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PROPERTIES OF NEUTRINOS DERIVED FROM EXPERIMENTS IN NATURE

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ABSTRACT

Our early work on the role of neutrinos of finite rest mass in astrophysics and cosmology is reviewed briefly. The main results are:

a) The neutrinos are sufficiently stable against radiative decay to survive for periods much longer than the age of the Universe, or more explicitly, $\tau_o > 10^{23}$s if $m_\nu \sim 1$ eV/c^2, $\tau_o > 10^{24}$s if $m \simeq 20$ eV/c^2 and $\tau_o/m_\nu c^2 > 10^{17}$s/eV irrespective of their mass.

b) The sum of the masses of all the various types of neutrinos is less than 35 eV/c^2.

c) If indeed any of the neutrinos should have a finite rest mass of $\sim$ 1-10 eV/c^2 it can be responsible for causing the apparent virial mass discrepancy in large clusters of galaxies.

These early ideas are extended to the case of the newly discovered τ-neutrino to show that it also conforms to the limits indicated above.

INTRODUCTION

Astrophysics is usually the science in which the well-known laws of physics are assumed to hold good even on the macroscopic scale of the astronomical bodies and on this premise we try to understand the nature and properties of the celestial objects. However, the inverse situation where studies are done outside the man-made laboratories in nature, has yielded many insights into basic physics and, in particular, into the properties of the fundamental particles. I have been motivated by this idea and today I shall review some of my early work on the neutrinos of finite rest mass specific to the interest of this conference.

Neutrinos of finite rest mass have fascinated physicists for a long time and we are probably indebted to Markov (1964), Gerstein and Zeldovich (1967) and to Bahcall, Cabibbo and Yahil (1971) for some of the very first applications to astrophysics and cosmology. Markov and also Bhadman (1974) had reviewed the field until then and suggested the possibility of neutrino stars.[1,2] Gerstein and Zeldovich[3] presented the first qualitative ideas and showed that the measured age of the universe restricts the mass of a neutrino to be less than ∿ 140 eV, and Bahcall et al. argued that if neutrinos had a finite rest mass, they could decay into lighter neutrinos and bosons and used this as a possible basis for understanding the low counting rate in the "Solar-Neutrino" experiment.[4] My own interest in the neutrino-induced phenomena[5-8] in nature and their cosmological effects has been a longstanding one (Cowsik et al., 1963, 1964, 1966, 1969) and on the particular topic of neutrinos of finite rest mass, I have considered basically three questions.

i. The effect of massive neutrinos in cosmology, leading to an upper limit[9] on the sum of their masses (Cowsik and McClelland, 1972).

ii. Clustering of neutrinos[10] and the virial mass discrepancy in the Coma-cluster of galaxies (Cowsik and McClelland, 1973).

iii. The stability of neutrinos[11] and limits on their decay rate (Cowsik, 1977, 1979).

Topic i was also considered by Marx and Szalay[12] (1972), topic ii somewhat later, Marx and Szalay[13] (1976) and supportive arguments for topic iii were given by Falk and Schramm[14] (1978).

After this, many important developments took place concerning the question of massive neutrinos: Lee and Weinberg[15] considered the contribution of very massive neutrinos $m_\nu > 1$ MeV/e^2 to the cosmological energy density (see also Dicus et al.,[16] 1977), Gunn et al. (1978) discussed the astrophysical consequences of the existence of a very heavy neutrino[17] with $m_\nu \gtrsim 2$ GeV/c^2, Tremaine and Gunn[18] (1978) have emphasized the fact that neutrinos behave like collisionless gas and would therefore evolve without any change in density in phase space and finally Steigman and Schramm[18] (1980) have reviewed the most recent astronomical observations which lend support to the idea that neutrinos indeed are responsible for binding the large clusters of galaxies.

The theoretical implications of neutrinos of finite rest mass have been discussed by several authors,[20-23] notably by Gell-Mann et al. (1975), and reviewed with extensive references by Mohapatra and Senjanovic (1980), by Pakvasa (1980) and by Witten (1980).

With the news of a possible experimental detection of a finite rest mass for the neutrino in the range 14 eV/c^2 < m_ν < 46 eV/c^2 by Lubimov et al.[24] (1980) and oscillations of neutrinos with a mass difference $\Delta m^2 \sim$ 1-10 eV^2 by Reines et al.,[25] there has been a great resurgence of interest. The theoretical ideas and the astronomical data pertaining to the question of massive neutrinos are being scrutinized carefully again by De Rujula and Glashow, Stecker, Kimble et al., Cowsik and Shipman.[25-30]

In this talk, I shall mainly review my early work on the role of massive neutrinos stated in terms of the three basic questions i, ii, and iii, cited above. Near the end of the talk, I shall show how the τ-neutrino could also be included in these discussions.

UPPER LIMIT ON THE NEUTRINO REST MASS

The hypothesis that the neutrino mass is zero, though aesthetically very appealing, has had rather limited validation by experiments performed in the laboratory. By looking for cut-offs in the spectrum of β-decay electrons and muons on a Kurie plot, the following limits have been placed on the neutrino masses:[31-33]

$$m_{\nu_e} < 60 \text{ eV/c}^2; \quad m_{\nu_\mu} < 0.5 \text{ MeV/c}^2; \quad m_{\nu_\tau} < 250 \text{ MeV/c}^2 . \qquad (1)$$

These limits, particularly for ν_μ and ν_τ, certainly need further improvement to validate the canonical assumption of masslessness. The idea behind the astrophysical method is that if one has a sufficiently large number of neutrinos, one can weigh them, as the gravitational force due to a large ensemble of neutrinos and antineutrinos add up. The plan is therefore to detect the net gravitational interaction of the large number of neutrinos in the universe using the galaxies as test particles! In discussing this problem, we take the customary point of view that the universe is expanding from an initially hot condensed state as envisaged in the Big-Bang cosmology.[34]

We will now proceed to calculate the number density of neutrinos and see what effect they have on the motion of the galaxies. In the early phase of the universe when the temperature was greater than ∿ 1 MeV, processes of neutrino production[35] which have also been considered in the context of high temperature stellar cores would lead to a generation of various kinds of neutrinos. Some of these reactions are listed below:

$$\begin{aligned} e^+ + e^- &\to \nu + \bar{\nu} \\ e + z &\to e + z + \nu + \bar{\nu} \\ \gamma_{plasmon} &\to \nu + \bar{\nu} . \end{aligned} \qquad (2)$$

Since weak interactions can also proceed through neutral currents, one could have all the kinematically allowed neutrino pairs ($\nu_e\bar{\nu}_e$, $\nu_\mu\bar{\nu}_\mu$, etc.) on the right-hand side of the above reactions. The actual density of neutrinos then is to be obtained by balancing, in detail, the rates of such processes with the rate of destruction of neutrinos through mutual annihilation and such other processes. Fortunately, however, this horrendous task is simplified, at least for the ν_e and ν_μ (with m < 1 MeV/c^2), by the consideration that, in the early universe, the density of e^+ and e^- is so high that the reactions rapidly drive the neutrinos into thermodynamic equilibrium. With this realization, we can immediately write for the number density of ν_e or ν_μ and, in fact, for that of any of the fermions and bosons to be

$$n_{Fi} = \frac{g_i}{2\pi^2\hbar^3}\int_0^\infty \frac{p^2 dp}{\exp[E/kT(z_{eq})] + 1} \qquad (3)$$

and

$$n_{Bi} = \frac{g_i}{2\pi^2\hbar^3}\int_0^\infty \frac{p^2dp}{\exp[E/kT(z_{eq})] - 1} . \quad (4)$$

Here n_{Fi} = number density of fermions of the ith kind

n_{Bi} = number density of bosons of the ith kind

g_i = effective degeneracy of the spin states ($\sim$2)

$E = c(p^2 + m^2c^2)^{1/2}$

k = Boltzmann's constant

$T(z_{eq}) = T_{rad}(z_{eq}) = T_F(z_{eq}) = T_B(z_{eq})$
= common temperature of all the particles in thermal equilibrium at the epoch characterized by the red-shift z_{eq}.

Initially, to get the general idea of the method, let us consider only the light particles like ν_e and ν_μ with

$$kT(z_{eq}) \approx 1 \text{ MeV} > mc^2 . \quad (5)$$

Then Eqs. (3) and (4) integrate to

$$n_{Fi}(z_{eq}) \approx 0.0913\ g_i[T(z_{eq})/\hbar c]^3$$

and (6)

$$n_{Bi}(z_{eq}) \approx 0.122\ g_i[T(z_{eq})/\hbar c]^3 .$$

As the universe expands and cools down, the neutrinos and such other particles survive without annihilation because of the extremely small cross sections for the annihilation of light neutrinos. Anticipating our discussion in the second section, we will neglect also the decay of these particles so that their number density decreases with the universal expansion simply as

$$n(z) = n(z_{eq})\frac{V(z_{eq})}{V(z)} = n(z_{eq})\frac{(1+z)^3}{(1+z_{eq})^3} . \quad (7)$$

Since $(1+z) = T_{rad}(z)/T_{rad}(0)$, the number density at the present $(z = 0)$ is given by

$$n_{Fi}(0) = n_{Fi}(z_{eq})(1+z_{eq})^{-3} = 0.0913\ g_i\left[\frac{T_{rad}(0)}{\hbar c}\right]^3$$

$$n_{Bi}(0) = 0.122\ g_i\left[\frac{T_{rad}(0)}{\hbar c}\right]^3 . \quad (8)$$

Using $T_{rad}(0) = 2.7°$ K as the present-day radiation temperature of the universe, we get

$$n_{Fi}(0) \approx 150\ g_i\ \text{cm}^{-3};\quad n_{Bi}(0) \approx 200\ g_i\ \text{cm}^{-3} . \quad (9)$$

These numbers are huge in comparison with the number density of baryons[36] in the universe, $\lesssim 4 \times 10^{-6}$ cm^{-3}. Consequently, even if the neutrinos have a very small rest mass, they would dominate the dynamics of the universe.

We will now proceed to set an upper limit to the energy density in the universe, contained in all forms. The cosmological expansion of the universe as traced by the red-shift of the galaxies is described in terms of two parameters:

a) The Hubbles' constant which measures the rate of change of expansion velocity with distance, designated by $H_o \approx 50$ km/sec/Mpc and,

b) The deceleration parameter, the second derivative of the expansion velocity designated by $q_o \approx 0.94 \pm 0.4$. These quantities are taken from measurements by Sandage. There is a third quantity which pertains to the expansion of the universe, namely,

c) The age of the universe which is designated by t_o and is shown to be longer than 8×10^9 years using observations of old globular clusters. These three parameters are not independent of each other but are related through the equation[34]

$$t_o = H_o^{-1}q_o(2q_o - 1)^{-3/2}\left[\cos^{-1}\left(\frac{1}{q_o} - 1\right) - \frac{1}{q_o}(2q_o - 1)^{1/2}\right] . \quad (10)$$

Equation 10 implies $q_o \lesssim 1.2$ quite consistent with Sandage's[37] direct determination using plots of red-shifts of galaxies versus their magnitudes. The Hubble parameter H_o and the deceleration parameter q_o define clearly the dynamics of an isotropic, homogeneous, expanding universe with a density, ρ_{tot}, given by the expression

$$\rho_{tot} = \frac{3H_o^2q_o}{4\pi G} \lesssim 1.3 \times 10^{-29}\ \text{g cm}^{-3} \lesssim 8 \times 10^3\ \text{eV}/c^2\ \text{cm}^{-3} . \quad (11)$$

This 8 keV cm^{-3} is the total energy density in all forms in the universe. The weakly interacting particles along would contribute density ρ_{weak} given by

$$\rho_{weak} = \sum n_{Bi}\, m_i + n_{Fj}\, m_j$$
$$= \sum 200\ g_i\, m_i + \sum 150\ g_j\, m_j < \rho_{tot} . \quad (12)$$

Or, simply

$$\sum g_i\, m_i \lesssim 53\ \text{eV}/c^2 . \quad (13)$$

Taking $g_i = 2$ and keeping in mind that particle and antiparticle masses are the same,

$$\sum m_i \le 13 \text{ eV}/c^2 \ . \qquad (14)$$

This means that the sum of the masses of all the light neutrinos and any hypothetical weakly interacting bosons is less than 13 eV/c^2. However, in the calculations performed so far, we have neglected the fact that the positrons which were also created copiously would have annihilated and raised the temperature of the universe[34] by a factor of ~ 1.4. This means that we should have used $T_{rad} = (2.7/1.4)°K$ in the formula for the densities in Eq. (8). This leads to the slightly less restrictive limit

$$\sum m_i < 1.3 \times (1.4)^3 \text{ eV}/c^2 < 35 \text{ eV}/c^2 \ . \qquad (15)$$

If a neutrino of a particular type saturates this limit, the other neutrinos should have zero mass. This limit of 35 eV/c^2 is a considerable improvement over the limit of 60 eV/c^2 for ν_e obtained from the study of tritium-decay and certainly reduces the 0.5 MeV/c^2 limits on ν_μ dramatically.

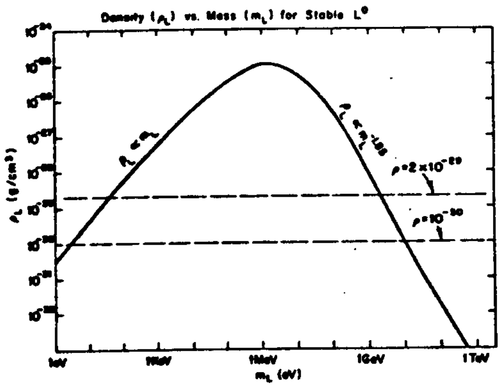

Fig. 1. The density of the universe contributed by neutrinos of various masses.

We have derived the above limit assuming that the masses of the neutrinos are smaller than ~ 1 MeV and that they were in thermodynamic equilibrium with the radiation field. If the mass of a neutrino is much larger than ~ 1 MeV, as could very well be true, a priori, for ν_τ, then such neutrinos would move away from thermodynamic equilibrium and their number densities at present would become progressively much smaller with increasing mass, than the numbers predicted by Eq. (9). The actual numbers are controlled by a detailed balancing of the reactions that produce them and their annihilation in a rapidly expanding universe. Lee and Weinberg[15] have made these estimates and were first to point out that unless m_ν is greater than 2 GeV, their cosmological number densities will be too large to be consistent with the value of ρ_{tot} defined in Eq. (11). Similar calculations[16] were also performed by Dicus et al. and their results are well exhibited in the following figure[17] taken from Gunn et al. Thus, the main results are

$$\sum m_{\nu i} < 35 \text{ eV}/c^2 \quad \text{for} \quad m_\nu < 1 \text{ MeV}/c^2$$

$$m_{\nu i} > 2 \text{ GeV}/c^2 \ . \qquad (16)$$

LIMITS ON RADIATIVE DECAY OF NEUTRINOS

In this section, I discuss the limits that can be placed on the radiative instability of neutrinos from a variety of observations. In this regard, I am motivated by several papers[4,38-40] which have considered the possibility that neutrinos could have finite rest mass and could, therefore, decay. One particular set of these[39,40] considers the mixing of ν_e and ν_μ and predicts observable widths for the lepton-number-nonconserving decays, such as $\mu \to e + \gamma$ and $\nu_\mu \to \nu_e + \gamma$. Independent of such theoretical considerations, it is worthwhile to study the observational limits on such processes.

The astrophysical environment provides excellent possibilities for such a study of very weak processes: Pathlengths of $\sim 10^{28}$ cm are available for the decay process to take place, huge in comparison with the $\sim 10^2$ cm available in most laboratory studies. Also, there are regions, such as the cores of very hot stars, where the weak process dominates, as the products of the competing electromagnetic channels are suppressed completely because of the enormous time scales needed for the diffusion of photons to the stellar surface.[42]

Recognizing that both neutral and charged currents are of comparable strength in weak interactions, one finds that there are several locations in nature where copious generation of ν_e and ν_μ, etc., takes place. Consider, at first, the possibility that the neutrino decays solely through the channel

$$\nu \to x + \gamma \qquad (17)$$

where x is any particle with a mass smaller than m_ν. The fraction of the neutrino energy carried by the photon is $\eta = (m_\nu^2 - m_x^2)/2m_\nu^2 \approx 0.5$ if m_x is small compared with m_ν. We can then make use of the observational limits on the photon intensities from the sources of neutrinos to place limits on the radiative stability of the neutrino. We shall proceed to derive these limits, considering sequentially, the cases where progressively longer times are available for the neutrinos to decay.

<u>Studies at Particle Accelerators</u>

Here the neutrino fluxes are generated by the decay of mesons and mesons produced by a high energy proton beam interacting with a target of nuclei. The neutrino fluxes at CERN have a mean energy of ~ 1 GeV and their decay will yield gamma rays of similar energy. These γ-rays would easily be detected in the spark chambers used in the "neutral current" experiment. The decay length available for the neutrinos is at least several meters ($\sim 10^2$ cm). We can safely assert that the number of gamma rays that would have been produced arriving in the direction of the beam due to the decay of neutrinos is less than the total number of neutral current events which have a similar signature. Equating the expected number of decays of neutrinos in a $\sim 10^2$ cm path with the number of ν-induced events ($\sigma \sim 10^{-39}$ cm^2) in a target of thickness $N_T \approx 10^{25}$ nuclei/cm^2, we have

$$\tau_\nu = \frac{F_\nu}{F_\gamma}\Delta t > \frac{1}{N_T\sigma}\frac{\ell}{c} \quad \text{or} \quad \tau_\nu > 3\times 10^6 \text{ s} . \qquad (18)$$

Since $\tau_\nu = \tau_o E_\nu / m_\nu c^2$ and $E_\nu \approx 10^9$ eV

$$\tau_o/m_\nu c^2 > 3\times 10^{-3} \text{ s/eV} . \qquad (19)$$

Atmospheric Neutrinos

Cosmic rays produce neutrinos[7,43] in the earth's atmosphere in a manner very similar to that described in the previous section and the events induced by these are recorded by instruments placed deep underground.[44] The mean energy of the neutrinos is again $\sim$ 1 GeV and the flux is $\sim 10^{-13}$ cm^{-2} s^{-1} sr^{-1}. The total event rate, R, is 10^{-13} cm^{-2} s^{-1} sr^{-1}. The decay length available at the apparatus is several meters. We can therefore write

$$\tau_\nu \frac{F_\nu}{R}\frac{\ell}{c} > 10^4 \text{ s} \quad \text{or} \quad \frac{\tau_o}{m_\nu c^2} > 10^{-5} \text{ s/eV} . \qquad (20)$$

Solar Neutrinos

If we make the eminently reasonable assumption that the energy generation in the sun is due to the synthesis of protons to helium nuclei, then at the earth we expect a neutrino flux of $\sim 10^{11}$ cm^{-2} s^{-1} irrespective of any specific solar models or details of nuclear reaction chains.[42] These neutrinos have a mean energy of $\sim$ 200 keV and their decay during the $\sim$ 500 s flight from the sun to the earth would result in an intense x-ray flux. The observations indicate that the x-ray flux from the quiet sun is below the level of detectability at $\sim 10^{-4}$ cm^{-2} s^{-1}; this yields

$$\tau_\nu > \frac{10^{11}\times 500}{10^{-4}} \approx 5\times 10^{17} \text{ s} \quad \text{or} \quad \frac{\tau_o}{m_\nu c^2} > 2\times 10^{12} \text{ s/eV} . \qquad (21)$$

White Dwarfs and Central Stars of Planetary Nebulae

It is well-known[45,46] that these stars cool rapidly by neutrino emission. An order of magnitude estimate of the neutrino fluxes emitted by such objects can be obtained by equating the gravitational energy released in their formation $\sim GM^2/R$, with the energy carried away by the neutrinos. With the assumptions $M = M_o = 2\times 10^{33}$ g and $R = 10^{8.5}$ cm, a total flux of $\sim 10^{58}$ neutrinos at $E_\nu \approx 100$ keV is radiated into the universe during the formation of such an object. The rate of formation of white dwarfs is ~ 1 yr^{-1} in a galaxy. With $M_{gal} \approx 10^{44}$ g and the mean density of the universe due to galaxies, $\rho_\mu \approx 10^{-31}$ g cm^{-3}, one will expect an x-ray flux from space of

$$F_\gamma \approx \frac{GM^2}{RE_\nu}\frac{\rho_\mu \tau_\mu R_\mu}{3\times 10^7 \times 10^{44}\,\tau_\nu} . \qquad (22)$$

Here $\tau_\mu \approx 10^{18}$ s is the "age" of the universe and $R_\mu \approx 10^{28}$ cm is its "radius". The observations of x-ray astronomers[47] yield a flux limit $F_{x\text{-ray}} < 10^{-1}$ cm^{-2} s^{-1} sr^{-1} which corresponds to

$$\tau_\nu > 10^{22} \text{ s} \quad \text{or} \quad \tau_o/m_\nu c^2 > 10^{17} \text{ s/eV} . \qquad (23)$$

Supernovae

These occur in a galaxy once in a hundred years and radiate energies about a few hundred times larger than emitted in the formation of a white dwarf. The mean energy of the radiated neutrinos is $\sim$ 10 MeV, and with the gamma-ray flux limits of $\sim 10^{-3}$ cm^{-2} s^{-1} sr^{-1} one obtains

$$\tau_\nu > 3\times 10^{23} \text{ s} \quad \text{or} \quad \tau_o/m_\nu > 3\times 10^{16} \text{ s/eV} . \qquad (24)$$

Falk and Schramm[14] have elaborated on this point lending further support to these results.

Neutrinos in "Big Bang" Cosmology

Noting that neutrinos of energy $\sim$ 100 keV live longer than 10^{22} s, one realizes that neutrinos generated during the condensed phase of the universe will have survived until the present epoch. Thus, the limits on the neutrino masses derived in the second section using the dynamical effects of the cosmological neutrinos on the present day expansion of the universe are valid. If m_ν is indeed larger than $\sim 10^{-3}$ eV/c^2, these neutrinos would have behaved as a non-relativistic gas during the expansion of the universe at redshifts $z < 1$ (i.e., during the relatively recent past of $\sim 3\times 10^{10}$ yr). This expansion would have slowed the neutrinos down to non-relativistic velocities and any hypothesis decay would yield photons with energy comparable to their rest mass. The number density of any one type of neutrinos and antineutrinos is ~ 400 cm^{-3} and yields a photon flux of

$$F_\gamma \approx \frac{n_\nu R_\mu}{\tau_\nu} \approx 10^{31}/\tau_o \qquad (25)$$

(assuming $\tau_o > \tau_\mu \approx 10^{18}$ s). The background photon flux has a maximum at the peak of the blackbody curve at 2.7° K, where the relic microwave should not distort the Planckian spectrum substantially, yields

$$\tau_o > 10^{19} \text{ s} \quad \text{if} \quad m_\nu \approx 10^{-3} \text{ eV}/c^2 . \qquad (26)$$

On the other hand, if $m_\nu \approx 1$ eV/c^2, their decay would yield a flux of optical photons. Using the observational limit on the background starlight flux of $\sim 3\times 10^8$ cm^{-2} s^{-1} provides the limit

$$\tau_o > 10^{23} \text{ s} \quad \text{if} \quad m_\nu \approx 1 \text{ eV}/c^2 . \qquad (27)$$

Such considerations are further worked upton recently by De Rujula and Glashow,[26] Stecker,[27] Kimble et al.[28] and by Shipman and

Cowsik.[30] In Table I, we summarize the limits derived in the last six subsections. Notice that the limits based on accelerators and atmospheric cosmic ray neutrinos are necessary to preclude the possibility that the neutrinos will decay inside the stellar sources themselves. The limits derived from the stellar sources allow us, in turn, to consider the neutrinos of cosmological origin and show that these neutrinos would have survived decay until the present epoch.

TABLE I - LIMITS ON NEUTRINO LIFETIMES

SOURCE	NEUTRINO DENSITY	E_ν	OBSERVATIONAL EVIDENCE	LOWER LIMIT τ_o/m_ν
Accelerators	10^8/pulse	∿GeV	Total no. of events	3×10^{-3} sec/eV
Cosmic Ray Neutrinos	$0.1 cm^{-2} sec^{-1} sr^{-1}$	∿GeV	$F_\gamma \lesssim 10^{-13} cm^{-2} sec^{-1}$	10^{-5} sec/eV
Sun	$10^{11} cm^{-2} sec^{-1}$	200 KeV	$F_\gamma \lesssim 10^{-4} cm^{-2} sec^{-1}$	$2\ 10^{12}$ sec/eV
White Dwarfs, etc	10^{58}/W.D	100 KeV	$F_\gamma \lesssim 10^{-1} cm^{-2} Sec^{-1} Sr^{-1}$	10^{17} sec/eV
Supernovae	4.10^{58}/SN	10 MeV	$F_\gamma \lesssim 10^{-3} cm^{-2} sec^{-1} sr^{-1}$	3×10^{16} sec/eV
Cosmos	$600\ cm^{-3}$	6×10^{-4} eV	$F_\gamma \lesssim 3 \times 10^8 cm^{-2} sec^{-1} sr^{-1}$ at 1eV	$\tau_o > 10^{23}$ sec
			$F_\gamma \lesssim 10^{13} cm^{-2} sec^{-1} sr^{-1}$ at 10^{-3} ev	$\tau_o > 10^{19}$ sec

Effect of Competing Channels of Neutrino Decay

Let us now consider, briefly, the consequences of the existence of competing processes to the radiative decay of the neutrino. For example, the very existence of the vertex $\nu \to x + \gamma$ implies a corresponding electromagnetic interaction of ν with the Coulomb fields of nuclei: $\nu + Z \rightleftarrows Z + x$. This interaction can possibly generate a comparatively rapid "stimulated decay" chain $\nu \to x \to \nu \to x \ldots$ during the transit of the neutrino from the stellar interiors to the surface. Since the stars do evolve by rapid loss of energy in some form, one can assert that the particles do escape from the star, still retaining a good fraction of their energy. Under these circumstances, the limits derived above are not affected substantially.

There is also the possibility, though theoretically unlikely, that other relatively rapid modes of decay besides $\nu \to x + \gamma$ exist, such as

$$\nu \to x + y + z + \ldots \qquad (28)$$

In this hypothetical decay, the sum of the masses of x, y, z,... should be less than m_ν and none of the particles can satisfy this condition, other than neutrinos and photons, of course. And yet, if any of x, y, z,... is easily observable, we may try to estimate the decay rate from the flux of the decay product and discuss its importance relative to radiative decay. If, however, the decay products have weak interactions alone, one may present the following arguments. Using the studies of ν-induced reactions at nuclear reactors and cosmic ray experiments deep underground, one can set $T_o^{\nu_e}/m_\nu c^2 \geq 3 \times 10^{-13}$ s/eV and $T_o^{\nu_\mu}/m_\nu c^2 \geq 3 \times 10^{-11}$ s/eV, where T_o is the lifetime due to all the decay modes. These limits are not very restrictive, and if $T_o/m_\nu c^2 < 10^{-5}$ s/eV, the neutrinos will decay inside the stars and the limits derived earlier will then apply to the radiative instability of the decay products x, y, z,...

On the other hand, one general consideration, the expected lifetime for the decay of neutrinos into only weakly interacting particles is rather long. Here I wish to consider two cases: (a) When the lifetime T for the competing process is longer than or comparable to the typical times involved in the discussions above, the limits remain essentially unaffected (i.e., if T > 500 s for the solar neutrinos and $T \geq 10^{18}$ s for the three previous cases. (b) If 10^{18} s < T < 1 s, then the neutrinos will escape the stellar regions ($T/m_\nu c^2 > 10^{-5}$ s/eV) and their radiative decay can become observable depending on the relative strengths of radiative decay and the other decay modes. Arguments presented above imply

$$\tau_o\ \nu_e/T_o^{\nu_e} > 10^{15} \quad \text{or} \quad \frac{\Gamma(\nu_e \to x + \gamma)}{\Gamma(\text{total})} < 10^{-15} \qquad (21')$$

$$\frac{\tau_o}{T_o} > 10^4 \quad \text{or} \quad \frac{\Gamma(\nu \to x + \gamma)}{\Gamma(\text{total})} < 10^{-4} \ \text{(all types)} . \qquad (23')$$

$$\frac{\tau_o}{T_o} > 3 \times 10^5 \quad \text{or} \quad \frac{\Gamma(\Gamma \to x + \gamma)}{\Gamma(\text{total})} < 3 \times 10^{-6} \ \text{(all types)} \qquad (24')$$

BINDING LARGE CLUSTERS OF NEUTRINOS WITH NEUTRINOS

We have shown in the second section that a very large number of neutrinos produced in the Big Bang fill the universe and that even if they should have a rest mass of only a few eV/c^2 they would dominate the gravitational dynamics of the universe. This is particularly true when we consider the fact that the cosmic density of all visible matter is only $\sim \rho_{crit}/100$. One consequence of this is that through their mutual gravitation interactions, neutrinos may have triggered the initial condensations that lead to the formation of clusters of galaxies. If this is true, then one may expect that in large clusters there might be substantial amounts of unseen mass in the form of neutrinos. This expectation is indeed borne out by the observations of several clusters of galaxies.[36,40] Several careful analyses of the peculiar motion of individual galaxies in the Coma cluster indicate that there is a substantial discrepancy between the sum of the masses of the individual galaxies (5×10^{48} g for $H_o \approx 50\ km\ s^{-1}\ m_{pe}^{-1}$) and the mass required to gravitationally bind the cluster ($\sim 4 \times 10^{49}$ g). These analyses also indicate that this discrepant mass is not a single large massive black hole but a smooth distribution sharply concentrated near the center following the general pattern traced by the galaxies themselves.

To check, qualitatively, the hypothesis that neutrinos of finite rest mass are responsible for this binding, we construct a simple model of the Coma cluster in the form of a gravitational potential well of constant depth extending over the core region of high galactic density ($R_c \approx 0.7\ m_{pc} \approx 2.1 \times 10^{24}$ cm). This well is then filled with neutrinos which are treated like a Fermi-Dirac gas at zero temperature. The total number of neutrinos that can be accommodated inside the well is given by

$$N_\nu = \frac{4V}{3\pi^2}\left(\frac{2m_\nu U_\omega}{\hbar^2}\right)^{3/2} . \qquad (29)$$

Here we have assumed two helicity states for each of ν_μ, $\bar{\nu}_\mu$, ν_e and $\bar{\nu}_e$. Also, m_ν = common rest mass assumed for each of the neutrinos; $V = 4/3\ R_c^3$ = volume of the potential well; $U_\omega = G M_c m_\nu/R_c \approx 1.3 \times 10^{17}$ erg g^{-1} = depth of the potential well = kinetic energy needed for the neutrinos to escape from the cluster. Notice that for M_c, we have used the total discrepant mass of the cluster $\approx 4 \times 10^{49}$ g.

Now, in order that the neutrinos may be responsible for binding the cluster, we set $M_c = N_\nu m_\nu$; substituting for U_ω in Eq. (29), we find

$$m_\nu^8 = \frac{3^4\pi^2 h^6}{2^{11} G^3 R_e^3 M_c} \approx 0.5\, \frac{h^6}{G^3 R_c^3 M_c} . \qquad (30)$$

Alternatively, following Landau and Lifshitz[50] the self-consistent equilibrium of a gravitating cloud of Fermi gas at zero temperature yields the result

$$m_\nu^8 \approx 6\, \frac{h^6}{G^3 R_c^3 M_c} . \qquad (31)$$

Because of the very high power of m_ν involved in Eqs. (28) and (29), they both yield essentially the same results: $m_\nu \approx 3 \times 10^{-33}$ g $\approx$ 2 eV/c^2. In Fig. 2, we show the surface density calculated in the self-consistent potential with the observed distribution of galaxies.[35]

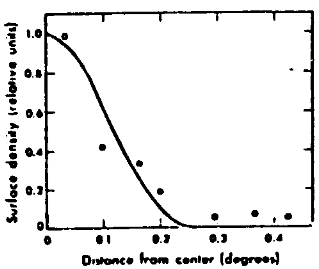

Fig. 2. The radial distribution of neutrino density compared with that of the galaxies. One has to include the effects of finite neutrino temperature to reproduce the tail in the distribution.

Notice the rather good fit between theory and observation in the central regions of the cluster. One should include effects of finite temperature of neutrinos to reproduce the long tail in the distribution of the galaxies. Tremaine and Gunn[18] have pointed out a very interesting point that the neutrinos with their weak interactions evolve as a collisionless gas and by Liouville's version, the thermal neutrinos will preserve phase space densities with occupation numbers 1/2 instead of 1 as implied in Eq. (29). Two points are worth noting in this regard: a) If the phase space distribution is lumpy to start with, then these lumps can be brought together through the dynamical friction with ordinary matter in the cluster which can radiate away energy and b) models with an occupancy number of 1/2 are not very different from the one that we have presented. Before closing this section, it is interesting to note that the total mass in neutrinos contained within the horizon at a red-shift z behaves as

$$M_H = \frac{4 \times 10^{53}(m_\nu c^2/1\ \mathrm{eV})}{(1+z)^{1.5}}\ \mathrm{g} . \qquad (32)$$

Equating M_H and the visual mass M_c of the Coma-cluster, we find $z \approx$ 400-1000 for $m_\nu c^2 \approx$ 1-10 eV. The kinetic energy of neutrinos at that time is $kT_\nu \approx 2 \times 10^{-7}\ (1+z)^2/(m_\nu c^2/1\ \mathrm{eV})$ and the Jeans mass M_J is of the order of M_H.

RADIATIVE STABILITY AND MASS OF THE TAU-NEUTRINO

Experiments performed at colliding e^+e^- [33] beams over the last few years have established the existence of a third sequential lepton, τ, at a mass of 1.78 GeV/c^2 along with an associated neutrino ν_τ. Though the properties of $\tau^\pm$ are being measured with progressively greater precision,[33,51,52] the only results[53,54] on ν_τ are that $m_{\nu_\tau} \leq 250$ meV/c^2 and its coupling has the usual strength and the V-A form of weak interactions. Further, there seems to be little hope of improving the mass limit in the foreseeable future. In order to delineate its properties, we therefore take recourse to the astrophysical methods, the efficacy of which are well demonstrated for the case of the other neutrinos. But, for the astrophysical arguments to be applicable, we have to first show that the ν_τ produced in the astrophysical setting lives long enough to have observable effects. To this end we utilize two accelerator experiments to show that $\tau_0/m_{\nu_\tau}c^2$ is longer than $\sim 10^{-6}$ s/eV. We then start the astrophysical discussions with a calculation of the effect of radiative decay on the primordial He and D abundance. Our discussion here is distinct from earlier work in that we consider the effect of photodissociation of the nuclei which indeed is the stronger and more direct effect as compared with the indirect effects of an increased radiation field discussed earlier. We then consider the emission of ν_τ from supernovae and show that their radiative lifetimes exceed the age of the universe. Thus, the large number density of ν_τ of cosmological neutrinos should conform to the upper limits on the mass expressed in Eq. (15).

Results from Accelerators

The generation and the decay properties of the τ lepton have been studied in detail by Bacino et al.[55] at SPEAR using the DELCO detector, which provides nearly a 4π coverage. They have measured the spectrum of electrons arising from $\tau^- \to e^- \bar{\nu}_e \nu_\tau$ and find an excellent fit to the theoretical expectations with standard V-A coupling for both the $\tau - \nu_\tau$ and $e - \nu_e$ vertices. If the ν_τ should decay inside their apparatus of dimensions ~ 20 cm giving rise to either a gamma ray or an electron, there would be a recognizable signal and it would contaminate and disturb the fine agreement between experiment and theory. Absence of any such effects indicate that the lifetime of ν_τ is longer than $\sim 10^{-9}$ s in the lab-frame; with the mean energy of ν_τ being ~ 500 MeV this corresponds to

$$\tau_o/m_{\nu_\tau}c^2 \geq 2\times10^{-18} \text{ s/eV} . \qquad (33)$$

This result is shown in Fig. 3 where cross-hatchings indicate forbidden regions in the $m_{\nu_\tau} - \tau_o/m_{\nu_\tau}$ plane.

We now turn our attention to the experiment performed by Heisterberg et al.[56] at Fermilab to measure the cross section for the process $\nu_\mu + e^- \to \nu_\mu + e^-$. The muon-neutrino beam was produced by allowing an immense fluence of 10^{19} protons at ~ 350 GeV to be incident on a target, one interaction mean free path thick. The pions and kaons which were produced in the interactions traversed a tunnel $\sim 5\times10^4$ cm long ($\ell/c \approx 1.7 \times 10^{-6}$ s) and the neutrinos generated in the decay of the mesons passed through a detector of cross-sectional area 10^4 cm^2 and depth $\sim 1.5 \times 10^3$ cm ($\ell/c \approx 5\times10^{-8}$ s). The detectors were sensitive to electrons and gamma rays and the cascades produced by the elastically scattered electrons were detected. In such a neutrino beam there would be a finite fluence of ν_τ also, generated mainly through the decay of $F\bar{F}$ mesons which are produced with roughly the same cross section as the $D\bar{D}$ mesons viz. $\sigma_{F\bar{F}} \approx \sigma_{D\bar{D}} \approx 100$ μb. Following the detailed discussion of Albright et al., we estimate the differential spectrum of $\nu_\tau + \bar{\nu}_\tau$ in the detector to be given by

$$F(E) \approx 3\times10^{11} \exp(-E/21.6 \text{ GeV})/\text{GeV} \quad \text{for} \quad E > 10 \text{ GeV}$$
$$= 0 \qquad\qquad \leq 10 \text{ GeV} . \qquad (34)$$

Now should ν_τ decay through either of the channels $\nu_\tau \to \gamma + x$ or $\nu_\tau \to e + x$, it would simulate $\nu_\mu e$-scattering events and would be recorded. An upper limit to the number of such events in their apparatus is given by the maximum number of background events of ~ 10. This leads to the inequality

$$\int_{15}^{E_{max}} \exp\left(\frac{-1.7\times10^{-6}\, m_{\nu_\tau}c^2}{\tau_o E}\right)\left[1 - \exp\left(\frac{5\times10^{-8}\, m_{\nu_\tau}c^2}{\tau_o E}\right)\right] F(E)dE < 10. \quad (35)$$

This translates into the result $\tau_o/m_{\nu_\tau}c^2 < 4\times10^{-19}$ s/eV or $\tau_o/m_{\nu_\tau}c^2 > 7\times10^{-7}$ s/eV, again displayed in Fig. 1. But in view of Eq. (1),

$$\tau_o/m_{\nu_\tau}c^2 > 7\times10^{-7} \text{ s/eV} . \qquad (36)$$

Limits from Cosmological He and D Abundances

Within the framework of the so-called "standard model" of the expanding universe,[34] neutrinos of mass in excess of 1 MeV go out of thermodynamic equilibrium very early and, as shown by Lee, Weinberg and Dicus et al., their density is controlled by a competition between production through a variety of channels and loss through annihilation.[15,16] Then at a temperature T_d corresponding to a redshift Z, they decouple with a density n_d and their subsequent evolution is controlled merely by expansion and possible decay. An empirical fit to the detailed calculations in the mass range 10 MeV < $m_\nu c^2$ < 1 GeV yields

$$n_d = 10^{32}\left(\frac{mc^2}{1 \text{ MeV}}\right)^{-0.45} \text{ cm}^{-3}$$
$$T_d = 5\times10^9\left(\frac{mc^2}{1 \text{ MeV}}\right)^{0.8} \text{ °K} \qquad (37)$$
$$(1 + Z_d) = 2.5\times10^9\left(\frac{mc^2}{1 \text{ MeV}}\right)^{0.8} .$$

The radiative decay of the neutrinos ($\nu \to \gamma + x$) would yield gamma-rays of energy $E_o \approx (m_\nu^2 - m_x^2)/2m_\nu \approx m_\nu/2$ and these gamma rays will suffer reduction in density due to effects of expansion and absorption mainly through the creation of e^+e^- pairs; their density at any time $t \gg t_d$ is given by

$$n_\gamma(> E_\gamma, t) = \int_{t_{min}}^{t} \frac{n_d}{\tau_o}\left\{\frac{1 + Z(t')}{1 + Z(t_d)}\right\}^3$$
$$\exp\{-(t' - t_d)/\tau_o - \sigma_\gamma c n_o \int_{t'}^{t} [1 + Z(t'')]^3 dt''\} dt' . \qquad (38)$$

Here, t_{min} is defined through the equation $E_o[1 + Z(t)] = E_\gamma[1 + Z(t_{min})]$. The total absorption cross sections of γ-rays, $\sigma_\gamma \approx 20$ mb and, the present day mean density of hydrogen in the universe, $n_o \leq 3\times10^{-6}$ cm^{-3}. In evaluating Eq. (6), it is convenient to use the empirical fits

$$[1 + Z(t)] \approx 5\times10^9\, t^{-1/2} \quad \text{(relativistic)};$$
$$[1 + Z(t)] \approx 10^{12}\, t^{-2/3} \quad \text{(non-relativistic)} \qquad (39)$$

corresponding to the early radiation dominated era and the later times dominated by non-relativistic matter, respectively.

Now $\sigma_{p.d}$, the cross section for the photodissociation of D and He is ~ 2 mb above the threshold energy E_T of ~ 4 MeV and ~ 20 MeV, respectively.[57] These cross sections are sufficiently large to produce substantial changes in composition before the thermalization of γ-rays. The effects of the photodissociation on the D and He abundance are indeed very complicated and have many interesting ramifications. But for the purposes of limiting the radiative decay of ν_τ we shall merely demand that the change effects in the abundance subsequent to the completion of nucleosynthesis at t_{syn} not be substantial. The logarithmic change in the abundance A of either nucleus is

$$\ell nA \approx - \int_{t_{syn}}^{t_u} c\, \sigma_{p.d}\, n_\gamma\, (> E_T, t)\, dt \tag{40}$$

where $t_{syn} \approx 300$ s and $t_u \approx 10^{18}$ s the present age of the universe. We can make the value of the integral in Eq. (8) small enough to be acceptable either by choosing $\tau_o \le t_{syn}$ or by choosing it very long, $\tau_o \ge t_u$. In these regions, we have

$$-\ell nA \ge 5\times10^8\, m^{-2.85}\, e^{-t_{syn}/\tau_o} \quad \text{for} \quad \tau_o \sim t_{syn}$$

$$\ge 2\times10^{24}\, \tau_o^{-1}\, M^{-2.85} \qquad \text{for} \quad \tau_o > t_u\,. \tag{41}$$

Assuming that the reductions in the abundances are not substantial, Eq. (9) yields the results: for $m_{\nu_\tau}c^2$ in the range 10 MeV to 250 MeV

$$\tau_o < 50 \text{ s} \quad \text{or} \quad \tau_o > 3\times10^{24}\, m^{-2.85}\,. \tag{42}$$

This region of restriction is again indicated in Fig. 1.

Limits from Studies of Supernovae

In a stellar collapse leading to a supernova explosion the core reaches temperatures of the order of 10-20 MeV and radiates away the gravitational binding energy of $\sim 10^{53}$ ergs in the form of neutrinos.[58-60] As previously discussed by us[11] and later by Falk and Schramm,[14] the observations of gamma-ray astronomy and the fact that they "do evolve by rapid loss of energy in some form" allows one to further restrict the radiative decay of the neutrinos. In estimating the emission of ν_τ we should keep in mind that as with ν_μ, only the processes involving neutral currents would be operative in the production process. These would be further suppressed if $m_{\nu_\tau} > 10$ MeV by a factor S which is approximately given by

$$S(m_{\nu_\tau}) \approx \frac{S_{th} \int_0^\infty p^2 [\exp \frac{\sqrt{p^2c^2 + m^2c^4}}{kT} + 1]^{-1}\, dp}{\int_0^\infty p^2 [\exp \frac{pc}{kT} + 1]^{-1}\, dp} \tag{43}$$

where S_{th} is a factor which takes into account departures from thermodynamic equilibrium. Taking $S_{th} \sim 1$ we estimate $S(mc^2 < 2kT) \approx 1$, $S(mc^2 \approx 5kT) \approx 0.125$, $S(mc^2 \approx 10kT) \approx 3.5\times10^{-3}$ and $S(mc^2 \approx 20kT) \approx 5\times10^{-7}$. Now should the ν_τ decay radiatively with an extremely short lifetime, then its energy will be fed back into the core and would be reradiated as ν_e and ν_μ. But if the lifetime is such that the decay takes place in the outer mantle between the radii of $\sim 10^7$ cm and $\sim 10^{12}$ cm the decay energy will heat up the mantle and will appear as kinetic energy of the debris. Observationally the debris has kinetic energies in the range 10^{49} - 10^{50} ergs and this precludes τ_o/m_{ν_τ} in the range $\sim 10^{-12}$ s/eV to $\sim 3\times10^{-3}$ s/eV for m_{ν_τ} below ~ 100 MeV. These regions are shown in Fig. 3.

Further, once $\tau_o/m_{\nu_\tau}c^2$ is longer than $\sim 10^{-6}$ s/eV, the neutrinos emitted from the supernovae will decay sufficiently far away from the core for the γ-rays to be observable. These γ-rays will have energies $\approx \frac{1}{2}m_{\nu_\tau}c^2$ or $\approx \frac{1}{2}kT \approx 5$ MeV, whichever is larger, and our earlier discussions[9] yield the result that

$$\tau_o/m_{\nu_\tau}c^2 > 10^{17} \text{ s/eV} \quad \text{for} \quad m_{\nu_\tau} < 20 \text{ MeV}/c^2$$

and

$$\tau_o/m_{\nu_\tau}c^2 > \left(\frac{m_{\nu_\tau}}{1 \text{ MeV}}\right)^{1.2} S(m_{\nu_\tau})\, 10^{17} \text{ sec/eV} \quad \text{for} \quad m_{\nu_\tau} > 20 \text{ MeV}/c^2 \tag{44}$$

since the flux of background γ-rays falls off as $E_\gamma^{-1.2}$ at high energies.

Limits from Radiation Background Below $\sim$ 20 eV

Now, notice that our arguments so far have precluded all values of τ_o/m_{ν_τ} shorter than $\sim 10^{17}$ s/eV. This means that if $m_{\nu_\tau} > 10$ eV, it will survive longer than 10^{18} s, the present age of the universe. Further, since the universe is transparent to photons up to a redshift of ~ 100, i.e., $t \approx 10^{15}$ s, the decay of neutrinos of $m > 10^{-2}$ eV will generate a background of radiation in the universe. By attributing the observed background radiation to the decay of the cosmological neutrinos, we can set a lower limit to the lifetime of the neutrinos. Noticing that neutrinos of such masses would have slowed down to non-relativistic velocities furing the expansion of the universe and following the analysis of Cowsik,[11] De Rujula and Glashow,[26] Stecker[27] and Kimble et al.,[28] we show in Fig. 1 the constraint that $\tau_o/m_{\nu_\tau} > 10^{23}$ s/eV at $m_{\nu_\tau} \approx 1$ eV[9] and the limit increases as $\sim \frac{1}{m}\exp(mc^2/4 \text{ eV})$ as the radiation background decreases exponentially. The limits are shown only up to a mass of ~ 35 eV/c^2 since larger masses are excluded unconditionally in any case.

The Limits on the Mass of the Tau Neutrino

The discussions in the previous sections have established that the lifetime of ν_τ is so long that the large numbers produced in the Big Band survive until today with their densities merely reduced by expansion of the universe. As discussed extensively before, if these neutrinos are massive, they would dominate the gravitational interactions in the universe. Following Lee and Weinberg[15] and Dicus et al.,[16] the mass region 1 MeV to 2 GeV is excluded and the

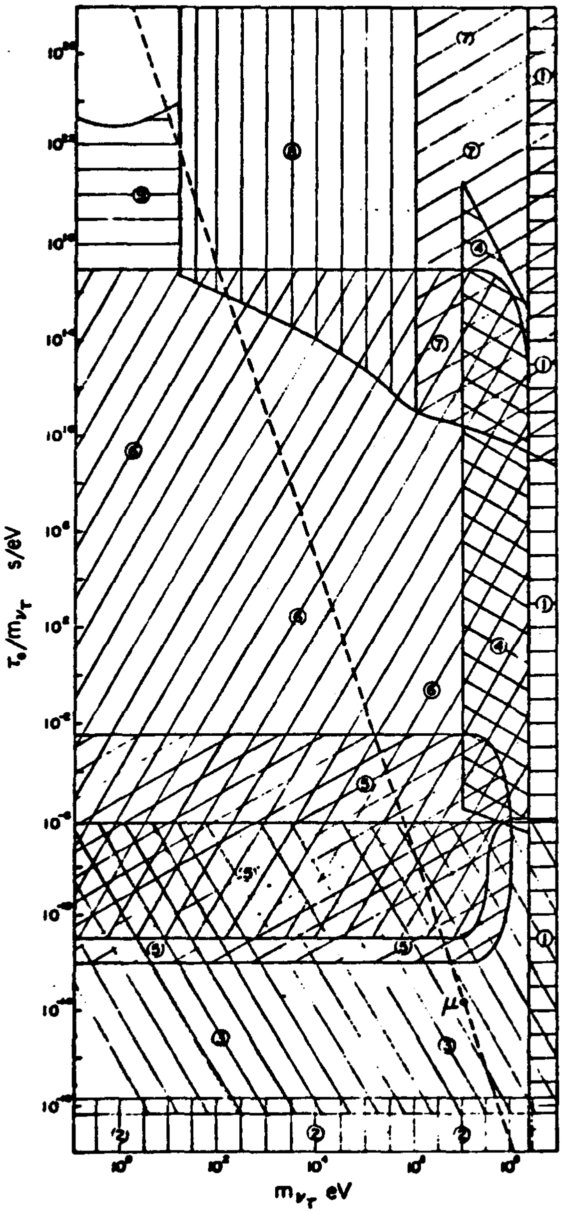

Fig. 3. Forbidden regions in the mass-lifetime plane of the τ-neutrino. 1) Decay kinematics of τ^-, 2) SPEAR experiment,[53] 3) Fermilab experiment,[54] 4) Photo-dissociation of cosmological D and He, 5) Supernova dynamics,[4] 6) γ-rays from supernovae, 7-8) Effects on universal expansion,[9,13,15,16] and 9) Optical and UV background radiation.

region 35 eV - 1 MeV is excluded by the neutrino density estimates of Cowsik and McClelland[9] and Szalay and Marx.[12] Thus we conclude

$$m_{\nu_\tau}c^2 < 35 \text{ eV} . \qquad (45)$$

For reasons of comparison, we have drawn a line which passes through the muon mass and lifetime and scaling as m^{-6} as expected in most models.

Thus, we have shown that the lifetime of ν_τ is longer than the age of the universe, which enables us to use the dynamical effects of cosmologically generated neutrinos to show that $m_{\nu_\tau} < 35$ eV/c^2.

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