SUMMARY REPORT OF THE
WORKING GROUP ON COLLIDING BEAMS

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1. Introduction

The working group considered the possibilities of p-$\bar{p}$, p-p and e-p colliding beams associated with a large proton synchrotron.

2. Assumptions

Our considerations are based on the performance parameters of the proton synchrotron shown in Table I.

Table I. Proton synchrotron performance parameters

Maximum energy	E	20	TeV
Proton flux	$\dot{N}_p$	10^{13}	s^{-1}
Repetition frequency	f_{rep}	1	min^{-1}
Normalized emittance	E_t	30π	µm
Peak magnetic field	B_M	10	T
Circulating current	I	0.5	A

The performance estimates of the p-$\bar{p}$ colliding beams are based on the parameters of the $\bar{p}$ factory shown in Table II.

Table II. Antiproton factory performance parameters

$\bar{p}$ flux	$\dot{N}_{\bar{p}}$	10^{12}	day^{-1}
Filling time	T_{fill}	1	day
Emittance	}	suitable	
Momentum spread			

These figures are close to the design values of the CERN p-$\bar{p}$ facility1). They might be improved by optimizing the energy and flux of the protons which are used in the $\bar{p}$ production.

In order to avoid lengthy discussions on the maximum proton current which can be stored in a storage ring, due to heating of superconducting magnets by continuous beam losses, beam abort systems, collective phenomena etc., we have assumed that the currents in the synchrotron and the storage ring are identical.

3. Results

The results of our discussions on p-$\bar{p}$, p-p and e-p colliding beams are summarized in Table V which has been compiled from the following more detailed documents:

- i) Bunched p-$\bar{p}$ and p-p colliding beams, E.D. Courant and E. Keil
- ii) Coasting p-p colliding beams, E. Keil and N.M. King
- iii) e-p colliding beams, T. Nishikawa and E. Keil
- iv) A bypass for p-$\bar{p}$ colliding beams, P. McIntyre.

3.1 p-$\bar{p}$ colliding beams

The performance is limited by the p and $\bar{p}$ currents and by the design of the low-β insertion. The stored currents are 10^{12} particles in each beam. The design of the low-β insertions is governed by the field at the edge of the aperture of the quadrupoles nearest to the crossing points and the possibility of correcting chromatic effects. The beam sizes and apertures are smaller than in machines like the FNAL and SPS synchrotrons. We did not study the tolerances associated with the small apertures and large machine size. At a total cross section of 100 mb, the event rate per collision is about 7, which may be uncomfortable for event analysis.

The scaling laws for the most important machine parameters with the machine energy γ are summarized in Table III.

Table III. Scaling laws for bunched p-$\bar{p}$ colliding beams

Assumed independent of γ :	
Number of p and $\bar{p}$	N
Beam-beam tune shift	$\Delta\nu$
Number of bunches	k_b
Quadrupole field at aperture limit	B_Q
Ratio of amplitude functions at crossing	β^*_y/β^*_x
Found proportional to $\gamma^{1/3}$:	
Vertical amplitude function	β^*_y
Horizontal amplitude function	β^*_x
Length of interaction region quadrupole	ℓ_Q
Free space around crossings	$\pm\ell_{int}$
Found independent of γ :	
Normalized emittances	E_x, E_y
Found proportional to $\gamma^{-1/3}$	
Luminosity	L
Quadrupole aperture	A_x

The interference between p-$\bar{p}$ colliding-beam and fixed-target operation can be reduced by bypassing the p-$\bar{p}$ interaction region for proton acceleration in the synchrotron. This may be particularly advantageous if the number of bypasses and their length are small, because of the large circumference of the synchrotron.

3.2 Bunched p-p colliding beams

If a proton storage ring is added to the synchrotron with roughly the same radius as the synchrotron, but exactly the same bunch spacing, bunched p-p collisions can be obtained.

If the current per bunch is kept the same as in the p-$\bar{p}$ colliding beams, the results of the p-$\bar{p}$ calculations on the machine parameters apply. However, in this case the number of bunches becomes $k_b = 2400$ in each beam, and the bunch spacing 25 m. This might give rise to multi-bunch instabilities and long-range tune shifts2).

In contrast to the p-$\bar{p}$ collisions where the beams always collide in the crossing regions in the absence of transverse electric fields, the p-p scheme requires careful beam alignment in order to ensure collisions. In the calculation, we have assumed head-on collisions although the beams must collide at a small angle in order to avoid the long-range tune shifts.

The main parameters are summarized in Table V. The event rate per collision, 7, is the same as for the p-$\bar{p}$ scheme.

3.3 Coasting p-p colliding beams

We have followed the standard procedure3) for designing coasting-beam p-p storage rings, and assumed the same total current as in the bunched p-p scheme above. This results in closed expressions for the luminosity L, optimum amplitude function β^* and crossing angle α (r_p is the classical proton radius, σ^* the rms beam radius at the crossing):

$$L \approx \frac{4}{3} \gamma \left(\frac{\pi I^3 \Delta\nu}{E_t \ell_{int} r_p e^2 c} \right)^{\frac{1}{2}} , \tag{1}$$

$$\beta^* \approx \left(\frac{\Delta\nu E_t \ell_{int} e c}{8 \pi I r_p} \right)^{\frac{1}{2}} , \tag{2}$$

$$\alpha \approx \left(\frac{2}{\pi} \right)^{\frac{1}{2}} \frac{I r_p \beta^*}{\Delta\nu \gamma \sigma^* e c} \tag{3}$$

The scaling laws for the main parameters are summarized in Table IV and the parameters themselves in Table V. It should be noted that the luminosity in the coasting-beam mode is only a factor of about three lower than in the bunched-beam mode. At a total cross section of 100 mb, the event rate is 46 MHz. A sketch of half an interaction region is shown in Fig. 1.

Table IV. Scaling laws for coasting p-p colliding beams

Assumed independent of γ :	
Number of protons	N
Normalized emittance	E_t
Free space $\pm\ell_{int}$ Amplitude function β^*	$\sim \gamma^*$
Luminosity	$L \sim \gamma^{-1}$
Crossing angle	$\alpha \sim \gamma^{-1}$

* imposed by chromaticity correction

By accumulating a few synchrotron pulses, using e.g. RF stacking, the luminosity can be increased in proportion to $I^{3/2}$ according to (1). Collective phenomena such as the longitudinal stability of the injected beam, the single-beam tune shift and the resistive-wall instability limit the performance well above the luminosity arrived at in Table V.

3.4 Bunched e-p colliding beams

The electron energy is assumed to be 140 GeV, obtained by rough scaling from LEP$^{4)}$. In order to simplify the estimate, we have assumed that the beam sizes at the crossing points are the same for electrons and protons, i.e.

$$\sigma_{xe} = \sigma_{xp} \qquad \sigma_{ye} = \sigma_{yp} \tag{4}$$

We further assume that the proton beam has the same bunch population as in the p-p and p-$\bar{p}$ schemes, that the electron beam has 'a' times that population and that there are k_b bunches in each beam. Then the total numbers are

$$N_p = \frac{1}{4} \cdot 10^{12}\, k_b \qquad N_e = \frac{1}{4} \cdot 10^{12}\, k_b\, a \tag{5}$$

The differences in energy and in the permissible beam-beam tune shifts ($\Delta\nu_p = 0.005$, $\Delta\nu_e = 0.06$) require that the following condition should hold for the amplitude functions at the crossing points in both planes:

$$\beta_e = 0.084\ a\ \beta_p \tag{6}$$

This condition looks feasible. The luminosity then becomes

$$L = 0.65\cdot 10^{30}\ k_b \qquad \mathrm{cm}^{-2}\mathrm{s}^{-1} \tag{7}$$

In order to reach the canonical luminosity $L = 10^{32}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$, k_b must be about 150. The synchrotron radiation power is about 110 MW.

The performance of a coasting proton beam colliding with an electron beam was also estimated. For about the same luminosity and synchrotron radiation power, the proton current is much higher.

4. Conclusions

The estimates show that adequate luminosities can be obtained for p-p and p-$\bar{p}$ colliding beams at 20 TeV, even with relatively conservative assumptions about the circulating p and $\bar{p}$ currents. The bunched p-p scheme has a slightly larger luminosity, but the coasting p-p scheme is much simpler and avoids the problem of several events per collision. Colliding e-p beams are also possible with good luminosity for electron energies above 100 GeV.

5. References

1) CERN Report, SPS/SI/PP/Int. Note/77-9.

2) E. Keil, C. Pellegrini and A.M. Sessler, Nucl. Instr. Methods 118, 165 (1974).

3) E. Keil, Proc. 8th Internat. Conf. on High Energy Accelerators, Stanford 1974, 660 (1974).

4) CERN/ISR-LEP/78-17.

Table V. Parameter list for p-$\bar{p}$, p-p and e-p colliding beams

	p-$\bar{p}$	p-p bunched	p-p coasting	e-p^* bunched
Energy/TeV	20	20	20	0.14/20
Bending magnet field/T	10	10	10	0.07/10
Number of particles	10^{12}	$6\cdot10^{14}$	$6\cdot10^{14}$	$4\cdot10^{13}$
Number of bunches	4	2400	-	160
Beam-beam tune shift	0.005	0.005	0.005	0.06/0.005
Hor. ampl. function β_x^*/m	25	25	-	2.1/25
Ver. ampl. function β_y^*/m	6.25	6.25	18.6	0.5/6.25
Crossing angle/μrad	0	0	47	small
Free space around crossing/m	±124	±124	±170	±20
Hor. norm. emittance/μm	48π	48π	30π	560/48π
Ver. norm. emittance/μm	12π	12π	30π	140/12π
Low-β quadrupole field/T	1	1	1	1
Low-β quadrupole half aperture/mm	23	23	65	35/23
Hor. rms beam radius at crossing/μm	120	120	-	120
Ver. rms beam radius at crossing/μm	31	31	84	31
Stored energy in beam/MJ	3.2	1900	1900	0.9/130
Synchrotron radiation power/MW	-	-	-	110/-
Luminosity/$cm^{-2}s^{-1}$	$2.6\cdot10^{30}$	$1.6\cdot10^{33}$	$4.6\cdot10^{32}$	10^{32}

* First number applies to electrons, second to protons.

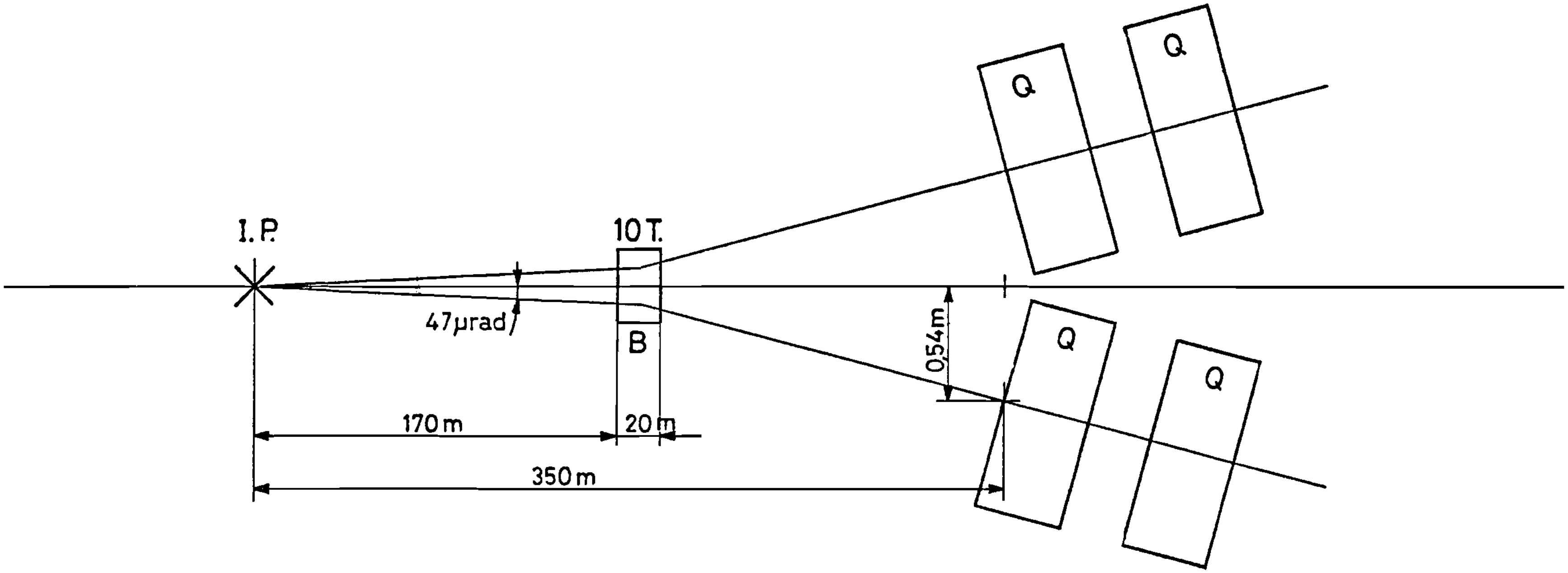

Fig. 1. Rough layout of half a coasting-beam p-p interaction region.