Note on ep Collisions

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I. Introduction

We consider a scheme for electron-proton colliding beams in which an electron beam of $E_e \approx 140$ GeV collides with a proton beam of $E_p \approx 20$ TeV. The parameters are chosen such as to achieve a luminosity of $L \approx 10^{32}\ \mathrm{cm^{-2}s^{-1}}$. The center of mass energy of the proposed system is 3.3 TeV. A possible configuration of ep-system is also proposed as an option of a $p\bar{p}$ colliding-beam plan.

Most ring parameters for ep collisions will be determined similar to the studies on electron and proton accelerators at Group 1 and Group 2.[1)2)] The bending radius of the proton ring is 6.7 km assuming a bending field of 10 T. For the electron ring, we consider two specific examples where the bending radii are scaled as $\rho \propto E_e^2$ and $\rho \propto E_e^{3/2}$ from existing designs. The first example is suitable to the storage ring operation while the latter may be used for a synchrotron operation. If we take the $\rho \propto E_e^2$ rule, then the same bending radius as for the protons corresponds to an electron energy of 120 GeV with the field strength of 0.06 T.

2. Luminosity Optimization

Because of the asymmetric nature of the two beams, the choice of parameters to optimize the luminosity for ep collisions is a more complicated problem than for e^+e^-, $p\bar{p}$ or pp collisions.

First we consider the case in which both the electron and proton beams are bunched, and then a burched electron beam colliding with a coasting proton beam. For simplicity, we assume a head-on collision and neglect the variation of beam radius along the longitudinal direction, z.

2-1. Case I (Bunched Proton Beam)

For bunched beam collisions, the luminosity is given by[3)]

$$L = f \frac{N_e N_p}{2\pi k_b} \frac{1}{\sqrt{\sigma_{ex}^2 + \sigma_{px}^2}\sqrt{\sigma_{ey}^2 + \sigma_{py}^2}} , \tag{1}$$

where f is the revolution frequency, k_b the number of bunches in each beam, N the total number of particles, and σ the r.m.s. beam radius of each beam in each direction. Suffixes e and p denote electrons and protons respectively, and x and y the horizontal and vertical directions.

The linear tune shifts are given by

$$\Delta\nu_{ex} = \frac{N_p r_e \beta_{ex}}{2\pi k_b \gamma_e (\sigma_{px} + \sigma_{py})\sigma_{px}} \tag{2}$$

$$\Delta\nu_{ey} = \frac{N_p r_e \beta_{ey}}{2\pi k_b \gamma_e (\sigma_{px} + \sigma_{py})\sigma_{py}} \tag{2'}$$

for electrons and

$$\Delta\nu_{px} = \frac{N_e r_p \beta_{px}}{2\pi k_b \gamma_p (\sigma_{ex} + \sigma_{ey})\sigma_{ex}} \tag{3}$$

$$\Delta\nu_{py} = \frac{N_e r_p \beta_{py}}{2\pi k_b \gamma_p (\sigma_{ex} + \sigma_{ey})\sigma_{ey}} \tag{3'}$$

for protons, respectively. Here the β's are the β-functions at the crossing point, γ's the relativistic energy factors, and r's classical radii of particles. Again, the suffixes identify the kind of particles and the directions.

To simplify the following discussions, we assume that

$$\begin{aligned} &\sigma_{ex} = \sigma_{px} \equiv \sigma_x \quad , \qquad \sigma_{ey} = \sigma_{py} \equiv \sigma_y, \\ &\Delta\nu_{ex} = \Delta\nu_{ey} = \Delta\nu_e = 0.06 \\ &\Delta\nu_{px} = \Delta\nu_{py} = \Delta\nu_p = 0.005 \ . \end{aligned} \tag{4}$$

These assumptions imply an appropriate choice of machine and beam parameters. For example, the β-functions for electrons and protons should satisfy the following relations;

$$\frac{\beta_{ex}}{\beta_{ey}} = \frac{\beta_{px}}{\beta_{py}} = \frac{\sigma_x}{\sigma_y} \tag{5}$$

and

$$\frac{\beta_{ex}}{\beta_{px}} = \frac{\beta_{ey}}{\beta_{py}} = \frac{r_p}{r_e}\frac{\gamma_e}{\gamma_p}\frac{\Delta\nu_e}{\Delta\nu_p}\frac{N_e}{N_p} \quad . \tag{6}$$

We can rewrite eq.(6) as

$$\frac{\beta_{ex}}{\beta_{px}} = \frac{\beta_{ey}}{\beta_{py}} = 0.084\ a, \tag{6'}$$

where $a = N_e/N_p$.

With the additional assumption that $\sigma_y \ll \sigma_x$, we obtain the standard expressions for the luminosity

$$L = \frac{N_p f \gamma_p \Delta\nu_p}{2 r_p \beta_{py}} = \frac{N_e f \gamma_e \Delta\nu_e}{2 r_e \beta_{ey}} , \qquad (7)$$

similar to those for symmetric collisions. Taking the proton parameters used for $p\bar{p}$ collisions[4] as

$$\frac{N_p}{k_b} = \frac{1}{4} \times 10^{12}$$

$$\beta_{py} = 6.2 \text{ m} \qquad (8)$$

and $\qquad f = 4.8$ kHz,

we obtain

$$L = 0.69 \times 10^{30}\ k_b\ \text{cm}^{-2}\text{s}^{-1}.$$

For $L = 10^{32}$ cm^{-2}s^{-1}, this requires $k_b = 145$. If we take $a = 1$, i.e. $N_e = N_p = 3.6 \times 10^{13}$, then eq.(6') yields

$$\beta_{ey} = 0.52 \text{ m},$$

which is a reasonable value for the β-function of the electron ring.

It follows from eq.(4), (5) and (6), that the luminosity for $\sigma_x \approx \sigma_y$ becomes twice that for $\sigma_x \gg \sigma_y$, if the other parameters remain the same.

2-2 Case II (Coasting Proton Beam)

From a technical point of view, a coasting proton beam system may be more practical. For this case, the luminosity is given by[3]

$$L = f \frac{N_e N_p}{\pi} \frac{\ell_{int}}{C} \frac{1}{\sqrt{\sigma_{ex}^2 + \sigma_{px}^2}\sqrt{\sigma_{ey}^2 + \sigma_{py}^2}} , \tag{9}$$

instead of eq.(1), where C is the total circumference and the beams are separated at $z = \pm\ell_{int}/2$.

The linear tune shifts for protons remain as given by eq.(3) and (3'), while the tune shifts for electrons are now given by

$$\Delta\nu_{ex} = \frac{N_p r_e}{\pi\gamma_e C(\sigma_{px} + \sigma_{py})\sigma_{px}} \int_{-\frac{\ell_{int}}{2}}^{\frac{\ell_{int}}{2}} \beta_{ex}(z)dz \tag{10}$$

$$\Delta\nu_{ey} = \frac{N_p r_e}{\pi\gamma_e C(\sigma_{px} + \sigma_{py})\sigma_{py}} \int_{-\frac{\ell_{int}}{2}}^{\frac{\ell_{int}}{2}} \beta_{ey}(z)dz. \tag{10'}$$

Here, we neglect the variation of the proton beam radius and the long-range interactions. The dispersion function is assumed to be zero in the interaction region. If we take that β_{ex} and β_{ey} vary as

$$\beta_{ex,y}(z) = \beta^*_{ex,y} + \frac{z^2}{\beta^*_{ex,y}} , \tag{11}$$

then we obtain

$$\Delta\nu_{ex} \approx \frac{N_p r_e \beta^*_{ex}}{\pi\gamma_e(\sigma_{px} + \sigma_{py})\sigma_{px}} \frac{\ell_{int}}{C} \left(1 + \frac{1}{12} \frac{\ell^2_{int}}{\beta^{*2}_{ex}}\right) \tag{12}$$

and

$$\Delta\nu_{ey} \approx \frac{N_p r_e \beta^*_{ey}}{\pi\gamma_e(\sigma_{px} + \sigma_{py})\sigma_{py}} \frac{\ell_{int}}{C} \left(1 + \frac{1}{12} \frac{\ell^2_{int}}{\beta^{*2}_{ey}}\right) , \tag{12'}$$

where β^{*}_{ex} and β^{*}_{ey} denote the values at the center of the interaction region. As a function of β^{*}_{ex} or β^{*}_{ey}, the tune shifts take a minimum[5)] when the conditions

$$\frac{\ell^2_{int}}{\beta^{*2}_{ex}} = 12 \qquad \text{or} \qquad \ell_{int} = 2\sqrt{3}\beta^{*}_{ex} \tag{13}$$

and

$$\frac{\ell^2_{int}}{\beta^{*}_{ey}} = 12 \qquad \text{or} \qquad \ell_{int} = 2\sqrt{3}\beta^{*}_{ey} \tag{13'}$$

are satisfied.

Now, the assumptions (4) also require

$$\beta^{*}_{ex} = \beta^{*}_{ey} \equiv \beta^{*}_{e}$$

and (14)

$$\sigma_{px} = \sigma_{py} = \sigma_x = \sigma_y$$

in addition to the condition (5).* Since we use a superconducting ring for protons, a circular cross section of the proton beam will be desirable. A strong coupling in horizontal and vertical oscillations of electrons is expected to result in $\beta_{ex} \approx \beta_{ey}$. Hence, the condition $\beta^{*}_{ex} = \beta^{*}_{ey}$ is also acceptable.[6)7)]

Taking these considerations into account, we can rewrite the tune shifts for electrons as

$$\Delta\nu_{ex} = \Delta\nu_{ey} = \frac{N_p r_e \beta^{*}_{e}}{\pi\gamma_e\sigma_x\sigma_y} \frac{\ell_{int}}{C} . \tag{15}$$

Let this be equal to the tune limit, i.e. $\Delta\nu_e = 0.06$. Inserting eq.(15) into eq.(9), we get the maximum luminosity as

* For $\sigma_x >> \sigma_y$, instead of eq.(5), $\beta^{*}_{ex}/\beta^{*}_{ey} = 2\ \sigma_x/\sigma_y$ is required with the condition (13').

$$L = \frac{N_e f \gamma_e \Delta\nu_e}{2 r_e \beta_e^*} \qquad (16)$$

Again, this is the standard expression as well as eq.(7). However, the value of the luminosity with the same choice of electron parameters will be halved compared to the bunched proton case since we assumed that $\sigma_x \approx \sigma_y$.

The condition (6), which is required from $\Delta\nu_e$ and $\Delta\nu_p$ limits, becomes

$$\frac{\beta_e^*}{\beta_p} = \frac{r_p}{r_e} \frac{\gamma_e}{\gamma_p} \frac{\Delta\nu_e}{\Delta\nu_p} \frac{N_e}{N_p} \frac{1}{4k_b} \frac{C}{\ell_{int}}$$

$$= 0.021 \ a \ \frac{C}{k_b \ell_{int}} , \qquad (17)$$

where we also assumed that $\beta_{px} = \beta_{py} = \beta_p$. The number of electron bunches is k_b and a is the ratio N_e/N_p. For $\ell_{int} \simeq 2$ m, we obtain from eq.(13) that $\beta_e^* \approx 0.6$ m. For $k_b \approx 150$, a small ratio a or a large β_p is required. From considerations on the radiation effect, the beam radius of electrons should be $\sim$0.1 mm.[6] In order to make the beam radii of protons and electrons equal, β_p must be about 10m for a reasonable value of the proton beam emittance. Consequently, the ratio $a = N_e/N_p \lesssim 0.014$, i.e. the number of protons in the coasting beam should exceed that of electrons by a factor of 70. However, the longitudinal density of the protons is still smaller than in the bunched-beam case. A larger number of electron bunches, $k_b \gtrsim 1000$ is an alternative which raises the ratio a and therefore reduces the total number of protons required. However, such a electron beam might suffer from multi-bunch instability.

3. Additional Remarks

3-1 Radiation from Electron Beam

The average energy U_0 radiated in one revolution of a single electron is

$$U_0 \approx 4.8 \text{ GeV} ,$$

and the total radiation power from 3.6×10^{13} electrons is

$$P_b \approx 130 \text{ MW}.$$

The effect of the beam strahlung[1)] can be discussed in the same way as for electron-positron collisions. However, the effect is smaller since the proton bunches are much longer.

3-2 Other Comments

More careful studies on lattice design, beam dynamics and instability as well as technical feasibility studies are necessary. The assumptions (4) for luminosity optimization were made to reduce the complexity caused by the asymmetric nature of the beams. The beam radii anticipated from the above choice of parameters are 0.1 mm or less. This would require a new technical procedure to align the colliding beams on the same line in crossing region. The synchrotron radiation from the orbital motions may provide a signal useful to automatic beam controls. However, we may find it wiser to use a larger beam size of protons than electrons in order to make the beam alignment feasible.

Design of the experimental insertions is of course particularly important. A very strong forward peaking of the secondary particles is expected at such a very high energy ep interactions, necessiating an extensive study of detector systems.

3-3 A Possible Configuration of ep-System

As an option of an existing $p\bar{p}$ ring, we propose a possible configuration of e-p system as shown in Fig.1. This fancy sketch includes possibilities of all types of colliding beam experiments as $p\bar{p}$, pp, $e^{\pm}p$, $e^{\pm}\bar{p}$, and $e^{+}e^{-}$ as well as fixed target experiments!

Finally the authors have greatly profited from discussions in the Group 3 of this Workshop.

References

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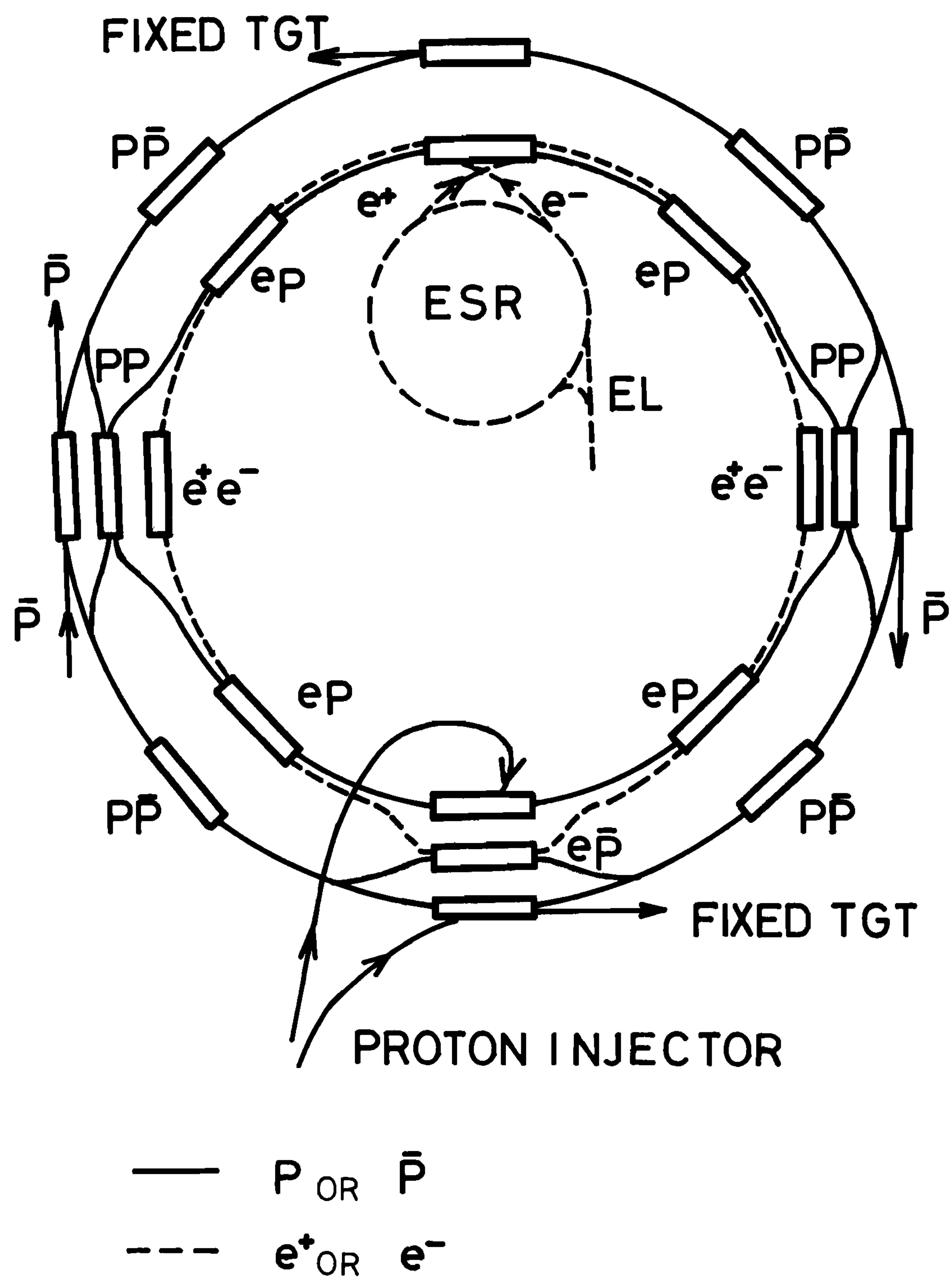

Fig. 1. A possible configuration of an ep system proposed as an option of a $p\overline{p}$ ring.

Experiments and Detectors