BUNCHED-BEAM p-$\bar{p}$ AND p-p COLLISIONS

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1. Introduction

In the following, we study collisions between bunched beams of protons and/or antiprotons. We assume, for simplicity, that the bunches collide head-on, and that the two beams have the same number of particles N and rms beam radii σ_x and σ_y at the crossing point.

2. Basic luminosity formulae

The luminosity L and beam-beam tune shifts $\Delta\nu$ are given by the standard formulae for e^+e^- storage rings[1]:

$$L = \frac{N^2 f}{4\pi k_b \sigma_x \sigma_y} \tag{1}$$

$$\Delta\nu_x = \frac{N r_o \beta_x}{2\pi k_b \gamma (\sigma_x+\sigma_y)\sigma_x} \tag{2}$$

$$\Delta\nu_y = \frac{N r_o \beta_y}{2\pi k_b \gamma (\sigma_x+\sigma_y)\sigma_y} \tag{3}$$

Here f is the revolution frequency, k_b the number of bunches in each beam, β_x and β_y the amplitude functions at the crossing point, r_o the classical proton radius and γ the usual relativistic factor.

If we make

$$\sigma_x/\sigma_y = \beta_x/\beta_y \tag{4}$$

as we shall assume in the following, the beam-beam tune shifts $\Delta\nu_x$ and $\Delta\nu_y$ become the same. Using (3) to eliminate one power of N in (1) yields, assuming that $\sigma_y \ll \sigma_x$:

$$L \approx \frac{N f \gamma \Delta\nu}{2 r_o \beta_y} \tag{5}$$

There is no reason not to build p-p colliding-beam machines with the highest possible magnetic fields in the dipoles B_M at all energies. Hence, the product $f\gamma$ is independent of the energy. We conclude that the luminosity does not depend explicitly on the energy of the machine, but only on the number of particles, the field in the dipoles and the value of β_y.

3. Stored beam intensity

The number of particles necessary for obtaining a given luminosity is obtained by solving (5) for N:

$$N = \frac{2Lr_o\beta_y}{f\gamma\Delta\nu} \tag{6}$$

The invariant emittances of the beams E_x and E_y are obtained from (2) and (3), again assuming $\sigma_y \ll \sigma_x$:

$$E_x = \frac{4\pi\gamma\sigma_x^{\ 2}}{\beta_x} = \frac{2Nr_o}{k_b\Delta\nu} = \frac{4Lr_o^{\ 2}\beta_y}{k_b f\gamma(\Delta\nu)^2} \tag{7}$$

$$E_y = \frac{4\pi\gamma\sigma_y^2}{\beta_y} = \frac{\beta_y}{\beta_x}\frac{2Nr_o}{k_b\Delta\nu} = \frac{4Lr_o^{\ 2}\beta_y^{\ 2}}{k_b f\gamma\beta_x(\Delta\nu)^2} \tag{8}$$

Because of the factor $f\gamma$, the invariant emittances do not depend on the design energy. In deriving (7) and (8) we have assumed that the crossings occur in regions without dispersion.

4. Limits on interaction region design

It is clear from (5) that the amplitude function β_y should be small in order to obtain a high luminosity. Lower limits on β_y are imposed by

(i) the bunch length

(ii) chromaticity correction

(iii) the strength of the nearest quadrupole.

The value of β_y should be large compared to σ_z, in order to avoid a reduction of the luminosity by the β_y-variation in the interaction region. The difficulty of chromaticity correction is proportional to ℓ_{int}/β_y where ℓ_{int} is the distance from the crossing point to the nearest quadrupole. In e^+e^- storage rings, ℓ_{int}/β_y must be less than about 50 to 100 in order to correct chromatic effects over a momentum range of about $\pm 1\%$.

The effect of the quadrupole strength on β_y is obtained by calculating the horizontal aperture of the first quadrupole A_x which must be a factor F_a larger than the rms beam size at the quadrupole entrance.

$$A_x = F_a \sigma_{xQ} \quad . \tag{9}$$

The beam size σ_{xQ} is given by:

$$\sigma_{xQ} = \sigma_x \left(\frac{\beta_{xQ}}{\beta_x}\right)^{\frac{1}{2}} \approx \frac{\sigma_x \ell_{int}}{\beta_x} \quad . \tag{10}$$

Substituting from (7) yields for the necessary quadrupole aperture:

$$A_x = F_a \ell_{int} \left(\frac{Nr_o}{2\pi\beta_x k_b \gamma\Delta\nu}\right)^{\frac{1}{2}} = \frac{F_a \ell_{int} r_o}{\gamma\Delta\nu} \left(\frac{L\beta_y}{\pi k_b f\beta_x}\right)^{\frac{1}{2}} \quad . \tag{11}$$

The quadrupole must have a focal length of about $\frac{1}{2}\ell_{int}$. Hence, its length ℓ_Q, magnetic field B_Q at a distance A_x from the centre ("poletip field") and the proton rigidity $B\rho$ are related by

$$A_x = \frac{B_Q \ell_{int} \ell_Q}{2B\rho} \tag{12}$$

Equating (11) and (12) yields an equation for ℓ_Q:

$$\ell_Q = \frac{e Z_o F_a}{2\pi B_Q \Delta\nu} \left(\frac{L \beta_y}{\pi k_b f\beta_x}\right)^{\frac{1}{2}} = \frac{e Z_o F_a}{2\pi B_Q} \left(\frac{N\gamma}{2\pi\beta_x k_b \Delta\nu r_o}\right)^{\frac{1}{2}} \quad . \tag{13}$$

Here e is the elementary charge and Z_o is the impedance of free space. Experience with the design of e^+e^- interaction regions indicates that there is a relation between ℓ_Q and ℓ_{int}, of the form

$$\ell_{int} = G_e \ell_Q \tag{14}$$

where the coefficient G_ℓ is of the order of 2 or more. In addition, chromaticity correction imposes a relation of the form

$$\ell_{int} = G_c \beta_y \tag{15}$$

By combining (13) to (15) we finally arrive at equations for β_y:

$$\beta_y = \frac{G_\ell F_a}{G_c} \frac{e\, Z_o}{2\pi B_Q \Delta\nu} \left(\frac{L\beta_y}{k_b f \beta_x} \right)^{\frac{1}{2}} \qquad (16)$$

$$\beta_y^{3/2} = \frac{G_\ell F_a}{G_c} \frac{e\, Z_o}{2\pi B_Q} \left(\frac{N\gamma\beta_y}{2\pi k_b \Delta\nu r_o \beta_x} \right)^{\frac{1}{2}} \qquad (17)$$

These equations include all the limitations on the interaction region design imposed by the poletip field B_Q, optical constraints (G_ℓ) and chromaticity (G_c); they contain only the ratio β_y/β_x on the right-hand side.

5. Application to p-$\bar{p}$ collisions

For p-$\bar{p}$ collisions we make the assumptions and obtain the scaling laws shown in Table 1.

Table 1. Assumptions and scaling laws for p-$\bar{p}$ collisions

Assumed independent of γ:	N, $\Delta\nu$, k_b, F_a, G_ℓ, G_c, B_Q, β_y/β_x, B_M
Proportional to γ^o:	E_x, E_y
Proportional to $\gamma^{1/3}$:	β_x, β_y, ℓ_Q, ℓ_{int}
Proportional to $\gamma^{-1/3}$:	L, A_x

A few comments are in order on the fixed parameters used in the detailed examples. The aperture allowance F_a includes the extra space required at the injection energy which is about twenty times smaller than the design energy. The chromaticity factor G_c is smaller than in e^+e^- storage rings because the tolerances on the chromaticity correction may be tighter in p-$\bar{p}$ schemes. The limit $\Delta\nu$ on the beam-beam tune shift is the conventional value for coasting proton beams. There are strong doubts whether it also applies to bunched proton beams.

The number of particles corresponds to that of the CERN p-$\bar{p}$ project[2]. It is used here as a lower limit of what might be achieved in an optimized $\bar{p}$-factory fed from a 20 GeV proton synchrotron with a flux of 10^{13} protons/s, or from one of its injectors with even higher fluxes.

When actual numbers are inserted into the equations above, it turns out that the apertures of the interaction region quadrupoles are rather small by comparison with present values. The physical reason for this is the constant invariant emittance which results in decreasing beam size with energy. In order not to deviate too much from present practice in quadrupole construction, we have assumed rather a low value of the poletip field B_Q.

The number of bunches is 4 in each beam, because we assume that the machine has 8 equidistant interaction regions. This choice avoids the complications of a higher number of bunches where the beams have to be separated in the unwanted crossings while they are in collision at the interaction regions. As a consequence of this choice, the total number of events in each collision between bunches increases as $\gamma^{2/3}$.

Table 2 shows the main machine parameters for three different different energies, namely about 200 GeV, 2 and 20 TeV. The 200 GeV machine has parameters which are fairly close to those of the CERN p-$\bar{p}$ project[2). It is included here as a useful check of the interaction region design procedure. There is fairly good agreement between the actual and the computed parameters.

As the design energy increases, the quadrupole aperture and beam size decrease, and the quadrupole length increases. The mechanical tolerances of the quadrupole must be roughly proportional to the aperture. Avoiding the field errors associated with the tolerances is the essential reason for the choice of B_Q. At the low value chosen, the windings of a superconducting quadrupole could be far away from the beam, thus relaxing considerably the tolerances on the coil position.

So far, we have tacitly assumed that the synchrotron can be modified to include p-$\bar{p}$ interaction regions with the properties described above. This must also hold during synchrotron operation, with injected beam sizes which are larger than those at the design energy. This problem can be alleviated by increasing the amplitude functions at the crossing points during synchrotron operation, or circumvented by taking the synchrotron beam through a by-pass around the interaction region[3).

Table 2. Parameters for p-$\bar{p}$ collisions at various energies

$N = 10^{12}$	$F_a = 40$	$B_Q = 1T$
$\Delta\nu = 0.005$	$G_\ell = 5$	$\beta_x/\beta_y = 4$
$k_b = 4$	$G_c = 20$	$C/2\pi\rho = 1.5$

γ	200	2×10^3	2×10^4	
β_y	1.33	2.9	6.2	m
β_x	5.3	11.4	25	m
ℓ_{int}	27	57	124	m
ℓ_Q	5.3	11.4	25	m
E_x	48π	48π	48π	μm
E_y	12π	12π	12π	μm
B_M	1	10	10	T
f	48	48	4.8	kHz
L	1.2×10^{30}	5.5×10^{30}	2.6×10^{30}	$cm^{-2}s^{-1}$
A_x	107	49	23	mm
σ_x	0.56	0.26	0.12	mm
σ_y	141	65	31	μm
W	0.032	0.32	3.2	MJ

We studied neither the RF system required for keeping the beams bunched, nor the magnet lattice outside the interaction regions and space-charge phenomena. We do not expect particular difficulties with the RF system. The magnet lattice is similar to that of the coasting-beam p-p collisions[4).]

A luminosity well above 10^{30} $cm^{-2}s^{-1}$ is within reach over the whole energy range. As the energy increases, the number of events in a single collision between bunches increases. It reaches about 7 at 20 TeV, assuming a total cross-section of about 100 mbarn. If this turns out to be too inconvenient for event analysis, th. . our concept of colliding a few bunches breaks down somewhere in the TeV range.

The only two alternatives are increasing the number of bunches and changing to coasting beams. Increasing the number of bunches implies beam separation at the unwanted crossing points at the same time as beam collisions in the interaction regions, and all the difficult tolerances associated with separation. Coasting beams imply a reduction of the luminosity to about $\mathcal{L} = 10^{28}$ $cm^{-2}s^{-1}$ at $N = 10^{12}$. Hence, in order to obtain a luminosity of about $\mathcal{L} = 10^{30}$ $cm^{-2}s^{-1}$, approximately 10^{13} protons and antiprotons must be stored.

6. Application to p-p collisions

If a storage ring is added to the synchrotron, p-p collisions can be obtained. We assume that the circumference and the number of bunches in the synchrotron and in the storage ring are chosen to be such that the bunch spacings in the two machines are identical.

For the time being, we also assume that no proton accumulation takes place in the storage ring, but that the available protons are arranged in a suitable number of bunches. It then seems natural to make the number of protons in a bunch the same as in the p-$\bar{p}$ scheme. In this case the beam sizes at the crossing points are the same as in the p-$\bar{p}$ scheme, and the conclusions on the interaction region design also apply to p-p collisions.

The number of protons is the same as in the synchrotron; we take $N = 6\times10^{14}$. The technical problems associated with the stored energy in the beam are all the same as in the synchrotron, provided that the beam transfer between the machines is clean enough, and can be considered solved. With 0.25×10^{12} protons in a bunch we thus arrive at 2400 bunches and a bunch spacing of about 25 m. The performance which might thus be achieved is summarized in Table 3.

Table 3. Parameters for p-p collisions at 20 TeV

$N = 6\times10^{14}$	$F_a = 40$	$\beta_Q = 1T$
$\Delta\nu = 0.05$	$G_\ell = 5$	$\beta_x/\beta_y = 4$
$k_b = 2400$	$G_c = 20$	$C/2\pi\rho = 1.5$
$\beta_y = 6.2$ m	$\ell_{int} = 124$ m	$E_x = 48\pi$ μm
$\beta_x = 25$ m	$\ell_Q = 5$ m	$E_y = 12\pi$ μm
$B_M = 10T$	$f = 4.8$ kHz	$L = 1.6\times10^{33}$ cm^{-2}s^{-1}
$A_x = 23$ mm	$\sigma_x = 0.12$ mm	$\sigma_y = 31$ μm
$W = 1900$ MJ		

Since head-on collisions are assumed, and the close bunch spacing is neglected, the luminosity figures must be considered optimistic.

Since the bunch population is the same as in the p-$\bar{p}$ scheme, the number of events for a single collision is again about 7. Hence the remarks made above also apply.

7. Conclusions

We have studied schemes for p-$\bar{p}$ and p-p collisions between bunched beams. With rather conservative assumptions about available $\bar{p}$ fluxes we obtain a luminosity in the 10^{30} cm^{-2}s^{-1} range which has rather a weak variation with the energy. The most natural scheme where the number of bunched beams is half the number of interaction regions, has the difficulty that the number of events in a single collision increases with the energy.

For p-p collisions, we make use of the full current available from the synchrotron. We find luminosities in the 10^{33} cm^{-2}s^{-1} range, and the same difficulties with the number of events in a single collision as in the p-$\bar{p}$ scheme. Since the luminosities are not much larger than for coasting p-p collisions[4)], there does not seem to be a good reason for bunched p-p collisions.

References

1) M. Sands, SLAC-121 (1970).

2) CERN Report, SPS/SI/PP/Int. Note/77-9.

3) P. McIntyre, contribution to this workshop.

4) E. Keil and N.M. King, contribution to this workshop.