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STOCHASTIC COOLING

reported by

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TABLE OF CONTENTS

0. Introduction

This is an attempt to give a "self-contained summary of recent theoretical and experimental work done at CERN on stochastic cooling". I hope that the names of those who contributed to this venture are adequately covered in the historical notes and in the list of references given below. My role here is that of a rapporteur.

The participants of this workshop on the future "world accelerator" (and the reader of this report) should like, I assume, to appreciate the beauty and to learn about the possibilities and limits of stochastic cooling, in order to understand the promises which phase-space cooling of "rare" particles may hold at the highest and lowest accelerator energies. I will try to satisfy this curiosity.

1. The principle

Stochastic cooling is in principle simple: a sensor measures the error in some property of each successive sample of beam particles (say, the error in transverse position <x>). The sample length is determined by the resolution i.e. by the rise time T_s - or if you prefer by the bandwidth $W = \frac{1}{2T_s}$ of the system.

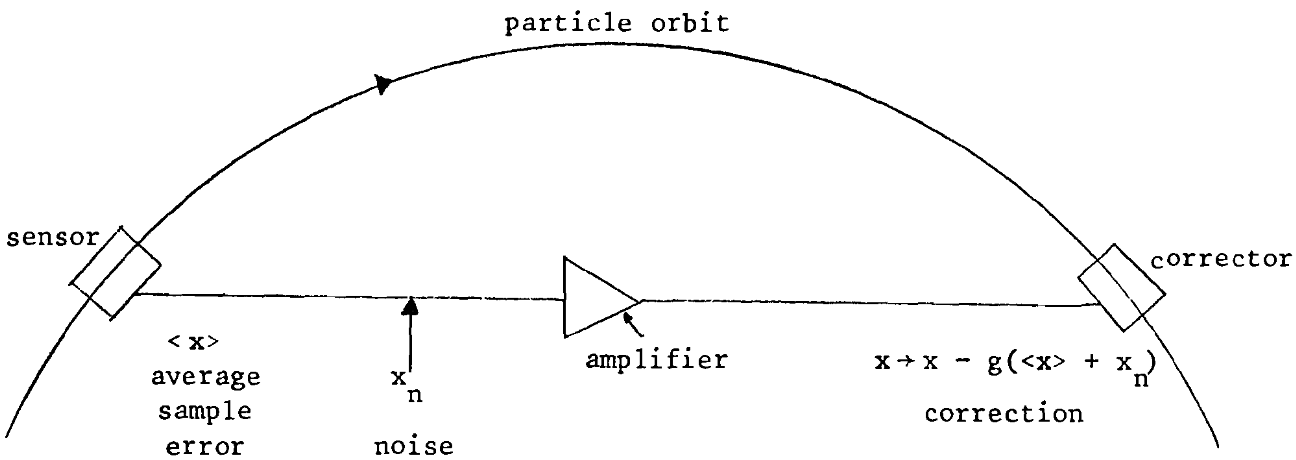


Figure 1

A correction signal is derived and applied on a corrector (transverse kicker). The system can only detect and correct the average error of the samples (centre of gravity <x >). The corresponding beam signals are called Schottky noise.

For zero energy spread cooling would stop once the average sample errors are corrected. However, due to the dispersion in revolution frequencies, particles will migrate between samples (mixing), the error will reappear and correction continues until ideally all particles have zero error.

A few equations may illustrate the principle. At the corrector each sample member gets its error changed by

$$x_c = x - \underbrace{g\,(\langle x\rangle}_{\text{correction}} + x_n) \qquad (1.1)$$

- x_n — noise
- $g\,\langle x\rangle$ — correction
- x — error of i-th sample member before passage of corrector
- x_c — error after passage of corrector

Here $\langle x \rangle$ is the average sample error, x_n the system noise and $g \leq 1$ the fractional correction per passage; g depends on the amplification, the number of particles and other system parameters.

As a very rough approximation, assume that each particle interacts only with itself whereas the mutual influence "averages out". Then, with a sample population of $N_s = N\,T_s / T_{rev}$

$$\langle x\rangle = \frac{1}{N_s}\left(x_{test} + \sum_{\substack{\text{all}\\ \text{others}}} x\right) \approx \frac{1}{N_s}\, x_{test}$$

In this approximation the change at the corrector $\Delta x = x_c - x$ becomes for any sample member

$$\Delta x \approx -\frac{g}{N_s}\, x$$

and the cooling rate for f_{rev} passages per second

$$\frac{1}{\tau} = \frac{1}{x}\frac{dx}{dt} \approx f_{rev}\frac{\Delta x}{x} \approx \frac{f_{rev}\, g}{N_s} = \frac{2Wg}{N} \qquad (1.2)$$

This simple result overestimates $1/\tau$ at most by a factor 2. However, it does not show up the heating due to noise and other particles.

To include these effects a slightly more elaborate evaluation of (1.1) is needed. The approach (due to Hereward) is summarized in Table 1. The result for the cooling of the rms beam error x_{rms} is

$$\frac{1}{\tau} = \frac{f_{rev}}{N_s}\left[g - \frac{g^2}{2}\left(1 + \frac{x^2_{n\,rms}}{x^2_{rms}}\right)\right] \qquad (1.3)$$

- $\frac{x^2_{n\,rms}}{x^2_{rms}}$ — heating by noise
- 1 — heating by other particles
- g — coherent effect (cooling) eq.(1.3)
- N_s — sample population, given by bandwidth and beam population N because $N_s = N\,T_s / T_{rev} = N f_{rev} / 2W$

Note that the derivation assumes perfect mixing between consecutive turns and no mixing between sensor and corrector.

TABLE 1 : EVALUATION OF COOLING RATE

Change at corrector for one passage (eq.(1.1))

$$x_c = x - g\,(\langle x\rangle + x_n)$$

Work out $\Delta x^2 = x_c^2 - x^2$:

$$\Delta x^2 = -\,2\,g\,x\,(\langle x\rangle + x_n) + g^2\,(\langle x\rangle + x_n)^2$$

Take the sample average

$$\langle \Delta x^2\rangle = -\,2\,g\,\langle x\rangle^2 - 2\,g\,\langle x\rangle\,x_n + g^2\,(\langle x\rangle^2 + 2\langle x\rangle\,x_n + x_n^{\,2})$$

For many passages, replace these quantities by their expectation values for random samples (mixing) of the beam. For $N_s \gg 1$

$$E\,(\Delta\langle x^2\rangle) = x^2_{rms.c} - x^2_{rms} = \Delta x^2_{rms}$$

$$E\,(\langle x\rangle^2) = \frac{1}{N_s}\,x^2_{rms}$$

$$E\,(x_n\langle x\rangle) = 0 \quad \text{(no correlation between noise and correction)}$$

$$E\,(x_n^{\,2}) = \frac{1}{N_s}\,x^2_{n\,rms}$$

where all rms are the beam rms values.

Hence

$$\frac{\Delta x_{rms}}{x_{rms}} \approx \frac{1}{2}\,\frac{\Delta x^2_{rms}}{x^2_{rms}} = \frac{-g}{N_s}\left[1 - \frac{g}{2}\left(1 + \frac{x^2_{n.rms}}{x^2_{rms}}\right)\right]$$

TABLE 2 : HISTORY (with emphasis on work at CERN)

Prehistory

Liouville	ca 1850	Invariance of phase space
Schottky	1918	Noise in DC electron beams

History

van der Meer	1968	Idea of stochastic cooling
ISR staff (Borer, Bramham, Hereward, Hübner, Schnell, Thorndahl)	1972	Observation of proton beam Schottky noise
van der Meer	1972	Theory of emittance cooling
Schnell	1972	Engineering studies
Hereward	1972-74	Refined theory, low intensity cooling
Bramham, Carron, Hereward, Hübner, Schnell, Thorndahl	1975	First experimental demonstration of emittance cooling
Palmer (BNL) Thorndahl	1975	Idea of low intensity momentum cooling
Strolin Thorndahl	1975	$\bar{p}$ accumulation, schemes for ISR using stochastic cooling
Rubbia	1975	$\bar{p}$ accumulation, schemes for SPS
Thorndahl	1976	Experimental demonstration of p cooling
Thorndahl	1977	Filter method of p cooling
Sacherer, Thorndahl van der Meer	1977-78	Refinement of theory; imperfect mixing, Fokker-Planck equations
ICE Team	1978	Detailed experimental verification
Herr	1978	Demonstration of bunched beam cooling.

2. History

For long the idea of stochastic cooling was regarded as too far fetched to be practical. A first experimental demonstration was tried and succeeded only 7 years after the invention (3 years after the first publication).

The inventor and the early workers had (mainly?) emittance cooling of high intensity beams in mind in order to improve the luminosity in the ISR. A new era began in 1975 when Strolin (coming back from a visit to Novosibirsk) and Thorndahl realized the interest of stochastic cooling, both in emittance and momentum, of low intensity $\bar{p}$ beams for the purpose of stacking. Stochastic cooling at low intensity is different from the original van der Meer cooling and the extension of the theory (to $g < 1$ in eq. 1.1) first done by Hereward and Thorndahl as well as the design of the momentum cooling hardware (Thorndahl, Carron) are perhaps as fundamental as the original invention and the earlier feasibility studies (van der Meer, Schnell, ...).

Following this broadening of the scope, Strolin and Thorndahl worked out in 1975 $\bar{p}$ collection schemes for the ISR using stacking in momentum space and Rubbia et al. made first proposals of the $\bar{p}$-p scheme for the SPS using similar techniques of stochastic cooling and accumulation. This work has given new life to the idea at a time when the ISR was routinely stacking such high proton currents that proton beam cooling became unnecessary or even impossible. Further mile stones since 1975 are the invention of the filter method of momentum cooling, the refinement of the theory and of the stacking schemes, the CERN $\bar{p}$-p proposal which has now become a project and the encouragement from the results of the cooling experiment ICE including the discovery of bunched beam cooling.

Table 3 : Experimental results

Year	Machine	Type of cooling	No. of Particles	Cooling Time
1975	ISR	vertical	$\approx 10^{13}$	100 h
1976/77	ISR	vertical	$\approx 10^{8}$	2 h
1978	ICE	vertical	3×10^{8}	4 min
		longitudinal	10^{7}	15 s
1980	AA	longitudinal	$2,5 \times 10^{7}$	1 s
		longitudinal	6×10^{11}	$\sim$ 1 h
		vert./horiz.	6×10^{11}	$\sim$0,5 h

3. Some limitations

A few fundamental limits are obvious from eq. (1.3)

a) Bandwidth limit

In the best of all cases $g = 1$ and

$$\frac{1}{\tau} = \frac{W}{N} \qquad (3.1)$$

Let for technical reasons $W < 1$ GHz. Then, particles can be cooled at best at a rate

$$\frac{dN}{dt} \approx \frac{N}{\tau} = W < 10^9/s \qquad (\simeq 8 \times 10^{13}/\text{day})$$

b) Noise limit

It follows from (1.3) that with noise fastest initial cooling is for $g = g_o$ given by

$$g_o = \frac{1}{1 + x_n^2/x^2} \qquad (3.2)$$

and the cooling rate (3.1) is reduced by 1 + signal power/noise power.

Further since the heating term contains the instantaneous rms beam error (x), cooling for any g will stop once x is small. In fact, $1/\tau \to 0$ for

$$x^2_{final} = \frac{x_n^2}{2/g - 1}$$

and for $g = g_o$

$$x^2_{final} = \frac{x_n^{\ 2}}{1 + 2x_n^2/x^2_{initial}} = \begin{cases} x_n^2 \ , & x_{initial} \gg x_n \\ \dfrac{x^2_{initial}}{2} \ ; & x_{initial} \ll x_n \end{cases}$$

Hence the larger g (the faster the initial rate), the smaller the final reduction of the error. For the fastest initial cooling, the asymtotic reduction in the presence of large noise is only $\sqrt{2}$. A remedy is of course dynamic gain, decreasing g (and $1/\tau$) as the error gets smaller.

c) Mixing limit

Interpreting g as the fractional correction per turn, we may intuitively require mixing (rms migration by one sample length $T_s = \frac{1}{2W}$) in at least 1/g turns

$$\Delta T_{rev} \frac{1}{g} \geq \frac{1}{2W} \qquad (3.3)$$

As an example (AA precooling) take

$$\Delta T_{rev} = \eta T_{rev} \frac{\Delta p}{p} \approx 0.1 \times 550 \text{ ns} \times 5 \times 10^{-3}$$

$W = 500$ MHz to find $g \lesssim 0.25$

$\frac{N}{\tau} \leq 0.25\ W = 1.2\ 10^8/s \quad (10^{13}/\text{day})$

[In fact, the mixing limit (3.3) is approximate. A rigorous calculation leads to a modification of the "heating by other particles" term (the "1" in the inner bracket in (1.3)) by a sum over the Schottky bands involded (see ref. 10, 11, 12), which for momentum cooling with constant g is written as

$$1 \rightarrow \Gamma = \frac{1}{\ell} \sum \frac{f_{rev}}{\Delta f_n} \geq 1 \qquad (3.4)$$

where ℓ is the number of Schottky lines in the pass-band f_{min} to $f_{min} + W$ and Δf_n the width of the nth line due spread in the beam. Hence "optimum" $g = g_o = \frac{1}{\Gamma + x_n^2/x^2}$; $g_o = 1/\Gamma$ for negligible noise.

For ideal momentum cooling and non overlapping bands

$$\Gamma \simeq \frac{1}{W\Delta T_{rev}} \ln (f_{max}/f_{min})$$

For betatron cooling with two Schottky bands per revolution harmonic, the appropriate modification is ½ of (3.4) as long as adjacent bands do not overlap. For overlapping bands ($\Delta f_n > f_{rev}$), the corresponding terms in (3.4) have to be replaced by 1 both for p and betatron cooling.]

d) Power limit

The larger the gain, the higher the power consumption of the amplifier. Thus the available broadband power may further restrict the cooling rate by requiring $g \ll g_o$.

[For (Palmer) momentum cooling, the damping rate may in this case be expressed [6,19] as

$$\frac{1}{\tau} \lesssim \frac{4ef_{rev}W}{(\Delta p\ c/e)_{rms}} \sqrt{n_{PU}\ R_{PU}\ n_g\ R_g} \sqrt{\frac{P}{P_n}}$$

This holds provided, one is limited by amplifier noise as is the case at low intensity. Here $P_n = 10^{v/10}\ kT_nW$ ($\approx 10^{-20}$ Watt/Hz x W at $T_n = 290^oK$ room temperature) is the preamplifier noise power, n_{PU} (200) the number of pick-ups, R_{PU} (50 Ω) their impedance, n_g (200) and R_g (50 Ω) are the corresponding characteristics of the correction gaps. The assumption is that the signal from the n_{PU} pick-ups is added, sent through the same amplifier and then split and distributed onto the n_g gaps; Δp (in eV/c) is the rms momentum spread of the beam.

Taking W = 0.5 GHz, P = 10 kW, $\Delta p = 10^{-2} \times 3.5$ GeV/c and $f_{rev} = 2$ MHz (figures corresponding roughly to the AA precooling), we find

$$\frac{1}{\tau} \leq 1\ s$$

In a similar way the transverse cooling rate [6] may be limited to

$$\frac{1}{\tau} \lesssim \frac{2}{3} \frac{e\, f_{rev}\, W}{(p\, c/e)\, \frac{\varepsilon}{\pi}} \sqrt{n_{PU}\, R_{PU}\, n_k\, R_k\, L_k^2} \sqrt{\frac{P}{P_n}}$$

Here L_k is the length, R_k the impedance and n_k the number of correction kickers, ε is the beam emittance.

Note that for constant $\Delta p/p$ and for constant emittance ε, these power limited cooling rates decrease linearly with increasing momentum, P .]

As a result of the different limitations discussed above, we may conclude that it is hard to cool more than, say, 10^{13} particles per day.

4. Refinements of the theory

4.1 Bad mixing

The appropriate generalization of the basic equation (1.3) has been obtained in ref. 23. Assuming a rectangular distribution dN/df of particles revolution frequencies of total width Δf , the modification mentioned above and generalized to include frequency dependent g is to replace "1" in the inner bracket of (1.3) by

$$\Gamma = \frac{1}{\ell\,\bar{g}^2} \sum_n \frac{g^2(f)\, f_{rev}}{\Delta f_n} \geq 1 \qquad (3.4\,a)$$

$$(\Delta f_n < f_{rev}\,,\ \text{else replace}\ f_{rev}/\Delta f_n \rightarrow 1)$$

This modification has been explained in ref. 26 by the fact that the noise density in the nth Schottky band is larger by $f_{rev}/\Delta f_n$ as compared to white noise (for non overlapping betatron bands, reduce (3.4) by 2). The consequence of (3.4) is stronger heating by the presence of the other particles.

4.2 Fokker-Planck equations

All cooling equations discussed so far were concerned with the rms - width (second moment) of the particle distribution. To obtain more detailed information about the evaluation of the distribution (tails, stacking etc.) a Fokker-Planck type of equation can be derived by working out higher order moment equations from the single particle dynamics in the presence of feedback and noise.

$$\text{Let}\ \psi(x,t) = \frac{dN}{dx}$$

be the distribtion function. The Fokker-Planck equation used by Thorndahl, van der Meer and Sacherer was written in the form:

$$(4.1) \qquad \frac{\partial\psi}{\partial t} = \frac{-\partial}{\partial x}\left\{\frac{\Delta x_c}{T_{rev}}\,\psi\right\} + \frac{1}{2}\,\frac{\partial}{\partial x}\left\{\frac{\Delta x^2{}_{ic}}{T_{rev}}\,\frac{\partial\psi}{\partial x}\right\}$$

where $\Delta x_c/T_{rev}$ is the coherent correction per unit time for a single particle and $\frac{\Delta x_{ic}^2}{T_{rev}}$ is the mean square blow up due to noise (the diffusion term if you like). Both quantities are in general functions of the error x.

Note that usually the Fokker Planck equation is derived in the form

$$\frac{\partial\psi}{\partial t} = \frac{\partial}{\partial x}\left(\alpha_1\psi\right) + \frac{1}{2}\frac{\partial^2}{\partial x2}\left(\alpha_2\psi\right) \tag{4.2}$$

It seems that for cooling problems the relation between $\alpha_1 = \langle\delta x\rangle/T_{rev}$ and $\alpha_2 = \langle(\delta x)^2\rangle/T_{rev}$ the average of the change and of the square of the change expected per time interval, permits to write (4.2) in the "diffusion form" (4.1). Equation (4.1) can be used to work out the detailed evolution of the stack including x dependent gain. It can be amended to include walls (losses) and particle influx (stacking).

4.3 Feedback via the beam

If mixing is imperfect a coherent modulation imposed on the beam by the corrector will remain to some extent. The effect is a reduction of the heating terms by the modulus $|T|$ of some complex transfer function T and of the cooling term by Re(T). The function T depends on the frequency spread within the beam. The corresponding reduction of Schottky noise has been observed both on the ISR and on ICE.(Fig. 2). On first sight the effect is beneficial since the heating is stronger reduced than the cooling. For more details we have to refer to the work of Sacherer [26].

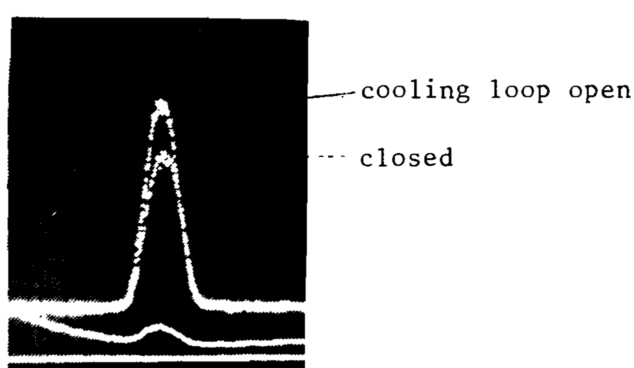


Fig. 2 : Reduction of Schottky noise in ICE

5. Practical systems

Systems proposed and tested are summarized in Table 4.

Table 4 : Different cooling systems

Type	Sensor	Corrector	Optimum Spacing	Tested on
Betatron cooling horizontal or vertical	Difference pick-up	Transverse kicker	$(2k+1)\lambda_\beta/4$	ISR ICE
Momentum cooling Palmer type	Horizontal difference pick-up	RF gap (acceleration/ deceleration)	$(2k+1)\lambda_\beta/2$ for simultaneous horizontal betatron cooling	ISR
Momentum cooling filter type	Longitudinal (sum) pick-up + comb filter	RF gap	$(2k+1)\lambda_\beta/2$ between 1st and 2nd corrector	ICE

We shall discuss the three systems and some results in more detail.

5.1 Betatron Cooling

Let the betatron oscillation of a particle be described by

$$y_i = a_i \sin\left(Q_i\ S/R + \varphi_i\right)$$

$$\tilde{y}_i = y_i' \ R/Q = a_i \cos\left(Q_i\ S/R + \varphi_i\right)$$

The kicker located at a phase advance $Q_i\ S/R = \theta_i$ downbeam of the pick-up (pu) corrects the angle

$$\tilde{y}' \rightarrow \tilde{y}' + g\ \langle Y\rangle_s$$

where $\langle Y\rangle_s$ is the sample average of the position error at the PU. Referred back to the PU the correction is

$$y \to y - g(\langle y\rangle + y_n) \sin\theta$$

$$\tilde{y}' \to \tilde{y}' + g(\langle y\rangle + y_n) \cos\theta$$

Repeating for both of these equations Hereward's analysis leading from (1.1) to (1.3) one obtains the cooling rate for the rms amplitude $A = \sqrt{y^2_{rms} + \tilde{y}'^2_{rms}}$

$$-\frac{1}{A}\frac{dA}{dt} = \frac{W}{N} g \left[\sin\theta - \frac{g}{2} \left(1 + \frac{A^2 n}{A^2}\right) \right] \qquad (5.1)$$

Optimum conditions are for sin θ = ±1 (-1 with phase inversion such that $g < 1$) i.e. if the betatron advance between PU and kicker is an odd multiple of a quater wavelength. Note also the factor 2 difference between (5.1) and (1.3) which is due to the fact that the kicker corrects only angle i.e. on average half the amplitude error of a particle.

As an example of a practical design we analyse the vertical cooling system in ICE (Fig. 3). The approach (due to Thorndhal) is to start from the beam Schottky current (I_{PU}) induced on the difference PU, to compare it to the preamplifier noise current I_n to determine the optimum

$$g = g_o = \left(\Gamma + I_n^2/I_{PU}^2\right)^{-1} \qquad (5.2)$$

which determines the "optimum" cooling rate.

To work out the amplifier power one determines: the rms sample displacement σ_s = rms beam height / N_s ; the correction per turn $g_o\, \sigma_s$. Details are summarized in table 5.

The actual system in ICE consists of 2 pick-ups and 2 kickers [20] with signal addition in front of the amplifier and splitting at the exit. Assuming ideal signal combination one expects twice the optimum cooling rate. Hence one calculates:

Cooling rate	$1/\tau_o = (4.3\ \text{min})^{-1}$
Amplifier power	$P \approx 0.5$ Watts

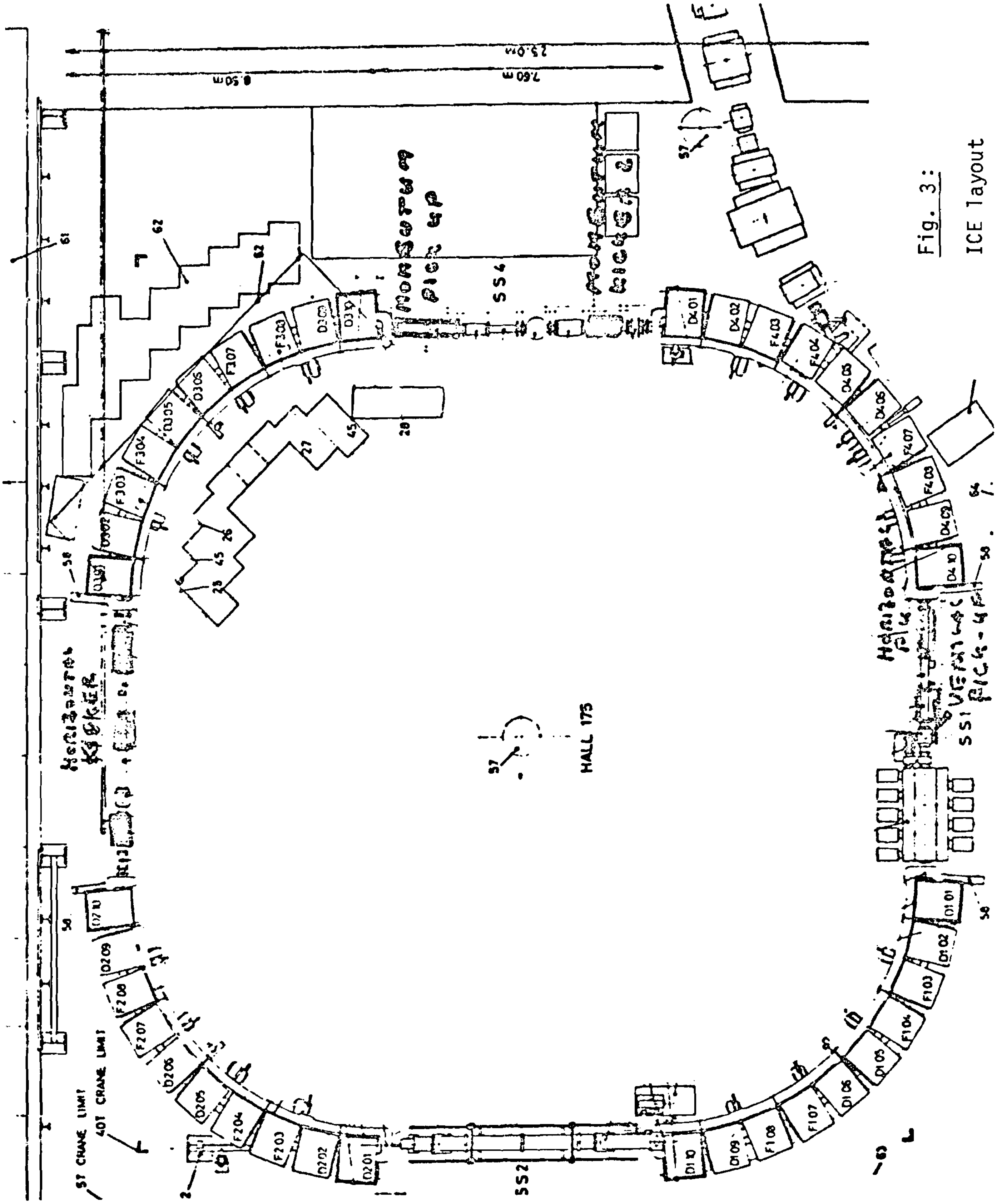


Fig. 3: ICE layout

TABLE 5

VERTICAL COOLING IN ICE

Particle momentum	1.7 GeV/c
Revolution frequency	$f_{rev} = 3.5$ MHz
Number of particles	$N = 3.5 \times 10^8$
Circulating current	$I_o = Nef_o = 196\ \mu A$
Momentum spread	$\Delta p/p \simeq \pm 2 \times 10^{-3}$
Bandwidth	$W = 250$ MHz (250 - 500 MHz)
Sample length	$T_s = \frac{1}{2W} = 2$ ns
Number of particles/sample	$N_s = NT_s/T_{rev} = 2.4 \times 10^6$
Longitudinal Schottky noise	$I_{SC} = \sqrt{2e\ I_o\ W} = 125$ nA
rms betatron amplitude	$a_v = 7$ mm
Effective beam height	$h \simeq 2\ a_v = \pm 14$ mm
rms dipole moment	$D_{rms} = a_v\ I_{SC}/\ 2 = 620$ nA mm
Spacing of PU plates	± 30 mm
PU sensitivity (60% at edge)	$S = 0.6$
Pick-up current	$I_{PU} = S\ D_{rms}/h = 12.5$ nA
Phase shift PU kicker	$1.5\ \pi$
Noise current ($v = 1.5$ dB noise figure at 300^0 K and $R = 50\ \Omega$)	$I_n = \sqrt{10^{v/10} kTW/R} \simeq 170$ nA
Mixing parameter	$\Gamma = \frac{\ln f_{max}/f_{min}}{2W\ \Delta T_{rev}} \simeq 1.8$
Optimum $g = g_0$ ($\Gamma \ll 1/g_0$)	$g_o \approx I_{PU}^2/I_n^2 = 0.0054$
Optimum cooling rate	$\frac{1}{\tau_o} = \frac{g_0 W}{2N} = (8.5\ \text{min})^{-1}$
rms centre of gravity displacement of sample	$\sigma_s = \frac{a_v/\sqrt{2}}{\sqrt{N_s}} = 3.2 \times 10^{-3}$ mm
Corresponding angle at $\beta \simeq R/Q = 10$ m	$\theta_s = \delta s/\beta = 3.2 \times 10^{-7}$ rad
Correction per turn	$\theta_c = g_0\ \theta_s : 1.7 \times 10^{-9}$ rad
Length of kicker plate	$\ell = 200$ mm
Plate spacing	$d = \pm 30$ mm
Kick per voltage U on kicker plates (transmission line kicker $1+\beta$ because of E and H deflection)	$\theta \simeq (1+\beta)\frac{U}{2d}\frac{\ell}{p\beta}$ $\theta/U = 4.2 \times 10^{-9}$ rad/V
Voltage for θ_c	$U_c = 0.41$ V rms
rms voltage due to noise	$U_n = g_o^{-1/2}\ ; U_c = 5.6$ V rms
Power on 50 Ω	$P = (U_n^2 + U_c^2) / R \simeq 0.52$ W
Amplification	$\alpha = P/I_n^2 R = 3.4 \times 10^{11}$ (115 dB)

The initial cooling rate measured on ICE at about 3.5×10^8 was $(4\ \text{min})^{-1}$ which agrees well with the results of table 5. It should however be mentioned that beam properties (N, σ_v) entering critically into the calculation are only known with limited accuracy.

Finally we mention the Fokker-Planck equation for betatron cooling. One has, neglecting for simplicity particle noise($\Gamma < x_x^2/x^2_{PU}$ as assumed above)

$$\Delta a_c / T_{rev} = \left(\frac{W}{N} g(a) \sin \theta \right) a$$

$$\Delta a^2_{ic} / T_{rev} = \frac{W}{N} g^2(a)\ a^2_n$$

Hence, if we believe the recipe (4.1):

$$\frac{\partial \psi}{\partial t} = \frac{W}{N} \frac{\partial}{\partial a} \left[-(ag \sin \theta)\ \psi + \frac{1}{2} g^2 a^2_n \frac{\partial \psi}{\partial a} \right] \qquad (5.3)$$

For constant g the final equilibrium distribution $\left(\frac{\partial \psi}{\partial t} = 0 \right)$ following from (5.3) is a Gaussian ψ (a) with rms width $A^2 = \frac{1}{2}\, g\, a^2_n / |\sin \theta|$ or $A^2_{final} = \frac{1}{2} A^2_{initial} / |\sin \theta|$ for $g = g_o \equiv a^2_{initial} / a^2_n$. This shows once again that the density increase is small if the cooling is fast.

5.2 Palmer Cooling

The single particle displacement at the PU is

$$x_i = \underbrace{x_{\beta i}} + \underbrace{\alpha_p \left(\frac{\Delta p}{p} \right)_i}$$

$\alpha_p \left(\frac{\Delta p}{p} \right)_i$ — displacement due to momentum error

$x_{\beta i}$ — due to betatran oscillation

The single passage correction (referred back for the PU at a betatron phase θ upbeams)

$$\frac{\Delta p}{p} \rightarrow \frac{\Delta p}{p} - \frac{g}{\alpha_p} \langle x \rangle$$

$$x_\beta \rightarrow x_\beta + g \ \langle x \rangle \cos \theta$$

$$\tilde{x}'_\beta \rightarrow \tilde{x}'_\beta + g \ \langle x \rangle \sin \theta$$

Repeating Hereward's procedure we obtain the cooling rates for rms momentum error $\Delta = (\Delta p/p)_{rms}$ and rms betatron amplitude A in the case of perfect mixing:

$$\frac{1}{\Delta}\frac{d\Delta}{dt} = \frac{2Wg}{N}\left[1 - \frac{g}{2}\left(1 + \frac{A^2}{\alpha_p{}^2\Delta^2} + \frac{\Delta_n{}^2}{\Delta^2}\right)\right]$$

$$\frac{1}{A}\frac{dA}{dt} = \frac{Wg}{N}\left[\cos \theta - \frac{g}{2}\left(1 + \frac{\alpha_p^2\,\Delta^2}{A^2} + \frac{A_n{}^2}{A^2}\right)\right]$$

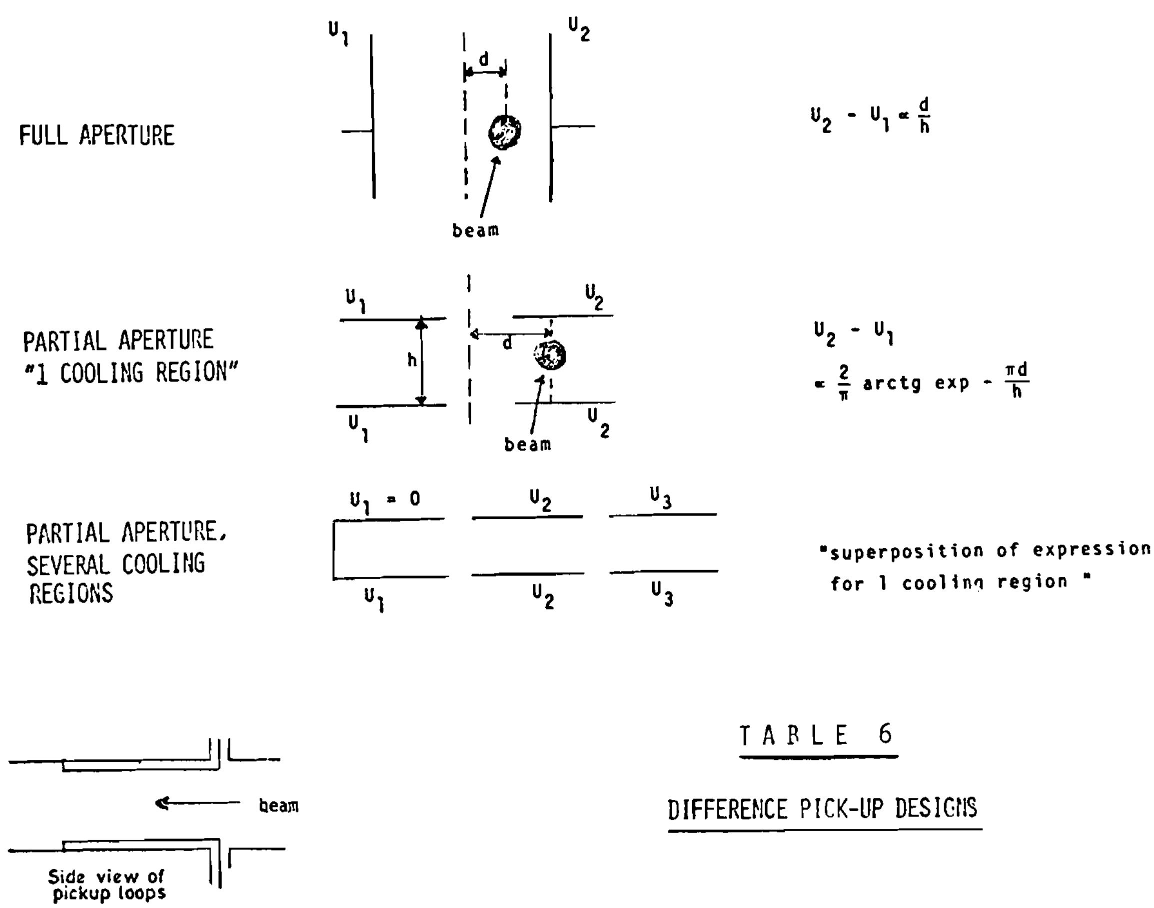


TABLE 6

DIFFERENCE PICK-UP DESIGNS

With partial aperture pick-ups (Table 6) and similar kickers, one can have several cooling regions with a position dependent gain. This behaviour can be emphasized by using filters producing a frequency , i.e. position dependent gain (via the relation between frequency and position). In this way small g can be produced at dense parts of the stack where heating by other particles is important and high gain at the low density and where fast cooling is required [29].

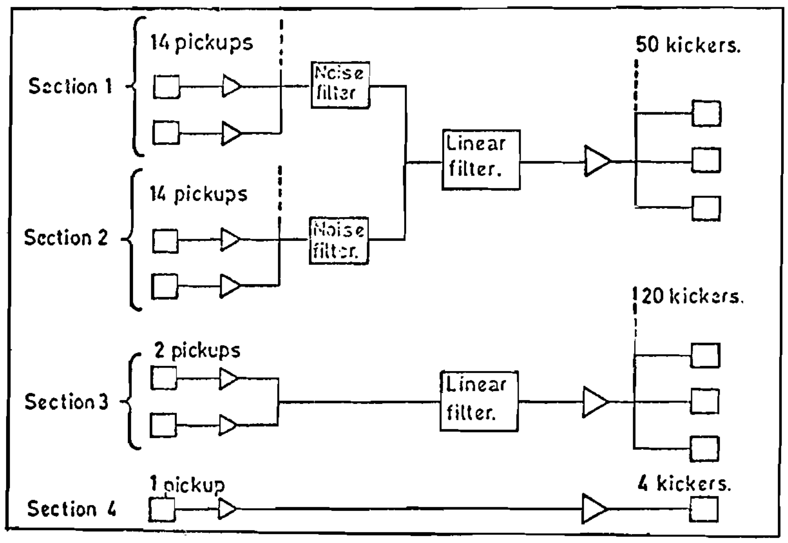


Fig. 5 :
Block diagram of the stack cooling system

The stack cooling in the AA ring [29] will in fact use a 3 or 4 stage system of Palmer type including filters to shape the gain versus position profile (Fig. 5-7).

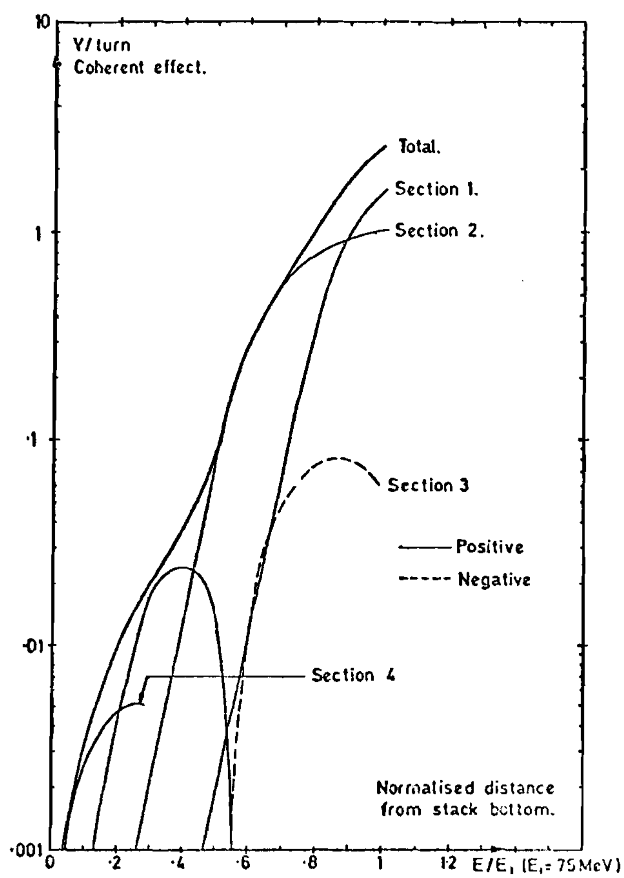


Fig. 6 : Gain curves for stack cooling system.

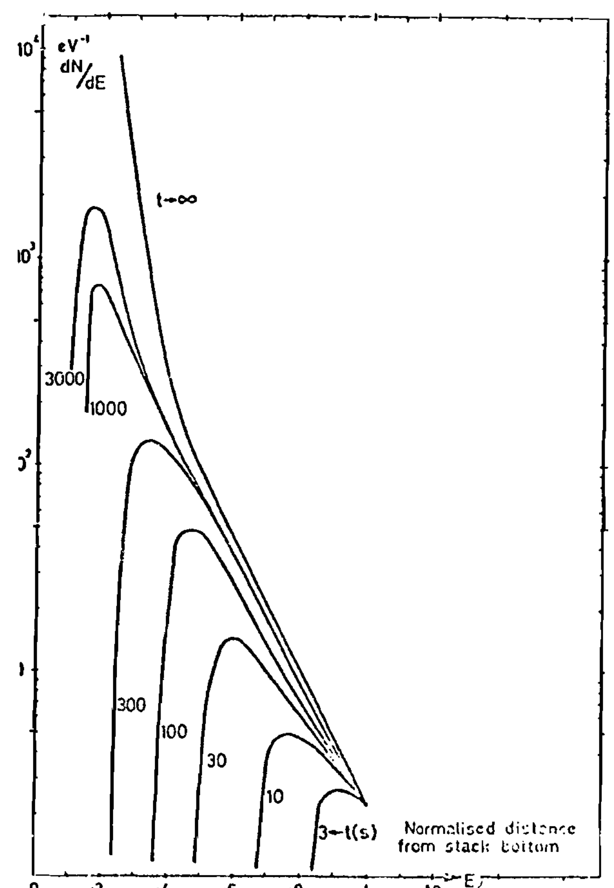
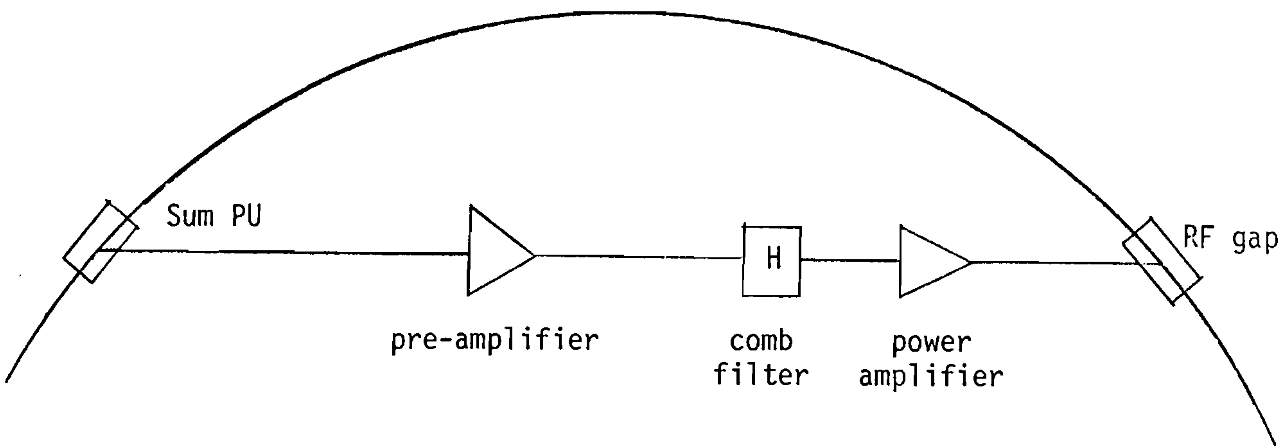


Fig. 7 : Density profile of the stack for a constant particle flux towards the stack bottom.

MOMENTUM COOLING, FILTER METHOD [22,24]



In the simplest case the filter is a transmission line shorted at the far end and with a length corresponding to half the circumference of the cooling ring. The notches at the harmonics of the revolution frequency are produced by $\lambda/2$ resonances where ideally the input impedance is zero and the phase changes sign. Due to this phase and amplitude characteristics, particles with a slightly too low momentum (to high frequency above transition) are accelerated and those with too high momentum decelerated until ideally all particles are "fallen into the notches".

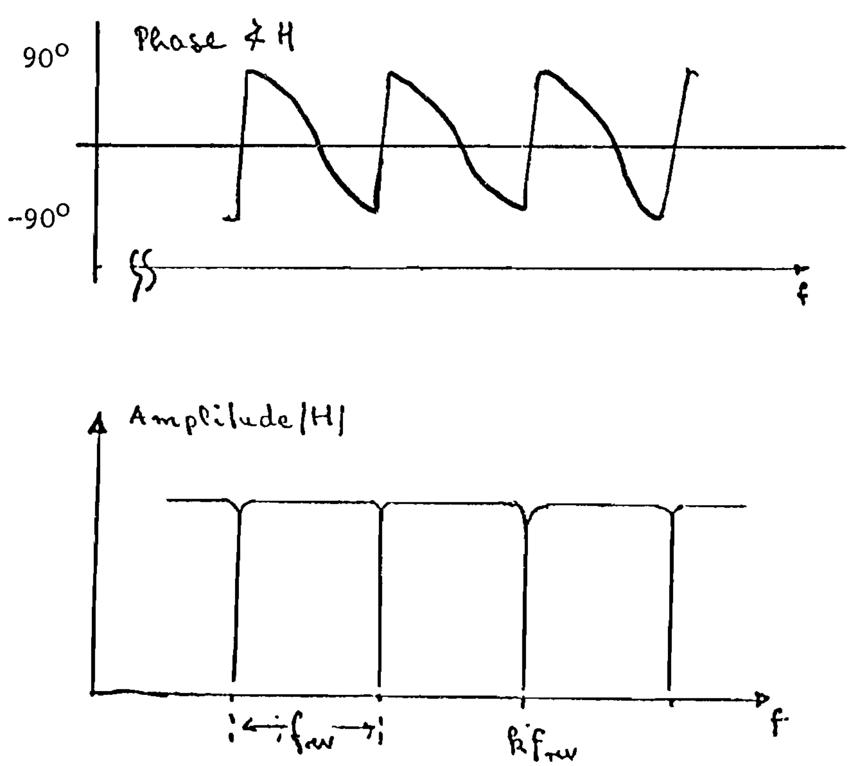


Fig. 8

Idealised characteristics of notch filter

If you prefer to look at the process in time domain: the pulse sent through the system by a particle of the nominal revolution frequency will be cancelled by its pulse from the previous turn reflected at the end of the line (T_{rev} changes slowly even for a strong RF). For too slow or too fast particles, the cancellation is imperfect and deceleration or acceleration will result.

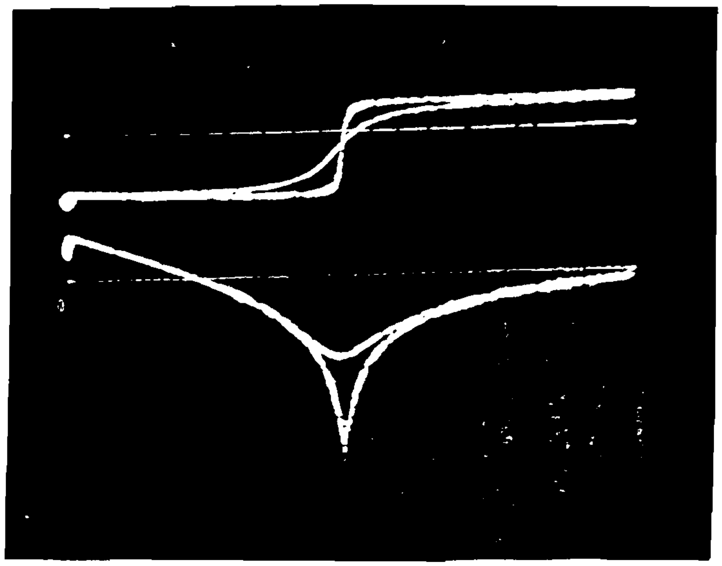


Fig. 9

Phase and amplitude at a notch of the ICE momentum cooling system. Steepening of the notch by a compensator circuit

Additional elements can be added to the filter to sharpen the notches and to reduce the gain between harmonics in order to filter out the preamplifier noise. The filter method is preferable for low intensity beams where high gain is needed and the amplifier noise becomes important. Moreover the sum pick-up produces as large a signal as can be obtained over the aperture and the use of ferrite rings gives sufficient wideband impedance even with a very short pick-up (Fig. 10).

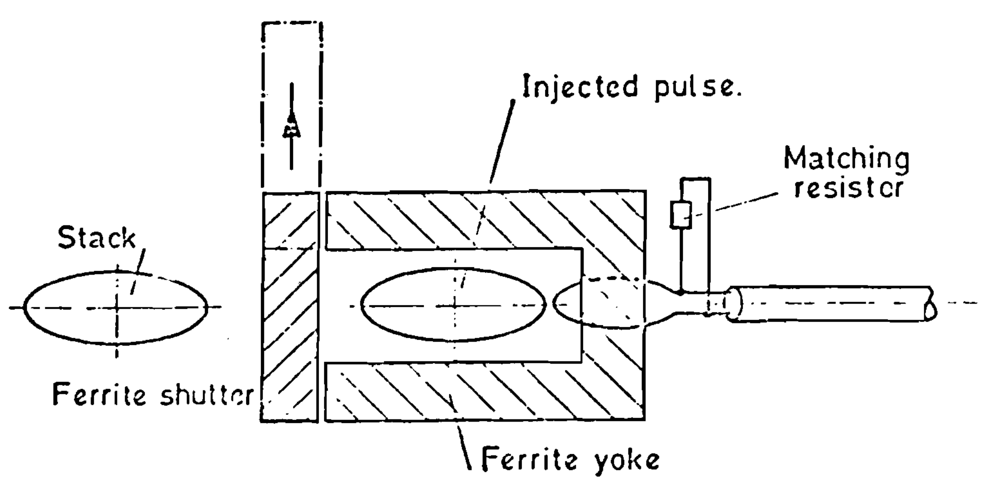


Fig. 10 : Schematic cross-section of sum Pick-up for momentum precooling in the Antiproton Accumulator

Following Sacherer[26)] we use the F.P. equation (4.1) to analyze the Thorndahl cooling. Let E be the energy error and $G^2(E) = R_{gap} I^2_{gap}/R_{PU} I^2_{PU}$ the power transfer function from PU to the gap. The coherent energy correction of a single particle for one passage is

$$(\frac{1}{e})\ \Delta E_c = 2\,e\,f_{rev}\ R \sum \mathrm{Re}\ G_n(E) \qquad \text{(Volts)}$$

i.e. the current per band multiplied by the mean of PU and gap impedance $R = \sqrt{R_{gap}\ R_p}$ (assumed to be purely resistive) and by the real part of the transfer function G(E) taken at the revolution harmonics and summed over the Schottky bands involved.

Similarly, assuming white amplifier noise of $P'_n = e^{v/10}\ KT_n$ ($\approx 10^{-20}$ W/Hz) and Schottky noise due to other particles of $2\ e^2\ f_o\ N\ R_{PU}$ Watts. in each band the mean square energy change per turn due to noise is

$$\left(\frac{1}{e}\right)^2 \Delta E^2_{ic} = P'_n\ f_{rev}\ R_{gap} \sum |G_n|^2 + e^2 f_{rev}\ R^2 \frac{dN}{df_o} \sum \frac{|G_n|^2}{n}$$

For a given system the filter characteristics

$$\sum R_e(G_n) = f_1(E),\ \sum |G_n|^2 = f_2(E)$$

and

$$\sum \frac{|G_n|^2}{n} = f_3(E)$$

can be measured and inserted into the F.P. equation for numerical integration. This has been done for ICE and results agree with measurements.

For analytical calculations, it is useful to expand G(E) near the notches assuming small losses. We use

$$G\left(\frac{\Delta f}{f_n}\right) \approx G_o\left[\frac{\Delta f}{f_n} + \frac{1}{2i\,q_n}\right] = G'\left[E + i\,\varepsilon\right]$$

where q_n is the quality factor ($\frac{f}{\delta f}$ with δf the distance of the $\pm\ 45^o$ degree phase points) of the n-th notch. One may write

$$\sum R_e(G_n) = \frac{W}{f_{rev}} G'E$$

$$\sum |G_n|^2 = \frac{W}{f_{rev}} G'^2(E^2 + \varepsilon_1{}^2)$$

$$\sum \frac{|G_n|^2}{n} = \frac{W}{f_{rev}} G'^2(E^2 + \varepsilon_2{}^2) \sum \frac{1}{n}$$

where ε_1 and ε_2 are related to the losses.

Further, following van der Meer, we normalize E and ψ to their initial values E_i and $\psi_i = N/E_i$ assuming an originally rectangular distribution, and we use a suitable normalization of time to reduce the number of variables.

Last but not least, we neglect the amplifier noise which turns out to be of no concern in the AA at $N \geq 10^7$ and in ICE at $N \geq 10^8$ (noise figure $v \lessapprox 3$ dB at $T_n = 290^{\circ}$K). We can then write

$$\boxed{\frac{\partial \psi}{\partial t_n} = \alpha \frac{\partial}{\partial E}\left(E\psi + \alpha(E^2 + \chi^2)\ \psi\ \frac{\partial \psi}{\partial E}\right)} \qquad (5.5)$$

where ψ, E and t_n are now the normalized variables. The "gain parameter" is

$$\alpha = \frac{e^2\ R\ G'\ N \sum \frac{1}{n}}{2\ T^2_{rev} |n| (\frac{\Delta p}{p})_i}$$

and the "loss parameter"

$$\chi = \frac{1}{q|n|(\frac{\Delta p}{p})_i}$$

with $$q = \left(\sum \frac{1}{n q_n^2} \Big/ \sum \frac{1}{n}\right)^{-1/2} ,$$

some sort of "average quality factor" of the filter line.

Finally, the normalization factor for time is

$$\frac{t}{t_n} = \frac{N \sum \frac{1}{n}}{4\, T_{rev}\, W^2 |n| \left(\frac{\Delta p}{p}\right)_i}$$

The advantage of (5.5) is that standard solutions can be calculated (numerically) as a function of the two parameters α and χ. Optimum conditions are found for $\alpha \approx 0.15 - 0.2$ provided that χ is small ($\chi < 0.05$).

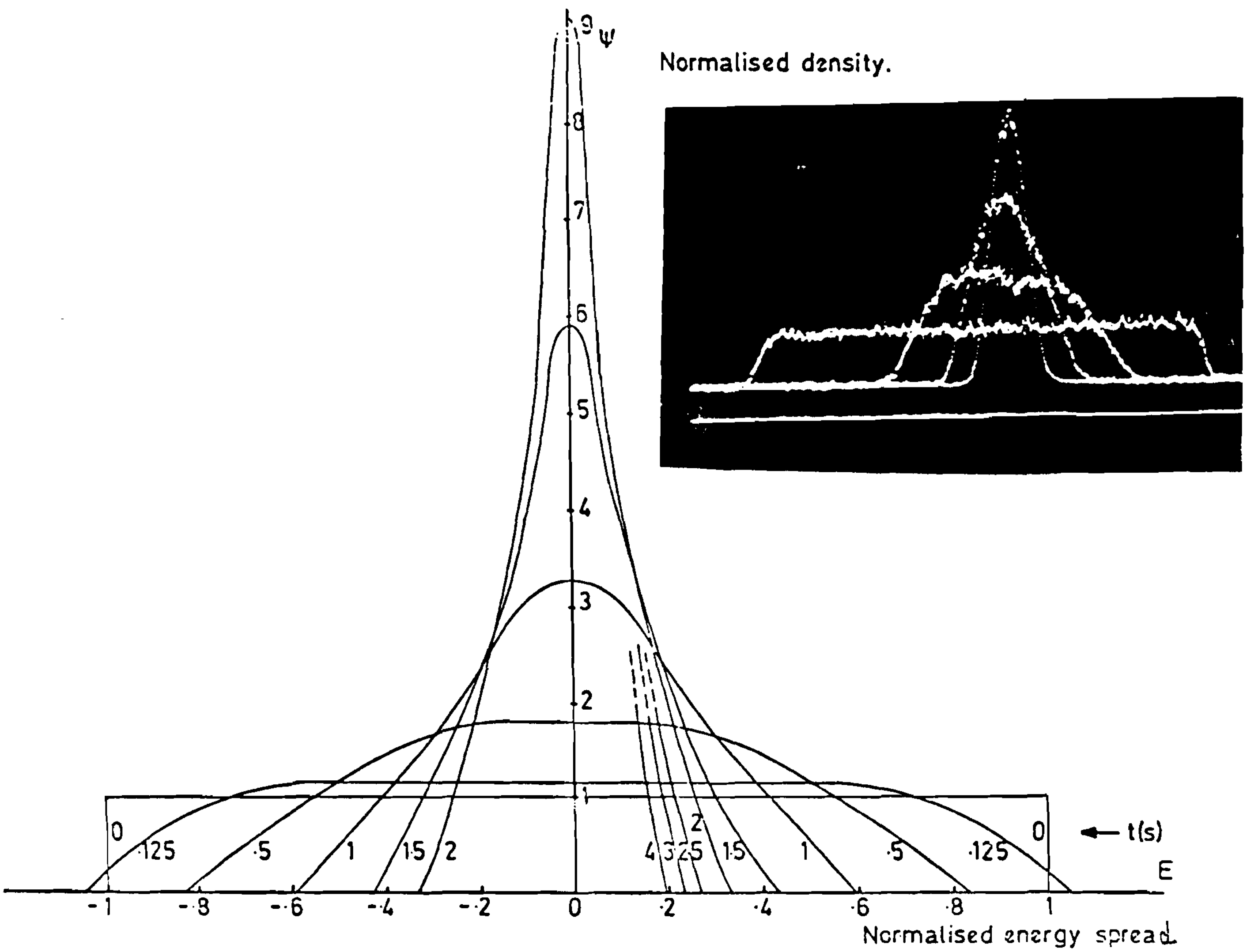


Fig. 11 : Precooling curves for the Antiproton Accumulator calculated from (5.5) for $\alpha = 0.15$, $\chi < 0.01$, and results from ICE (for a different set of parameters.)

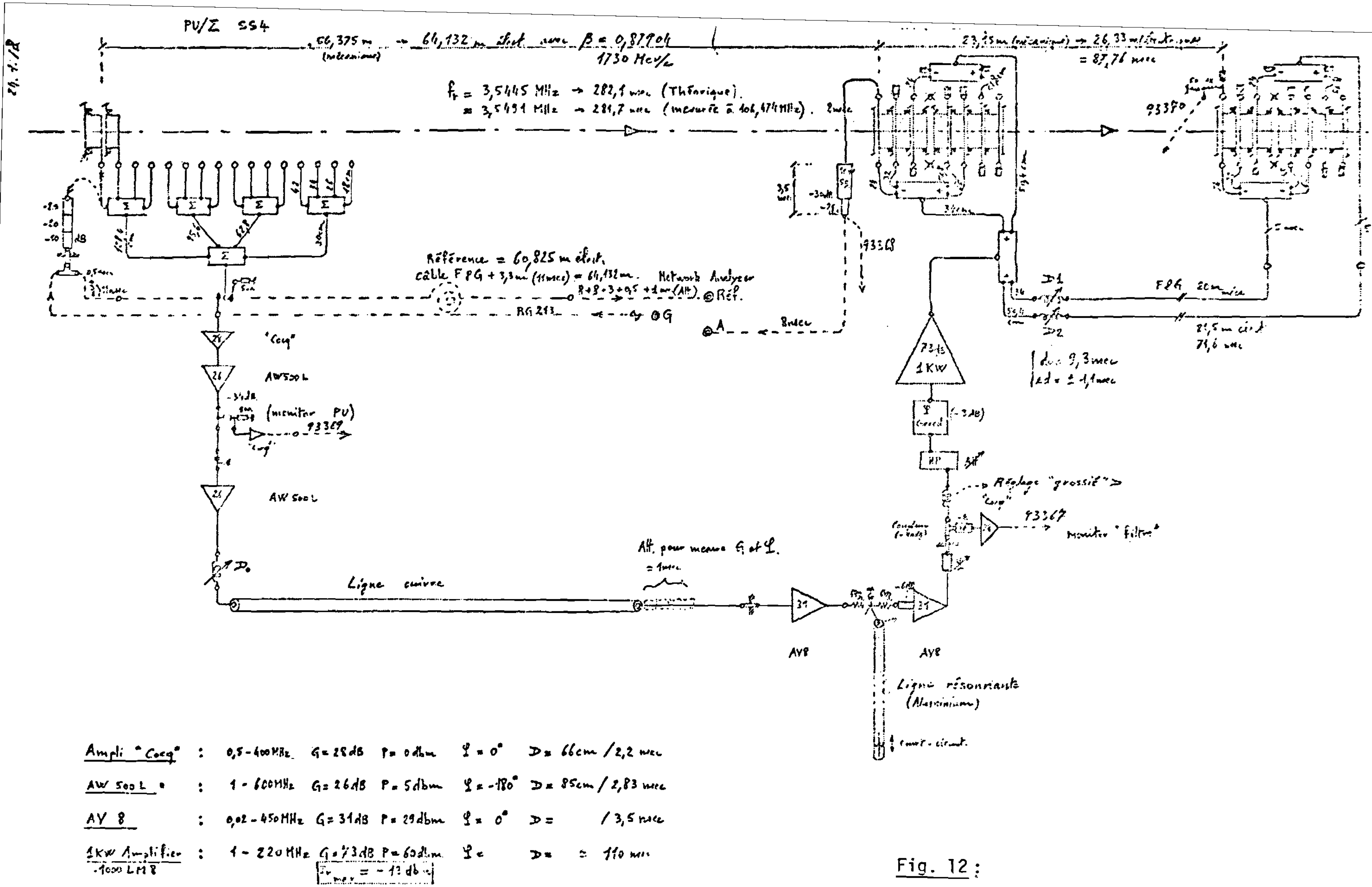
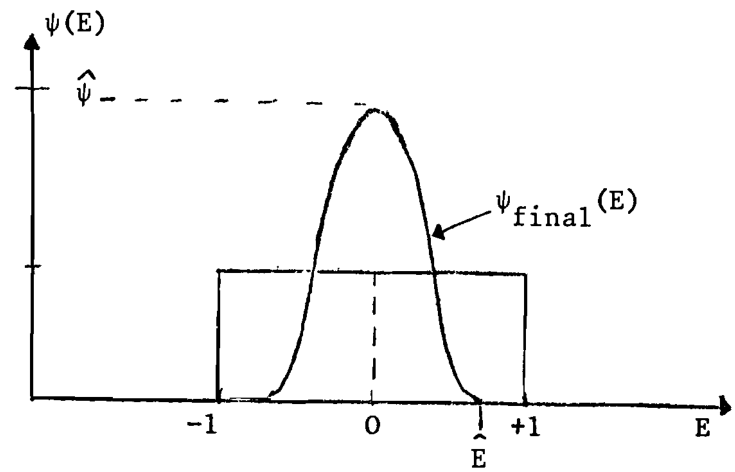


Fig. 12:
ICE momentum cooling system

As a result of the losses, an asymptotic distribution will be reached



which can be obtained from (5.5) putting $\partial\psi/\partial t = 0$:

$$\psi_{final}(E) = \begin{cases} \frac{1}{2\alpha} \ln \frac{\chi^2 + \hat{E}^2}{\chi^2 + E^2} & |E| < \hat{E} \\ 0 & |E| > \hat{E} \end{cases} \tag{5.6}$$

with $\hat{E}$ from $\hat{E} - \chi \arctan \hat{E}/\chi = \alpha$

PARAMETER	ICE	AA	
p	1.7	3.5	GeV/c
T_{rev}	280	540	ns
$\eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}$	0.65	0.1	
$(\Delta p/p)_{initial}$	3×10^{-3}	1.5×10^{-2}	total
W	110	250	MHz
f_{max}	180	400	MHz
f_{min}	70	150	MHz
N	3.5×10^{8}	2.5×10^{7}	
ψ_i	75	0.49	eV^{-1}
t/t_n	12.5	0.12	
$t(t_n = 17)$	210	2	s
$q\chi = \frac{1}{\eta(\frac{\Delta p}{p})_i}$	512	667	
q for $\chi = 0.01$	5.1×10^{4}	6.7×10^{4}	

Table 7 :

ICE and AA parameters

Equation (5.6) has been used to compare the results obtained in ICE to the theory. In Fig. 13 we plot the inverse of the assymptotic width ($\hat{E}^{-1}$) as a function of the asymptotic height.

One concludes that in the early measurements the points follow a line with $\chi \simeq 0.3$ and in the later measurements after improvement of the filter with $\chi \simeq 0.05 - 0.1$. For very large density increase, the points seem to follow a curve of decreased losses. This is probably explained by the reduction of noise in the closed loop case which was mentioned above.

The Q values of the notches were measured and q was found to be about 7000 giving for ICE parameters $\chi \simeq 0.07$. With the optimum $\alpha = 0.15$ one expects an increase in peak density by about 8 for $\chi = 0.05$ or

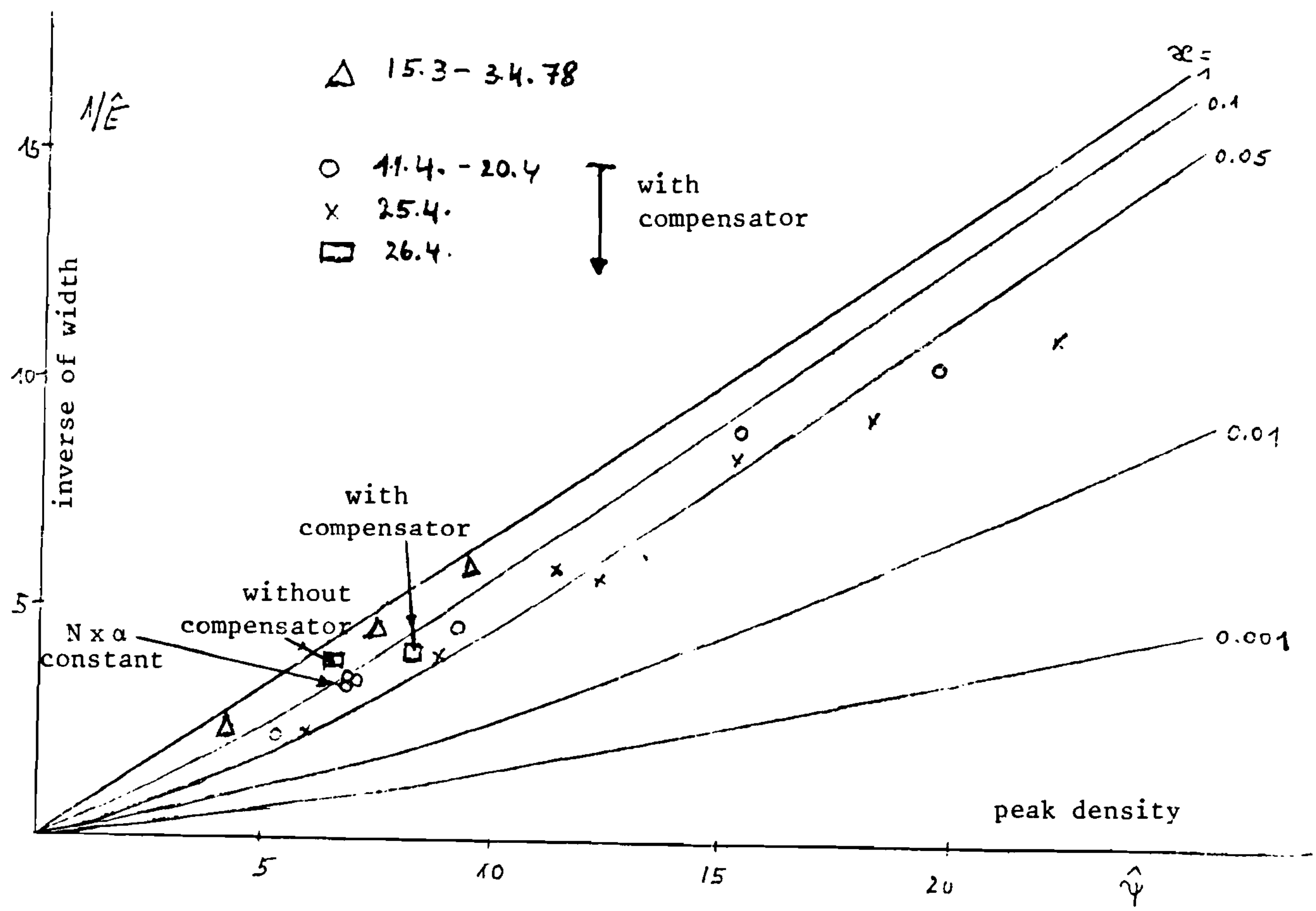


Fig. 13 : Asymptotic peak density and width
Theoretical curves and ICE results

by 6 for $\chi = 0.1$, both in a normalized time of $t_n = 17$, corresponding to 3.5 min in ICE at 3.5×10^8 p. The measured density increase under these circumstances was about 7. Scaled to precooling in the AA ring, this corresponds to the same density increase at the same χ or to the required increase by 9.5 in 2 s if $\chi \leq 0.01$ is ensured. Again some of the ICE parameters entering critically into the comparison could only be determined with limited accuracy at the time when these measurements were done. (A new run is presently under way).

With these reservations, we can conclude that the ICE results confirm the theory and the assumptions gone into the design of the AA precooling system where lower filter losses are foreseen.

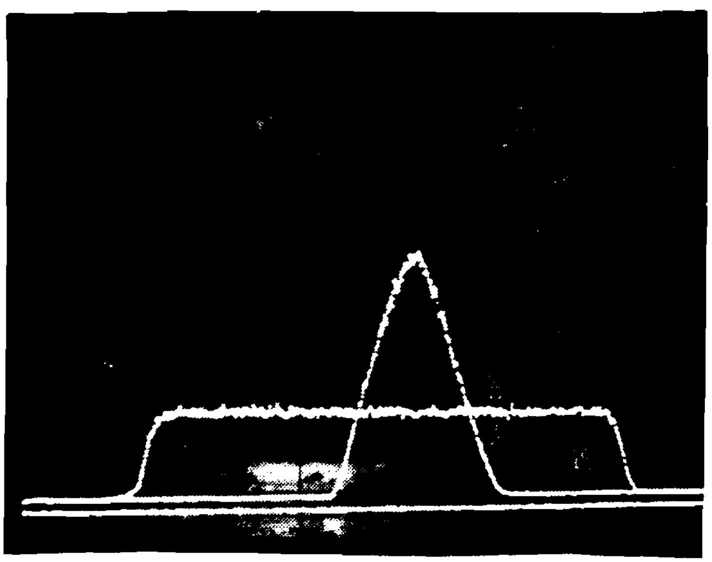


Fig. 14 :
Schottky scan of momentum distribution initial and after 3.5 min of cooling. 3.5×10^8 p. The horizontal scale is momentum ($\Delta p/p = 0.5 \times 10^{-3}$/div) The vertical scale is proportional to the square root of particle density.

6. Applications of stochastic cooling

This is my (incomplete) list of applications:

1. Stacking of $\bar{p}$ and other rare particles.
2. Physics with highly monochromatic and sharply collimated beams.
3. High density heavy ion beams.
4. Increase of beam life time (compensation of multiple scattering effects, high order resonances, beam-beam interactions).
5. Non destructive observation of low intensity beams (50 circulating particles can be seen in ICE after cooling to $\Delta p/p = 10^{-5}$).
6. Bunched beam cooling at high energy (ICE results).
7. Stochastic trapping (i.e. cooling a coasting beam into a bucket).

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