HAS THE NEUTRINO A NON-ZERO REST MASS?*
(Tritium β-Spectrum Measurement)

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ABSTRACT

The high energy part of the β-spectrum of tritium in the valine molecule was measured with high precision by a toroidal β-spectrometer. The results give evidence for a non-zero electron antineutrino mass.

Fifty years ago Pauli introduced the neutrino to explain the β-spectrum shape. Pauli made the first estimate of the neutrino mass ($E_{\beta\ max} \cong$ nuclei mass defect): it should be very small or maybe zero. Up to now the study of the β-spectrum shape is the most sensitive, direct method of neutrino mass measurement.

For allowed β-transitions, if $M_\nu = 0$, then $S \simeq (E-E_0)^2$. The Kurie plot is then a straight line with the only kinematic parameter being $E_k = E_0$ (total β-transition energy). If $M_\nu \neq 0$, then $S \simeq (E_0-E)\sqrt{(E_0-E)^2-M_\nu^2}$. The Kurie plot is then distorted, especially near the endpoint.

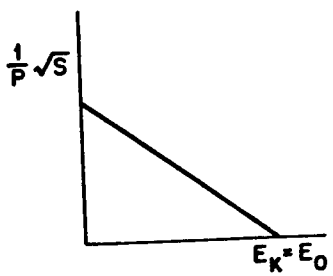

Fig. 1. Kurie plot for $M_\nu = 0$.

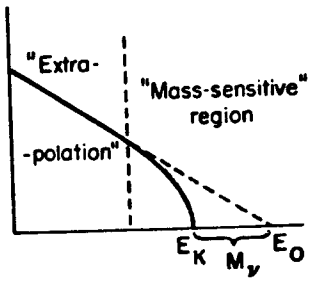

Fig. 2. Kurie plot for $M_\nu \neq 0$.

The method for the neutrino mass measurement is to obtain E_0 from the extrapolation and obtain E_k from the spectrum intercept. Then $M_\nu = E_0 - E_k$. Qualitatively, $M_\nu \neq 0$ if the β-spectrum near the endpoint runs below the extrapolated curve.

*Paper presented by Oleg Egorov.

In real life things are more complicated. The apparatus resolution R(E,E') strongly affects the spectrum endpoint and rather weakly affects the spectrum slope.

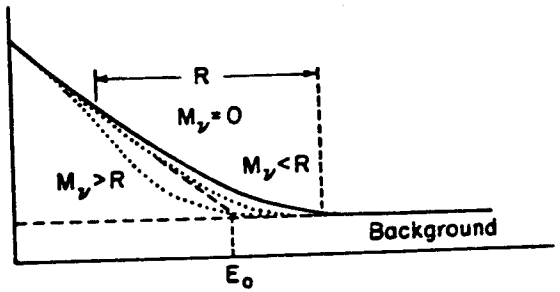

Fig. 3. Realistic Kurie plot.

E_0 can still be obtained by extrapolation. However, we are unable to get E_k. If $M_\nu > R$, then once again the lack of counts near the endpoint would indicate that $M_\nu \neq 0$. If $M_\nu \leq R$, the changes due to non-zero mass and the influence of R are indistinguishable. For M_ν determination the knowledge of R is compulsory. The background determines the statistical accuracy near the endpoint, i.e., in the region of the highest sensitivity to the ν mass. So: 1) R should be $\sim M_\nu$, 2) the smaller M_ν is, the smaller the background ($\sim M_\nu^2$) must be and the higher the statistics ($\sim M_\nu^{-3}$) must be. For example, suppose that for $M_\nu = 100$ eV we need resolution R, background Q, and statistics N. If $M_\nu = 30$ eV, to achieve the same $\Delta M/M$ they should be R/3, Q/10, and N × 30, respectively.

The shorter the β-spectrum, the less it is spread due to R (as $R \sim \Delta p/p = \text{const.}$). A classical example is ^3H β-decay, which has 1) the smallest $E_0 \sim 18.6$ keV, 2) an allowed β-transition, simple nucleus, and simple theoretical interpretation, 3) highly reduced radioactivity. The first experiments with ^3H were by S. Curran et al. (1948) and G. Hanna, B. Pontecorvo (1949). Using ^3H gas in a proportional counter, they obtained $M_\nu \leq 1$ keV. Further progress required magnetic spectrometer development. This allowed the resolution to be improved considerably, and L. Langer and R. Moffat (1952) obtained $M_\nu \leq 250$ eV. The best value was obtained by K. Bergkvist (1972): $R \sim 50$ eV and $M_\nu \leq 55$ eV.

The ITEP spectrometer is of a new type: ironless, with toroidal magnetic field (E. Tretyakov, 1973). The principle of the toroidal magnetic field focusing systems was proposed by V. Vladimirsky et al. (An example is a "Horn" of ν-beams.) It turns out that a rectilinear conductor (current) has a focusing ability for particles emitted perpendicular to the rotation axis. This system has infinite periodical focusing structure. The ITEP spectrometer is based on this principle.

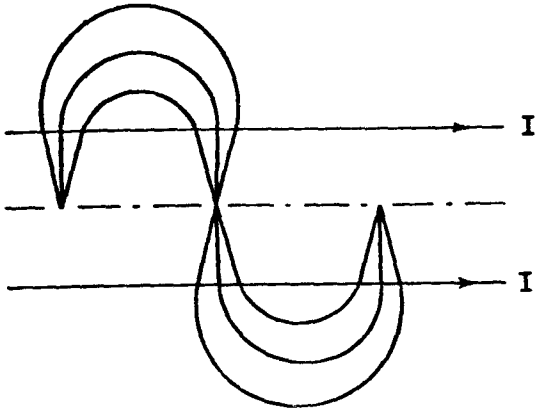

Fig. 4. Toroidal magnetic field focussing system.

The spectrometer. The solid angle is ± 7.5° (from a normal); 2 × 120° (azimuthally). The optical resolution is $\Delta p/p = 0.03\%$ for point-like (.5 mm) source and detector. The dispersion is $D = \Delta z/\frac{\Delta p}{p} = 3700$ mm. At a displacement of $\Delta z = 1$ mm, the energy changes by $\Delta E = 10$ eV ($E = 18500$ eV). At 18.4 keV for the used source and a detector size of 2.7 mm, the optical resolution is $\Delta p/p = 0.12\%$ or 45 eV at the spectrum endpoint. The focal plane size is 20×20 mm^2. Within these limits there is no significant change of resolution.

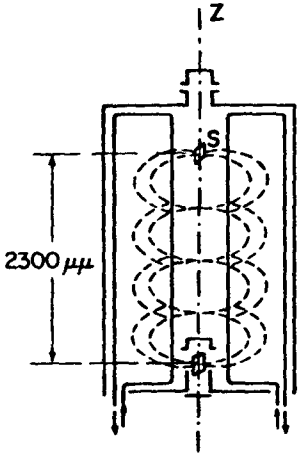

Fig. 5. Schematic view of the spectrometer.

The main (working) source. The substance used was valine ($C_5H_{11}NO_2$) enriched by tritium with 2 atoms of T per mole (120 g/mol.). Its specific activity was 10^{-3} Ci/cm^2, and its thickness ~ 2 μg/cm^2. T-valine was put on the Al foil (2.8×20 mm^2) on both sides by vacuum evaporation. To increase the activity, the 9 identical sources were put along the z-axis (step 3 mm) so the whole focal plane size was used. The sources were put under additional electric potentials in order to ensure focusing from different source planes into the same detector gap (Bergkvist method). At a source thickness of $\sim 2.10^{-6}$ cm, the potential difference between source and support due to β-decay is negligible ($< 10^{-3}$ v).

The detector. The window size was 2.7×20 mm^2 for all detectors. In the first part of the experiment (published in 1976), a Geiger counter was used (window thickness: $\delta = 75$ μg/cm^2; pressure 120 torr). In the second, a proportional counter was used ($\delta = 150$ μg/cm^2; pressure 750 torr). In the last, three sense wire proportional counters (proportional chamber) were used. The pulse height information from the proportional detectors allowed the background to be reduced (by an order of magnitude). The three-wire detector, using the whole focal plane, permitted statistics to be collected at three energies simultaneously.

The calibration. The internal conversion electron lines of Yb^{169} were used for the apparatus calibration. There are many lines in the 10 - 60 keV range. Some of them are very near to the endpoint of ^3H (the M-lines of $E_\gamma = 20.4$ keV: 18.4, 18.6, 18.8 keV). The calibration accuracy was ~ 2 eV. The relative widths of the lines do not depend on the energy in the 10 - 60 keV range ($\Delta p/p = \text{const}$).

The apparatus time drift. To control the stability, the M_I Yb (18.4) line from the Yb + .5L T-valine source was measured once a day.

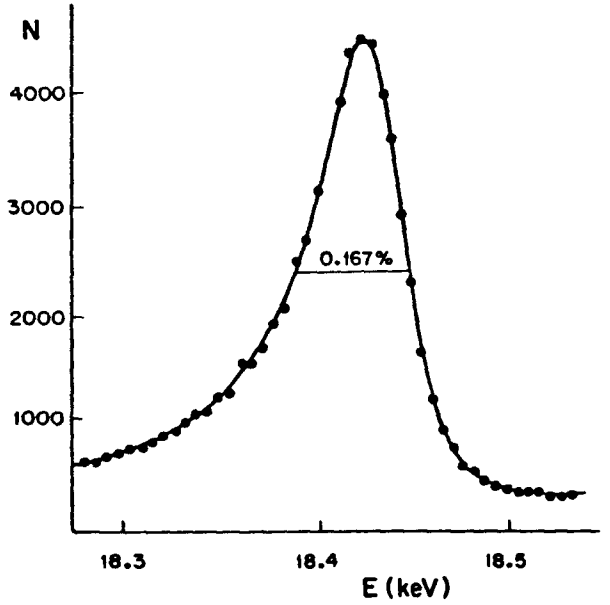

Fig. 6. M_I line from the Yb + .5L T-valine source.

In Fig. 6 the solid line was averaged over eight days of running (one "standard" sample). The circles are the first-day line.

Total resolution function of the apparatus with the working source. The total resolution function R is determined on the one hand by the optics ($\Delta p/p = 0.12\%$) and on the other hand by distortions due to electrons traversing the source body. The knowledge of R is of principal importance since in our case $M_\nu < R$. A question arises: Why not try to make the resolution of the spectrometer still better and search for an effect which is within the apparatus resolution? The answer is: Due to rearrangement of atomic levels after a β-decay $^3H \to {}^3He$, 70% of decays only end up in the ground state, whereas 30% go into excited states, which are effectively separated by $\geq$ 40 eV from the ground one. This means that the resolution would be ultimately determined by the physics of the level structure, and it does not help much to go with R below 40 eV. So, the problem of M_ν being less than R is unavoidable.

The β-emission from the working source may be expressed as a sum of the particle yields from separate layers:

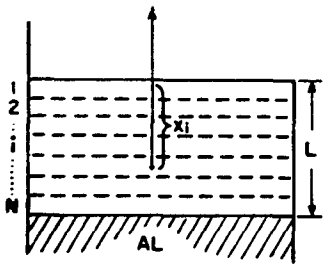

Fig. 7. Schematic of a general source.

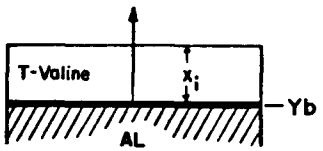

Fig. 8. Schematic of the source with a layer of T-valine on Yb.

$$R = \frac{1}{N} \sum W(X_i, p \to p') \ . \qquad (1)$$

$W(X_i, p \to p')$ may be determined by a separate measurement. A number of sources were made with different thicknesses of T-valine put on an Yb source. Thus the shape of the Yb (M_I, 18.6 keV) line was measured for different thicknesses X_i. On the other hand, the β-spectrum of T-valine was measured and its intensity gave the layer thickness as a fraction of the thickness of the working source. As seen in Fig. 9, the shape change is a linear function of the thickness, and therefore irregularities of a layer thickness do not affect the line shape:

$$R = \frac{1}{N} \sum W(X_i) = W(0.5\ L) \ . \qquad (2)$$

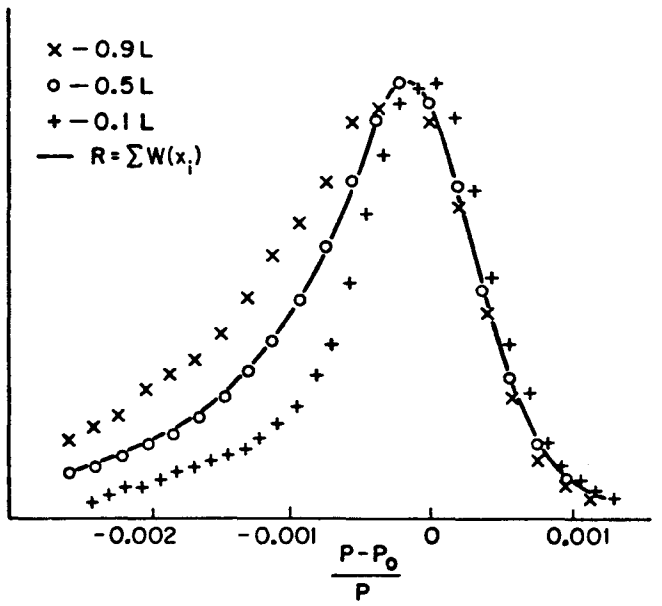

Fig. 9. Shape of the Yb(M_I,18.6 keV) line for different thicknesses of T-valine.

β-spectrum measurements. Sixteen data samples were taken during the four-year data-taking period (1975-78). Each sample is in fact an independent complete experiment (with calibration, R determination, etc.) The results of the first four samples ($M_\nu \leq 35$ eV) were published in 1976. The present total statistics are 1.5×10^7 counts or $\sim 10^5$ counts in the mass-sensitive region of $\sim$ 100 eV. A "standard" sample consists of 112 spectrometer current points. The measured interval is $\Delta E \sim 720$ eV (15 points beyond the endpoint - for the background). The measurements are from point to point ($\sim$ 100 sec/point) to and fro. The total measurement time per point in a sample is $\sim$ 6000 sec. Thus one sample means a 60-fold spectrum measurement (so the possible apparatus drift was averaged over many cycles).

The background. As a consequence of the 720° rotation angle and the use of proportional detectors, we have: 1) low background level ($\sim$ cosmic rays): 0.03 - 0.05 counts/sec, 2) the background independent of the spectrometer current, 3) no change with the source in or out.

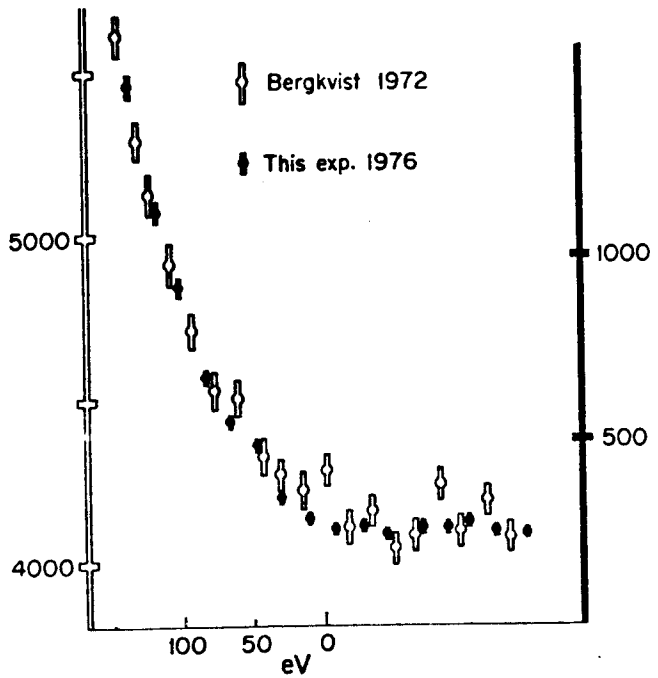

Fig. 10. β-spectrum measurements.

The data analysis. For $M_\nu < R$, the distortion of the spectrum due to $M_\nu \neq 0$ and R is indistinguishable. The mass effect cannot be seen directly by the eye. Our aim was to make the analysis independent of experimental conditions as well as the unknown atomic level structure of the T-valine molecule (our β-source).

To illustrate our approach, let S_{true} be the ^{3}H-in-valine β-spectrum in nature

$$\underset{\text{(unknown)}}{S_{true}} \xrightarrow{\text{measurement}} \underset{\text{(known)}}{\boxed{R}} \rightarrow \underset{\text{(measured)}}{S_{exp}} \tag{3}$$

One takes a theoretical model:

$$S_{model} \rightarrow \boxed{R} \rightarrow S_{theory} . \tag{4}$$

By comparing S_{theory} with S_{exp}(χ^2-minimization), one gets the unknown parameters (including M_ν). In this case the obtained parameters will have a physical meaning only if S_{model} is an adequate representation of S_{true}. The latter has to be proved. Instead of trying to prove this and then answering the question: "What is the value of the neutrino mass?", we are rather going to answer the question: "Is $M_\nu = 0$ compatible with our experimental data?"

We subdivide S_{model} into "known" and "unknown":

$$S_{model} = S_{known} \cdot \psi_{unknown} . \tag{5}$$

In $\psi_{unknown}$ we take the extreme values which yield the minimal M value: $\psi_{unknown} \rightarrow \psi_{limit}$.

$$S_{known} \cdot \psi_{lim} \rightarrow \boxed{R} \rightarrow S_{lim} . \tag{6}$$

In other words, we are not going to measure M_ν, but instead set a lower limit and find whether the neutrino has a non-zero mass at all.

The β-spectrum model. With one final state E_o:

$$S_{model}(p) = S_S(p) \cdot \psi(p) . \tag{7}$$

$S_S(p)$ is known (for $M_\nu = 0$), while $\psi(p)$ is unknown.

$$S_S(p) = F(z,E) \cdot p^2 (E_o - E) \sqrt{(E_o - E)^2 - M_\nu^2} \tag{8}$$

where F(z,E) includes the effects of Fermi motion. After "the measurement":

$$S_{theory}(p) = A' \int S_S(p') \cdot \psi(p') \cdot \varepsilon(p') \, R(p',p) \alpha p + \Phi \tag{9}$$

where $\varepsilon(p')$ is the efficiency. Let us make an expansion of $\psi(p') \cdot \varepsilon(p')$ and sum:

$$S_{theory}(p) = A \sum_i F(E_i) \cdot p^2 \cdot (E_o - E_i) \sqrt{(E_o - E_i)^2 - M_\nu^2} \times \times (1 + \alpha(p_i - \tilde{p})) \cdot R_{ik} + \Phi . \tag{10}$$

So we have five parameters: M_ν, E_o, A, ϕ, α. And finally, one should take into account the effect of the rearrangement of the atomic level structure

$$S_{theory} = 0.7 \, S_{theory}(E_o) + 0.3 \, S_{theory}(E_o - 43 \text{ eV}) . \tag{11}$$

This is correct for the atomic state of ^{3}H.

The compatibility. First of all, one should check whether the model is compatible with the data. To find the parameters, minimize the χ^2:

$$\chi^2(H_\nu, E_o, \alpha, A, \phi) = \sum_k \left(\frac{S^k_{exp} - S^k_{theory}}{\sqrt{S^k_{exp}}} \right)^2 . \tag{12}$$

$\sqrt{S^k_{exp}}$ is the minimal possible error = $\sqrt{N}$. In reality, $\sigma = r\sqrt{N}$. (The parameters do not depend on r.) The hypothesis may be accepted if r is not far from 1.

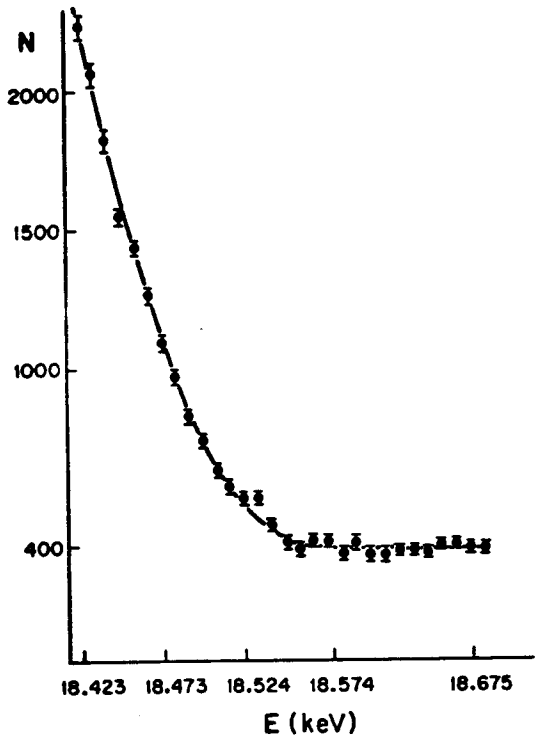

Fig. 11. One sample of β-spectrum measurements.

One sample of β-spectrum measurements (from 16) is shown in Fig. 11. It gives the parameters M_ν = 42 eV, E_o = 18578, ϕ = 392, χ^2 = 128, and N_D = 112 - 5 = 107. So $\chi^2_N = \chi^2/N_D$ = 1.20. Figure 12 shows the histogram of χ^2_N for all sixteen samples of β-spectrum measurements. The histogram width is in accordance with the expected (from N_D) χ^2-distribution, but shifted: $\overline{\chi^2_N}$ = 1.16. From this, we obtain r ≤ 1.08.

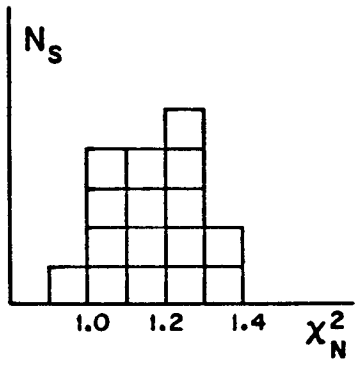

Fig. 12. Histogram of χ^2_N.

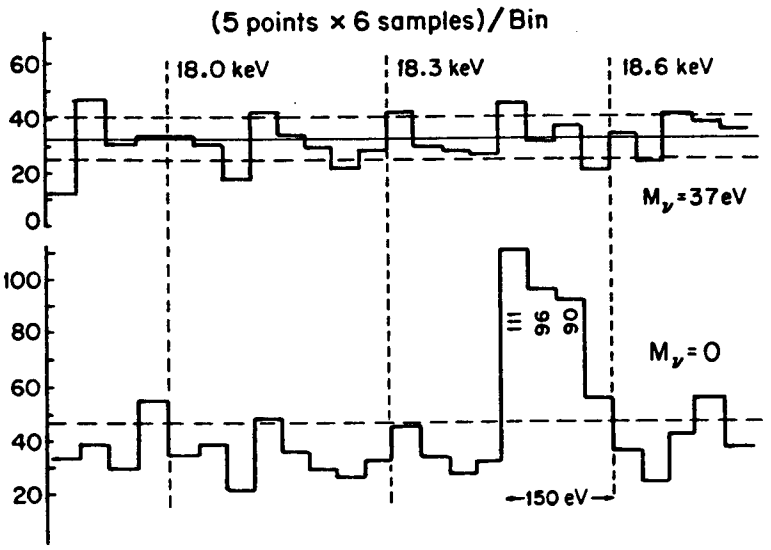

Fig. 13. χ^2 distribution versus electron energy from 6 samples with 5 points per bin. For M_ν = 37 eV, $\sum\chi^2$ = 8.2 and $\overline{\chi^2}$ = 33 ± 8. For M_ν = 0, $\sum\chi^2$ = 1053 and $\overline{\chi^2}$ = 36 ± 8.

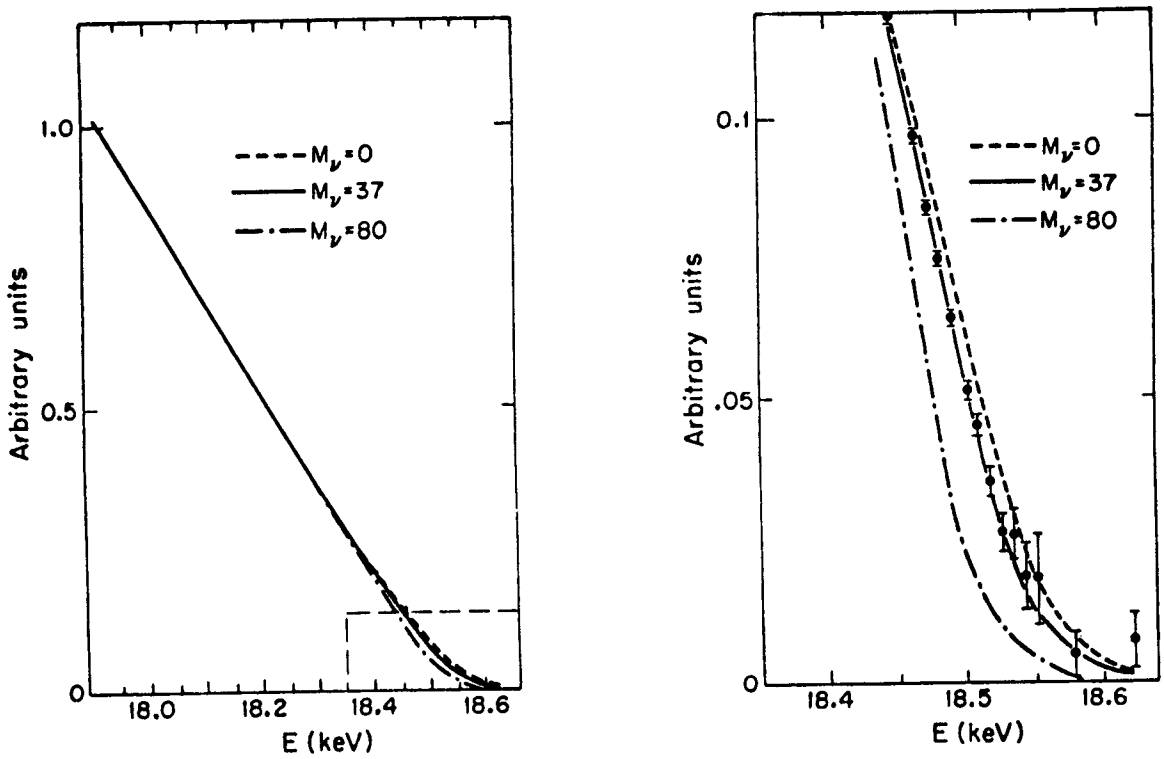

Fig. 14. Kurie plot with half of the statistics.

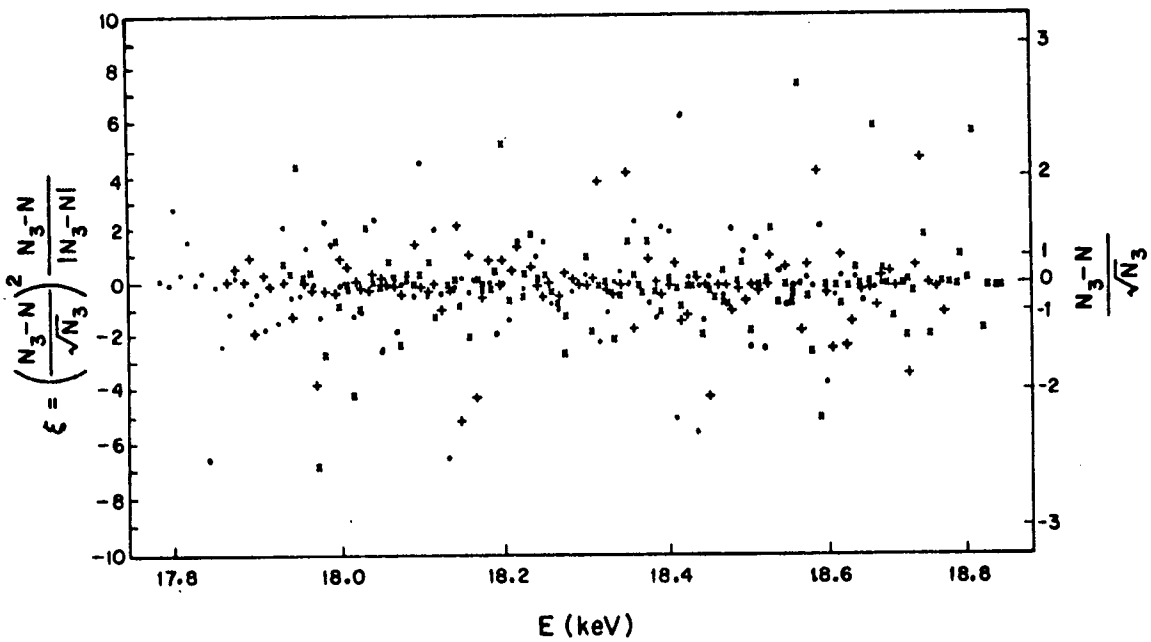

Fig. 15. Residuals as a function of energy.

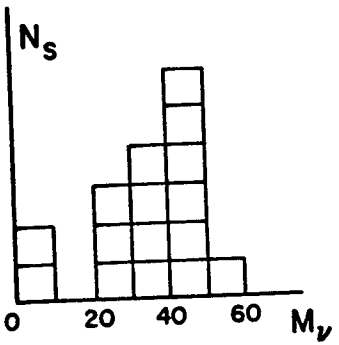

Fig. 16. Distribution of the parameter M_ν in 16 samples.

The behavior of the residuals along the energy axis in Fig. 15 shows no systematic deviation in any part of the energy range. Our conclusion is that the model is compatible with the experimental data, and that the data quality is satisfactory (r-1 ≤ 8%).

Sixteen experimental samples were analyzed independently, each sample being in fact a complete experiment (with calibration and R determination). The sixteen samples have already enough statistics to give the parameter distribution: $\overline{M}_\nu$ = 34 eV, σ(one sample) = 14 eV and $\sigma(\overline{M}_\nu)$ = 3.5 eV. Thus $\overline{M}_\nu$ is different from zero by ∿ 10 standard deviations.

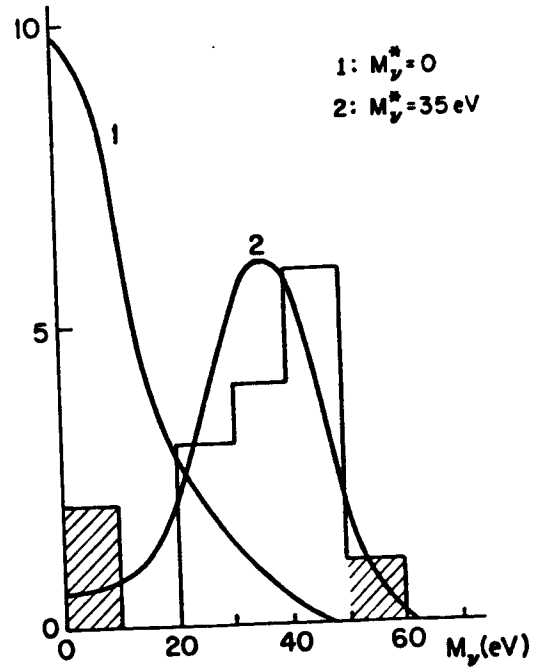

Fig. 17. $P(M_\nu|M_\nu^*)$ for M_ν^* = 0 and M_ν^* = 35 eV superimposed on the M_ν histogram.

$P(M_\nu|M_\nu^*)$, the Monte Carlo expected function for M_ν with M^* fixed (by the device R), is shown in Fig. 16 for M_ν^* = 0 and M_ν^* = 35 eV. The compatibility of $P(M_\nu|M_\nu^*)$ with the M_ν histogram is displayed in Fig. 17. χ^2_{min} = 8 is attained at M_ν^* = 35 eV with N_D = 6.

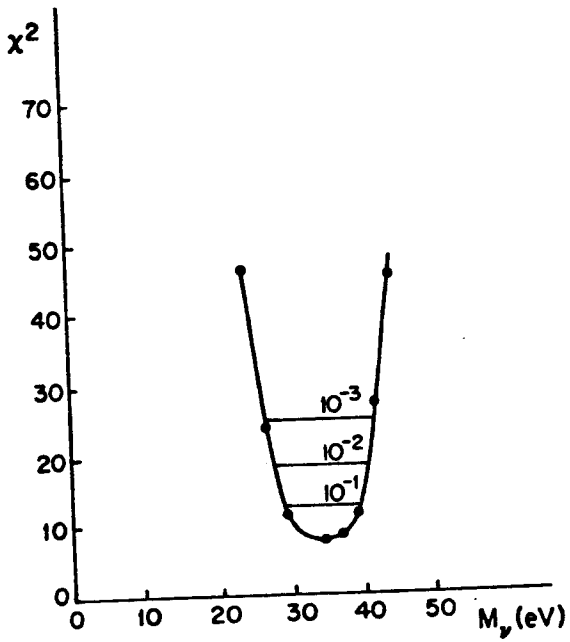

Fig. 18. χ^2 for the compatibility of $P(M_\nu|M_\nu^*)$ with the M_ν histogram as a function of M_ν^*.

We obtain the limits 28 ≤ M^* ≤ 41 eV at the 99% confidence level.

How much is M_ν changed when one varies R? We note that if R gets narrower, then M_ν becomes smaller. We have analyzed the data using R corresponding to the source with thickness one-half of the working one. In this case $\overline{M}_\nu$ (or M_ν^*) became smaller by 7 eV. The difference (R-R$_{\frac{1}{2}}$) is higher by an order of magnitude than we can assume from any experimental inaccuracy in R. (See R-W(0.5L) difference.)

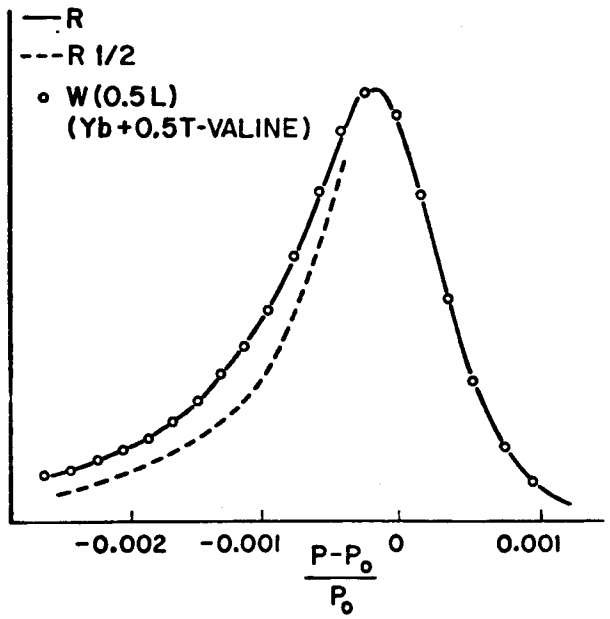

Fig. 19. Resolution function for two different source thicknesses.

M_ν^* is statistically incompatible with zero. What is the physical meaning of this statement? Our conditional conclusion is that the real neutrino mass is $28 < M_\nu^* < 41$ if the model is an adequate representation of the true β-spectrum of the ^{3}H-in-valine molecule, or if the T-valine spectrum is the same as the atomic tritium spectrum (accepted model). However, the latter statement we cannot prove.

The neutrino mass lower limit. Let us ask ourselves: are there any effects which can imitate the neutrino mass? Is it possible that $M_\nu = 0$ in nature, but that in the experiment we have some non-zero value? Let us look at where there can be a difference between S_{model} and S_{true}. There are two equivalent ways to account for final states of the system emerging from β-decay $(\omega_i, \Delta E_i)$: The first, which is just what we did before, is:

$$S_{theory} = (\sum \omega_i \, S(\Delta E_i)) * R \, . \qquad (13)$$

The second is:

$$S_{theory} = S(E_o)(\sum \omega_i \, R(\Delta E_i)) = S(E_o) * R^{SUM} \, . \qquad (14)$$

$S(E_o)$, the spectrum from the ground state transition, is adequately represented by $S_{true}(E_o)$.

As far as the valine final states spectrum $(\omega_i, \Delta E_i)$ is concerned, it is unknown. This means that R^{sum}_{valine} may be wider as well as narrower than $R^{sum}_{atom\ ^3H}$. But we know that the narrower R is, the smaller the resulting M_ν. The narrowest possible R^{sum} arises if in the spectrum $\omega_i, \Delta E_i$ we take only one final state:

$$R^{sum}_{min} = R \qquad (15)$$

This extreme situation is the same for whatever chemical composition or physical state (solid, crystal, etc.) of the material is used. Having built the S^{limit}_{theory}, we get a lower limit for M_ν. If the $P(M_\nu|M_\nu^*)$ are not completely adequate for the experiment, then the tails are the first to be affected. We have made the analysis with the extreme channels of the histogram being disregarded. The result is shown by the dashed curve in Fig. 20.

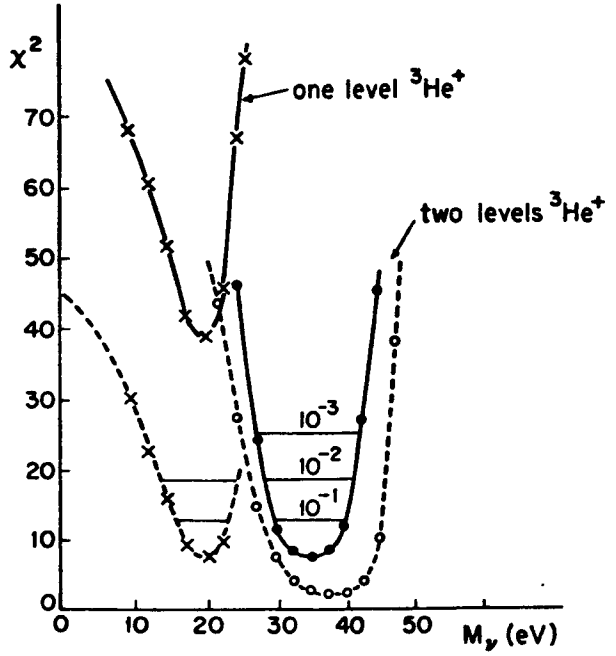

Fig. 20. χ^2 for the compatibility of $P(M_\nu|M_\nu^*)$ with the M_ν histogram, disregarding the extreme channels.

Conclusions. The conclusions that are independent of the source material are:

1. The $M_\nu = 0$ hypothesis is incompatible (statistically at a high confidence level) with our experimental data. This indicates that at least one neutrino has a non-zero mass.

2. $14 < M_{\tilde{\nu}_e} < 46$ eV at a 99% confidence level if $\tilde{\nu}_e$ is a mass eigenstate.

The conclusions that are dependent on the source material are:

3. If the final state spectrum of ^{3}He in the source corresponds effectively to the one final state spectrum*, then

$$14 < M_{\tilde{\nu}_e} < 26 \text{ eV} \quad \text{at a 99\% confidence level} \tag{16}$$

if $\tilde{\nu}_e$ is a mass eigenstate.

4. If the final state spectrum of ^{3}He corresponds effectively to the atomic tritium spectrum ($\omega_2 = 0.3$; $\Delta E_2 = 43$ eV), then

$$24 < M_{\tilde{\nu}_e} < 46 \text{ eV} \quad \text{at a 99\% confidence level} \tag{17}$$

For the time being we do not see any effects which could have essentially changed these limits.

*In a complex system (like valine) there may be numerous levels. However, if $\Delta E_i < R$ and $\sum \Delta E_i \lesssim R$, then the system will effectively be almost a one-level system (at FWHM, $R \approx 54$ eV).

SOME PHENOMENOLOGICAL CONSIDERATIONS OF NEUTRINO OSCILLATIONS IN VACUUM AND MATTER

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ABSTRACT

Neutrino oscillation phenomena are reviewed, including indications from solar and reactor experiments, accelerator limits, CP violation tests, and deep mine possibilities for measuring vacuum oscillations and matter corrections.

INTRODUCTION

The weak interaction eigenstates of neutrinos ν_α (with $\alpha = e,\mu,\tau$) are related to the mass eigenstates ν_i (mass m_i with $i = 1,2,3$) by a unitary transformation

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}|\nu_i\rangle \ . \tag{1}$$

For a ν_α state of momentum p, the time evolution is

$$|\nu_\alpha\rangle = \sum_i e^{-iE_i t} U_{\alpha i}|\nu_i\rangle = \sum_{i,\beta} e^{-iE_i t} U_{\alpha i}U^*_{\beta i}|\nu_\beta\rangle \ . \tag{2}$$

At a distance $L \simeq t$ from a relativistic ν_α source, the $\nu_\alpha \to \nu_\beta$ transition probability is

$$P(\alpha\to\beta) = |\sum_i e^{-i\Delta_{in}} U_{\alpha i}U^*_{\beta i}|^2 \tag{3}$$

where

$$\Delta_{in} = \frac{(m_i^2-m_n^2)L}{2E} = \frac{\delta m_{in}^2 L}{2E} \ . \tag{4}$$

In units $\delta m^2(\text{eV}^2)$, L(m), and E(MeV), $\Delta/2 = 1.27\delta m^2 L/E$. In the special case of oscillations involving two neutrinos only (e.g., ν_e,ν_μ), the mixing matrix is real

$$U = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \tag{5}$$

and the transition probabilities are given by

$$\begin{aligned} P(e\to e) &= P(\mu\to\mu) = 1 - \sin^2 2\alpha \sin^2{}^1\!/_2\Delta \\ P(e\to\mu) &= P(\mu\to e) = \sin^2 2\alpha \sin^2{}^1\!/_2\Delta \ . \end{aligned} \tag{6}$$