show that this conclusion is strengthened if the upper limit on the primordial helium abundance, Y , is decreased. We will also review the arguments that big bang nucleosynthesis places on the number of types of neutrinos if neutrinos have a small rest mass.

This paper will in large part draw on the recent work of Schramm and Steigman¹ and Olive et al.²

For convenience we will express mass densities in terms of the critical density, $\rho_c = 3H_0^2 (8\pi G)^{-1}$, which separates those Friedman models (with $\Lambda = 0$) which expand forever ($\rho < \rho_c$) from those which eventually collapse ($\rho > \rho_c$). For each contribution to the total mass density, ρ_i , we introduce the density parameter, Ω_i , where

$\rho_i = \Omega_i \rho_c$. To allow for the large uncertainty in the present value of the Hubble parameter, we will use $H_0 = 100 h_0$ (kms⁻¹Mps⁻¹) where $0.5 \leq h_0 \leq 1$. In terms of Ω_i and h_0 we have

$$\rho_i = 2 \times 10^{-29} \Omega_i h_0^2 (\text{gcm}^{-3}). \quad (1)$$

One should note that not all combinations of h_0 and Ω_i are allowed in a standard Friedman cosmology since we know that the age of the universe must be greater than the mean age of the elements, 8.7×10^9 yr. This lower limit excludes values of $\Omega \geq 1$ for $h_0 > 0.75$ and approximately yields the constraint that $\Omega h_0^2 \leq 1$. The lower limit of 8.7×10^9 yr comes from the most conservative combined age constraints of ²³²Th/²³⁸U and ¹⁸⁷Re/¹⁸⁷O (ref. 3).

To obtain estimates of cosmological densities we will frequently use the mass to light ratio, M/L, for a particular class of objects. If we assume that this class of objects gives a good estimate of the bulk of the mass associated with the light of the universe we can estimate a cosmological mass density by multiplying the M/L by the average luminosity density of the universe, \mathcal{L} . From Kirshner, Oemler and Schechter⁴, $\mathcal{L} \approx 2 \times 10^8 h_0 L_\odot \text{Mpc}^{-3}$; (Note though that Gott and Turner⁵ used $\mathcal{L} = 1 \times 10^8 h_0 L_\odot \text{Mpc}^{-3}$ showing that there is at least a factor of 2 uncertainty here.)

As an example of how M/L can be used note that Peebles⁶ has estimated that M/L for the solar neighborhood is 1 to 2 (note that unlike other M/Ls this particular value is independent of h_0 since it is based only on local measurements) thus if this were the characteristic mass associated with the light of the universe

$$\rho_s = \frac{M}{L_{SN}} \cdot \mathcal{L} \approx 1.5 \times 10^{-32} \frac{M}{L_{SN}} h_0 \text{g/cm}^3 \approx 1.5 \times 10^{-32} h_0 \text{g/cm}^3$$

and thus

$$\Omega_s = \frac{0.0007}{h_0}$$

or for the limiting value of $h_0 \leq 1$ (ref. 7) we obtain

$$\Omega_s \geq 0.0007$$

Since we know there is mass not in the form of stars (e.g. interstellar gas and dust) this is clearly an extreme lower limit on Ω . It is also an extreme lower limit on the density of baryonic matter, Ω_b . Not all density estimates must be baryon matter. In particular, estimates of Ω from dynamics are not necessarily limiting the density of baryons.

DENSITY INFERRED FROM DYNAMICS

Let us now examine the mass density inferred from the dynamics of astrophysical systems of different scales. Here, we will rely extensively upon the data assembled in the excellent review article by Faber and Gallagher⁸.

The inner luminous parts of galaxies (mainly Ellipticals and SOs for our purposes) which are probably dominated by ordinary matter give

$$(M/L)_{E,SO} \approx 20 h_0$$

which yields

$$\Omega_{E,SO} \approx 3 \times 10^{-31} h_0^2 \text{g/cm}^3$$

or

$$\Omega_{E,SO} \approx 0.012.$$

There is, of course, good evidence that galaxies are considerably more massive than is inferred from studying their inner regions where most of the light originates. Indeed, from studies of the dynamics of Binary galaxies and Small Groups of galaxies, Faber and Gallagher⁸ obtain

$$(M/L)_{B,SG} \sim 70 \text{ to } 100 h_0$$

which yields

$$\rho_{B,SG} \approx 1 \text{ to } 1.5 \times 10^{-30} h_0^2 \text{g/cm}^3$$

and

$$\Omega_{B,SG} \approx 0.05 - 0.075$$

If we take into account the uncertainty in \mathcal{L} this might go as low as 0.025. As implied by the results of Gott et al.⁹ for reasonable values of H_0 on scales up to those of binaries and small groups, most of the inferred mass can be in ordinary matter. There is still a "missing mass" (or rather missing light) problem since the M/L implied from binary and group dynamics is greater than that implied by normal stellar population or that implied by internal galactic motions, however, this problem may be due to non-luminous ordinary matter. However, as mentioned by Schramm and Steigman (again, ref. 1), if the upper limit on Y is pushed below ~ 0.22 then baryonic matter would no longer be able to satisfy these mass constraints. We will see that for large clusters of galaxies we

may already have reached the point where baryonic matter cannot do the job.

After years of extensive investigation using dynamic arguments, the conclusion remains that clusters of galaxies are very massive^{6,10} with implied

$$(M/L)_c \sim 500 \pm 200 h_0$$

yielding

$$\rho_c \sim 7.5 \times 10^{-30} h_0^2$$

and

$$\Omega_c \sim 0.4$$

Within the factor of 2 uncertainty in \mathcal{L} and the uncertainties in M/L and other data contributing to this value of Ω , it is probably true that a value as low as 0.1 cannot be excluded (for example see arguments by Aarseth, Gott and Turner¹¹). (It is also probably true that if one really pushed the data in the other direction Ω as large as unity might not be completely excluded.) However, it is also clear that the current best fit values are somewhat above 0.1.

DENSITY INFERRED FROM NUCLEOSYNTHESIS

During primordial nucleosynthesis the light elements (D, ³He, ⁴He and ⁷Li) are formed by two body reactions whose rates depend on nuclear density (c.f. Schramm and Wagoner¹² and references therein). High nuclear density results in the production of more ⁴He and ⁷Li and less D and ³He since nucleons are conserved (for $T \lesssim 1$ MeV, baryon nonconserving processes are entirely negligible), an upper limit to the present density in nucleons may be inferred from an upper limit to the primordial abundance of deuterium. Since photons are also conserved (with account taken of the extra photons created when electron-positron pairs annihilate as the universe cools below m_e), it is convenient to compare nucleons and photons. For an upper limit to the primordial abundance by mass of ⁴He, $Y \lesssim 0.25$ (see arguments supporting this upper limit in Yang et al.¹³, hereinafter referred to as YS²R) and, for three, two-component neutrinos: ν_e, ν_μ, ν_τ the nucleon to photon ratio is limited by,

$$\frac{n_b}{n_{\gamma_0}} \lesssim 4.2 \times 10^{-10}$$

The number density of black body photons is $n_{\gamma_0} \approx 400 (T_0/2.7)$ which leads to an upper limit on $\Omega_b h_0^2$, where Ω_b is the fraction of

the critical density in the form of baryons.

$$\Omega_b h_0^2 \lesssim 0.014 \frac{T_0^3}{2.7}$$

As pointed out by YS²R¹³ this limit is identical with the one which follows from the deuterium abundance and is consistent with (but more restrictive than) the limit from ⁷Li. A firm upper limit to Ω_b follows using a lower limit to H_0 ($h_0 \gtrsim 0.5$)¹⁴ and an upper limit to T_0 ($T_0 \lesssim 3.0$ K)¹⁵.

$$\Omega_b \lesssim 0.12$$

This limit constrains any matter which was in the form of baryons when $T \sim 10^9$ K even if the matter is now locked up in blackholes.

Therefore, if the "best fit" mass inferred from clusters were (at the time of primordial nucleosynthesis) in the form of nucleons (e.g.: today they might be in black holes, neutron stars, etc.), too much ⁴He and ⁷Li and too little D would have been produced. Additional support for such a disparity is provided by the x-ray observations; the temperature of the x-ray emitting gas is a probe for the depth of the potential well in which the gas resides. Using the results of Lea et al.¹⁶ and assuming the universality of the results yields

$$\Omega_c \gtrsim 0.2$$

However, again one should be careful with regard to the filling of large clusters in the universe. Rather than looking for consistency in the limits of the uncertainties, in this paper we will accept the possibility that Ω_c may be significantly greater than Ω_b and we will assume that the difference is due to the existence of relic neutrinos with a small mass ($m_\nu \approx 5$ eV).

It should be noted that if the upper limit to the Helium abundance, Y , is found to be less than 0.25 as discussed by Stecker¹⁷ (and references therein) then the upper limit on Ω_b decreases which increases the discrepancy with Ω_c and argues more strongly for the existence of massive neutrinos or some other non-baryonic matter. It should also be noted as pointed out by Olive et al.² that with the lower limit to Ω_b inferred from the inner regions of galaxies yields no serious inconsistency with big bang nucleosynthesis with three neutrino species as long as Y is $\gtrsim 0.15$.

Our selection of low mass neutrinos as the solution to the dark matter problem is a natural consequence of the nucleosynthetic upper limit eliminating such things as low mass stars, rocks, planets, clouds and stellar mass black holes and radiation limits eliminating most reasonable mass ranges for primordial mini-black holes and magnetic monopoles. This leaves only the low mass neutrinos proposed by Cowsik and McClellan¹⁸ and Marx and Szalay¹⁹ and the high mass ($\gtrsim 106$ eV) neutrinos discussed by Gunn et al.²⁰

Since high mass neutrinos are more ad hoc and exclude the known ν_e , ν_μ , ν_τ we will concentrate here on low but finite neutrinos.

RELIC NEUTRINOS

During the early evolution of the universe, all particles, including neutrinos, were produced copiously (c.f. Steigman²¹). Here we focus on the known e^- , μ^- and τ^- neutrinos and entertain the possibility that they have a small but finite mass. Neutrinos with full strength, neutral current, weak interactions were produced by reactions of the type

$$e^+ + e^- \leftrightarrow \nu_i + \bar{\nu}_i; \quad i = e, \mu, \tau.$$

At high temperatures ($kT > m_\nu c^2$), these neutrinos were approximately as abundant as photons.

$$\frac{n_{\nu_i}}{n_\gamma} = \frac{3}{8} g_{\nu_i}$$

where g_{ν_i} is the number of neutrino helicity states. For massless spin $\frac{1}{2}$ neutrinos with $\bar{\nu} \neq \nu$, then $g_{\nu_i} = 2$. If the neutrinos have a mass then each spin $\frac{1}{2}$ particle would have 2 helicity states thus for $\bar{\nu} \neq \nu$, $g_{\nu_i} = 4$ however because known neutrinos appear to be only left handed then, if massive, they may be of the Majorana type ($\bar{\nu} = \nu$), in which case g_{ν_i} is still 2. As for the numerical factor, $3/4$ comes from the difference between Fermi-Dirac statistics (neutrinos) and Bose-Einstein statistics (photons); the remaining factor of $\frac{1}{2}$ is from the number of photon spin states ($g_\gamma = 2$).

For light neutrinos ($m_\nu \ll 1$ MeV), equilibrium was maintained until $T \approx 1$ MeV. At lower temperatures the weak interaction rate is too slow to keep pace with the universal expansion rate so that few new neutrinos are produced and, equally important, few annihilate. Thus, for $T \approx 1$ MeV, the neutrinos decouple; at this stage their relative abundance is given above. When the temperature drops below the electron mass, electron-positron pairs annihilate heating the photons but not the decoupled neutrinos. The present ratio of neutrinos to photons must account for the extra photons produced when the e^\pm pairs disappeared (c.f. Steigman²¹).

$$N_\gamma(T < m_e) = \frac{11}{4} N_\gamma(T > m_e); \quad \frac{n_\nu}{n_\gamma} = \frac{3}{22} g_{\nu_i}$$

From the present density of photons we obtain the present number density of neutrinos; multiplying by the neutrino mass we obtain the neutrino mass density (ρ_ν) which may be expressed in terms of the density parameter,

$$\Omega_{\nu_i} h_0^2 = \frac{g_{\nu_i} m_{\nu_i} T_0^3}{200 \cdot 2.7}$$

where m_{ν_i} is in eV and the sum is over all neutrino species with $m \ll 1$ MeV. We have implicitly assumed that the neutrinos still exist today and thus have a lifetime greater than the age of the universe for decay into anything other than neutrinos. Because $\Omega_b^2 \leq 1$, this limits the sum of masses

$$\sum m_i \lesssim 100 \text{ eV for } g = 2$$

and

$$\sum m_i \lesssim 50 \text{ eV for } g = 4$$

For $T \lesssim 3$ K and $h_0 < 1$ and assuming Majorana mass neutrinos with $g_\nu = 2$ we find that

$$\Omega_{\nu_i} \gtrsim 0.014 m_{\nu_i}$$

From our limit on Ω_b we obtain the relationship with $g_{\nu_i} = 2$

$$\frac{\Omega_\nu}{\Omega_b} \gtrsim \frac{\sum m_{\nu_i}}{10}$$

which is independent of h_0 and T . Therefore if neutrinos have masses of the order of 10 eV or greater than neutrinos are the dominant mass component of the universe today.

Massive neutrinos gravitate and they will have participated in gravitational clustering (see Gunn et al.²⁰). However, since neutrinos are non-interacting, their phase space density is conserved and they will cluster only in the deepest potential wells; the slowest moving (i.e.: the heaviest) will cluster most easily.

Tremaine and Gunn²² point out that neutrinos lighter than ≈ 3 eV will not cluster at all due to the fact that their velocities in clusters will be greater than that necessary to escape. Neutrinos with a mass between 3 and 10 eV will be in clusters of galaxies (the deepest potential wells) but not (significantly) contribute to the mass on smaller scales. But recall, the scale on which the missing light problem truly emerges is that of clusters of galaxies (and to the mass of the universe) is from relic neutrinos with a finite mass $m_\nu \approx 6 \pm 3$ eV.

Obviously, if $\Omega_\nu \approx \Omega_c$ were > 1 the Friedman universe would be closed by neutrinos. As mentioned before, current estimates put $\Omega_c < 1$ and thus Ω_ν would also be constrained to be < 1 however the

uncertainties are sufficiently large that closure by neutrinos can not be completely excluded. Note that if

$$\sum m_{\nu_i} > 100 h_0^2 \frac{2.7}{T_0} \text{ eV}$$

then the Friedman universe with $\Lambda = 0$ is closed.

It is intriguing to note that neutrinos with mass less than 3 eV do not get included in density estimates from cluster dynamics, thus they may provide a smooth background. However, the contribution to Ω from such low mass neutrinos is small. Thus, it is probably true that Ω_ν is constrained by Ω_c .

LIMITS ON NEUTRINO FLAVORS

YS²R¹³ (and references therein) showed that $Y \lesssim 0.25$ and a lower limit on

$$\frac{n_b}{n_\gamma} \text{ of } \sim 2 \times 10^{-10}$$

set a limit on the number of neutrino flavors N_ν to be $\lesssim 4$. It has been shown by Olive et al.² that as long as these limits on Y and n_b/n_γ are valid this limit on N_ν holds. However, as Olive et al.² point out the limit on n_b/n_γ used above comes from binary galaxies and small groups of galaxies. If these systems are dominated by low mass neutrinos with masses between 10 and 20 eV then one could no longer use that limit on n_b/n_γ . The limit on n_b/n_γ from the central regions of galaxies only yields $n_b/n_\gamma \gtrsim 3 \times 10^{-11}$ when all the uncertainties are included. This limit could allow N_ν to be infinite for $Y \lesssim 0.25$ or to still be extraordinarily large even for a limit on Y of $\lesssim 0.22$. A less certain lower limit on n_b/n_γ comes from the hot x-ray emitting gas in clusters. This yields $n_b/n_\gamma \gtrsim 10^{-10}$, which in turn limits N_ν to $\lesssim 5$. It is obvious from the above that the strength depends on the mass of the neutrino. If neutrinos have masses $\lesssim 10$ eV then Binaries and small groups of galaxies will not trap neutrinos and the old limits on $N_\nu \lesssim 4$ hold however if $m_\nu > 10$ eV. The lower limits on n_b/n_γ must be revised with a tentative limit on N_ν of ~ 5 but no firm limit being available. The argument for $m_\nu > 10$ eV would be enhanced if Y were found to be $\lesssim 0.23$ since then no 3 ν solution would exist for $n_b/n_\gamma \gtrsim 2 \times 10^{-10}$.

SUMMARY

Low mass neutrinos appear to be the most reasonable solution to the missing mass (light) problem in cosmology. The cosmological implications on m_ν place it between 3 eV with the best fit near 10 eV. With these massive neutrinos consistent big bang solutions

are found for Helium mass abundances $\gtrsim 0.15$. If neutrinos have mass $\gtrsim 10$ eV the cosmological limits on N_ν can be loosened.

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WHAT LIMITS (IF ANY) DOES BIG BANG NUCLEOSYNTHESIS
PLACE ON THE NUMBER OF NEUTRINO FLAVORS?

Michael S. Turner
The University of Chicago, Chicago, Illinois 60637

ABSTRACT

The mass fraction of ${}^4\text{He}$ synthesized in the big bang, Y_p , depends upon the neutron halflife, τ_n , the baryon-to-photon ratio, η , and the number of 2-component neutrino species, N_ν . New observational and experimental data have led to a re-examination of the constraints on particle physics and cosmology which follow from primordial nucleosynthesis. If baryons provide most of the mass which binds binary and small groups of galaxies, then N_ν must be ≤ 4 . However, if massive neutrinos (or other non-baryonic matter) provide this mass, then at present no firm limit can be placed on N_ν . In addition we find that η must lie in the range $10^{-9.9 \pm 1}$, implying that baryons alone cannot close the universe; the related baryon-to-specific entropy ratio must lie in the range $10^{-10.8 \pm 1}$. If the universe is dominated by non-baryonic matter, then there is no contradiction between the predictions of primordial nucleosynthesis and the observations of ${}^4\text{He}$ provided that $Y_p \geq 0.15$.

I. INTRODUCTION

There is an impressive body of evidence which supports the hot big bang theory. This evidence includes: (i) the expansion of the universe discovered by Hubble and others, (ii) the 3K cosmic microwave background discovered by Penzias and Wilson, (iii) the singularity theorems of Hawking and Penrose, (iv) the abundance of ${}^4\text{He}$ and several other light elements which were produced ~ 3 min after the big bang, and perhaps, (v) the presence of only matter in the universe rather than equal amounts of matter and antimatter. We have reason to believe that this is a result of a slight excess of baryons over antibaryons having evolved during the epoch of baryosynthesis ($t \sim 10^{-35}$ sec), and later when all the antibaryons and most of the baryons annihilated ($t \sim 10^{-6}$ sec) the ~ 1 baryon per 10^{10} photons we see today was left due to this excess.² The time-temperature relation in the standard hot big bang model (Friedman-Robertson-Walker cosmology) when the energy density of the universe is dominated by relativistic particles ($t \leq 10^{12}$ sec) is

$$t = 2.42 \times 10^{-7} \text{ sec } (100/N)^{1/2} T_{\text{GeV}}^{-2} \quad (1)$$

where N is the sum of the statistical weights of all the particle species present [$= \sum_{\text{bosons}} g_i + (7/8) \sum_{\text{fermions}} g_i$] and T is the temperature measured in GeV ($1.16 \times 10^{13} \text{ K} = 1 \text{ GeV}$). From (1) it is clear that at early times particle energies were very high. At the planck time ($t \sim 10^{-43}$ sec) particle energies were as large as