LIMITATIONS ON PERFORMANCE OF e^+e^- STORAGE RINGS AND LINEAR COLLIDING BEAM SYSTEMS AT HIGH ENERGY

J.-E. Augustin*, N. Dikanski†, Ya. Derbenev†, J. Rees‡, B. Richter‡, A. Skrinski†, M. Tigner**, and H. Wiedemann‡

Introduction

This note is the report of working Group I (J. Rees - Group Leader). We were assisted at times by U. Amaldi and E. Keil of CERN. We concerned ourselves primarily with the technical limitations which might present themselves to those planning a new and higher-energy electron-positron colliding-beam facility in a future era in which, it was presumed, a 70-GeV to 100-GeV LEP-like facility would already exist. In such an era, we reasoned, designers would be striving for center-of-mass energies of at least 700-GeV to 1-TeV. Two different approaches to this goal immediately came to the fore: one, a storage ring based on the principles of PEP, PETRA, and LEP and the other, a system in which a pair of linear accelerators are aimed at one another so that their beams will collide. We realized early in the study that a phenomenon which has been negligible in electron-positron systems designed to date would become important at these higher energies - synchrotron radiation from a particle being deflected by the collective electromagnetic field of the opposing bunch - and we dubbed this phenomenon "beam-strahlung." During the rest of the week we investigated the scaling laws for these two colliding-beam systems taking beam-strahlung into consideration.

* Orsay
† Institute of Nuclear Physics, Novosibirsk
‡ SLAC
** Cornell

We concluded that it would be necessary to depart from some of the common design features of PEP, PETRA, and LEP in order to push electron storage rings into the energy regime of interest. Either the stored bunches would have to be markedly longer, presumably requiring a radio-frequency well below the 350-500 MHz range, or the number of bunches in each beam would have to be increased as it could, for example, by providing separate storage rings for the two beams or by arranging discrete beam separations in a single ring. By taking such measures, however, it does appear possible to make electron-positron storage rings work in this energy regime.

Colliding linac beams are less sensitive to the effects of beam-strahlung. The feasibility of this technique appears to rest more on the achievable density of the beams in phase space. If the necessary densities are attained, we suggest that, for sufficiently high energies, the cost of the linac system will fall below that of an equivalent storage ring system.

In the following sections, we shall first review briefly the formulae relevant to beam-strahlung; then we shall apply them to storage rings to obtain scaling laws; and finally we shall derive some scaling laws for the system of colliding linac beams. Before embarking on that program, however, we wish to put the reader on notice that our intent throughout will be <u>to estimate</u>, not to calculate in detail. Our results are therefore approximate and should be taken in that spirit.

Beam-strahlung

As a particle of either beam passes through a bunch of the opposing beam at an interaction region, it is deflected by the collective electromagnetic field of that bunch to a degree and in a manner which is determined by the particle's coordinates entering the interaction region and by the spatial distribution of charge in the bunch. The net angular deflection itself, being a non-linear function of the particle's coordinates, is the source of the familiar incoherent beam-beam limit (Amman, 1973; Amman and Ritson, 1961) which has been regarded as a fundamental limitation on performance in the design of all single-ring electron-positron systems. (It should be remarked here that efforts are being made at Orsay, France, to circumvent this limitation by means of a multiple-beam scheme called DCI which, if successful, will vitiate the incoherent limit and the beam-strahlung effects about to be described.) As the particle is being deflected it emits synchrotron radiation, the properties of which are strongly dependent on particle energy. The radiated energy and the critical energy of the spectrum both increase rapidly with particle energy even though the beam-beam tune shift is being held constant, and at sufficiently high energy, the beam-strahlung process imposes its own limitations.

Figure 1 shows a typical particle incident on an opposing bunch. The particle density distributions are taken as tri-Gaussian with horizontal, vertical and longitudinal standard deviations of σ_x^*, σ_y^*, and σ_z, respectively. The typical incident particle's vertical position is taken as σ_y^* and its horizontal position is taken as zero. The deflection of the trajectory is characterized by the focal length F, considered to be large compared to the bunch length $2\sigma_z$. The bending radius during deflection is designated ρ_b.

$$\frac{1}{\rho_b} = \frac{1}{F} \frac{\sigma_y^*}{\sigma_z} \qquad (1)$$

The distance over which the bending takes place is only σ_z because the relative velocity between particle and bunch i_ 2c.

The total energy radiated by the particle in one such collision is

$$\Delta U = \frac{2}{3} mc^2 \gamma^4 \frac{r_e \sigma_z}{\rho_b^2}$$

$$= (14 \times 10^{-6}) \frac{E^4 \sigma_z}{\rho_b^2} \quad , \qquad (2)$$

where lengths are measured in meters and energies in GeV. The symbol r_e refers to the classical radius of the electron and mc^2 is the electron's rest energy. The critical energy of the beam-strahlung is

$$\epsilon_c = \frac{3}{2} \hbar c \frac{\gamma^3}{\rho_b}$$

$$= (2.2 \times 10^{-6}) \frac{E^3}{\rho_b} \quad . \qquad (3)$$

The mean number of photons emitted in the collision is (Sands, 1971)

$$n_\gamma = \frac{5}{2\sqrt{3}} \alpha\gamma \frac{\sigma_z}{\rho_b}$$

$$= 20 \frac{E\sigma_z}{\rho_b} \quad . \qquad (4)$$

where α is the fine-structure constant.

Effects of Beam-strahlung in Storage Rings

Whether the effects of beam-strahlung are important in any particular storage-ring design or not depends, of course, on the value of ρ_b. Returning to Eq. (1) we first relate the focal length to the beam-beam tune shift, the touchstone of the incoherent limit. A thin lens located at the interaction point where the vertical betatron function is β_y^* produces a tune shift

$$\Delta\nu = \frac{\beta_y^*}{4\pi}\,\frac{1}{F} \quad ,$$

approximately if the storage-ring tune, unperturbed by the lens, is not too close to an integer, so we can write

$$\frac{1}{\rho_b} = \frac{4\pi\Delta\nu}{\beta_y^*} \cdot \frac{\sigma_y^*}{\sigma_z} \qquad (5)$$

Note that we would have got the same value for ρ_b if we had chosen for our typical trajectory $y=0$, $x=\sigma_x^*$; because storage rings such as PEP, PETRA, and LEP have been designed to operate simultaneously at the vertical and horizontal incoherent limit where $(\sigma_y^*/\beta_y^*) = (\sigma_x^*/\beta_x^*)$.

The values of ρ_b computed from Eq. (5) for PEP, PETRA, and LEP are not very different. LEP parameters are taken from CERN/ISR-LEP/78-17. The values are of the order of 10^2m, being about 200 m for LEP at peak performance ($E=70$ GeV). This is not surprising since all of these machines have been designed to use their full apertures which themselves are similar, have similar β_y^*-values and have similar bunch lengths. In other words, the design practices currently in favor for large electron-positron storage rings tend to hold ρ_b roughly constant independent of energy. This fact leads us to observe that the energy loss ΔU and the critical energy ε_c will be rapidly increasing functions of energy as the machines are pushed to higher and higher energies.

Next let us note that the mean number of photons emitted per collision is small. In LEP at 70 GeV, $n_\gamma = 0.25$. Even at five or ten times that energy, the number is of the order of one, so we conclude that the beam-strahlung process will always involve strong fluctuations from collision to collision.

Evaluating Eqs. (2) and (3) for LEP at 70 GeV we get

$$\Delta U = 0.24 \text{ MeV}$$

$$\varepsilon_c = 3.5 \text{ MeV}$$

(See also Hofmann and Keil, LEP-70/85, Dec. 1978.) Inasmuch as the synchrotron radiation loss-per-turn in the bending magnets is 906 MeV compared to a total loss to beam-strahlung at all eight interaction regions of 2 MeV, we can neglect any contributions to radiation damping due to beam-strahlung. Thus beam-strahlung introduces an important source of energy fluctuations over and above those already present without bringing any new damping with it. The result will be an increased energy spread in the stored beams.

The total fractional energy spread (σ_e/E) can probably not be allowed to increase much above 10^{-3}, the approximate value it has in PEP, PETRA, and LEP, because the correction of chromatic effects has already, at that level of energy spread, become very difficult. This assertion is, of course, somewhat arbitrary and argumentative. Work is in progress in several centers on improved methods of chromaticity correction in large storage rings and accelerators. If this work bears fruit it may allow the accommodation of significantly larger energy spreads in these machines, and in that event, our findings here may simply serve to emphasize the importance of this work. Nevertheless, for our purposes we take the following as principles: (1) The equilibrium fractional energy spread due to the synchrotron radiation in the magnets (σ_{eo}/E) cannot be allowed to be greater than that allowed in LEP at 70 GeV, and (2) the additional energy spread created by beam-strahlung (σ_{eb}/E), which adds in quadrature, cannot itself be greater than (σ_{eo}/E).

$$\left(\frac{\sigma_{eb}}{E}\right)^2 \leq \left(\frac{\sigma_{eo}}{E}\right)^2 \leq \left(\frac{\sigma_e}{E}\right)^2_{LEP} \qquad (6)$$

To apply these criteria we need to collect formulae relating these quantities to the storage-ring parameters (Sands, 1971).

$$\left(\frac{\sigma_{eo}}{E}\right)^2 = \frac{55}{32\sqrt{3}} \frac{\hbar}{mc} \frac{\gamma^2}{J_\varepsilon \rho_o} \quad , \qquad (7)$$

where J_ε is the longitudinal damping partition number and ρ_o is the radius of curvature in the bending magnets, considered to be constant. We shall assume $J_\varepsilon = 2$ as it is in a separated-function lattice so that

$$\left(\frac{\sigma_{eo}}{E}\right)^2 = (0.7 \times 10^{-6}) \frac{E^2}{\rho_o} \quad . \qquad (8)$$

The contribution due to the beam-strahlung is estimated as follows. Let n_d be the number of orbital periods in a longitudinal damping time.

$$n_d = \frac{E}{U_0} = \frac{3}{4\pi} \frac{\rho_0}{\gamma^3 r_e}$$

$$= (1.13 \times 10^4) \frac{\rho_0}{E^3} \qquad (9)$$

The equilibrium energy spread due to beam-strahlung alone then would be

$$\sigma_{eb}^2 = \frac{1}{4} n_d \, n_{ir} \, n_\gamma \, \langle u^2 \rangle$$

where n_{ir} is the number of interaction regions where beam-strahlung is emitted and $\langle u^2 \rangle$ is the mean-square energy (Sands, 1971).

$$\langle u^2 \rangle = \frac{11}{27} \varepsilon_c^{\,2} .$$

$$\left(\frac{\sigma_{eb}}{E}\right)^2 = (1.1 \times 10^{-7}) \frac{E^2 \, n_{ir} \, \rho_o \, \sigma_z}{\rho_b^{\,3}} \tag{10}$$

Now we are prepared to apply the criteria of Eq. (6) which, combined with Eq. (8), gives

$$(0.7 \times 10^{-6}) \frac{E^2}{\rho_o} \leq \left(\frac{\sigma_e}{E}\right)^2_{LEP} = (1.2 \times 10^{-3})^2$$

We conclude that

$$\rho_o \geq 0.5 \, E^2 \tag{11}$$

The bending radius must scale as the square of the beam energy. Next we combine Eqs. (6), (10), and (11) to obtain

$$\frac{E^4 \, n_{ir} \, \sigma_z}{\rho_b^{\,3}} \leq 26 \tag{12}$$

We have chosen the equality in Eq. (11) in order to permit the smallest value of ρ_b. Now we could express this result in terms of the tune shift via Eq. (5), but our goal is to determine the limitation on luminosity as a function of energy imposed by the restriction we have adopted on beam energy spread, so we shall employ the usual luminosity and tune-shift formulae

$$\mathscr{L} = \frac{f_o N^2}{4\pi \; n_b \, \sigma_x^* \, \sigma_y^*} \tag{13}$$

and

$$\Delta\nu = \frac{N \, \beta_y^* \, r_e}{2\pi \, \gamma \, n_b \, \sigma_x^* \, \sigma_y^*} , \quad (\sigma_y^* << \sigma_x^*) \tag{14}$$

where N is the number of particles per beam and n_b is the number of bunches per beam. These two expressions, taken together with Eq. (5) yield

$$\frac{1}{\rho_b} = 4\sqrt{\pi} \, r_e \, \frac{\mathscr{L}^{1/2} \, \sigma_y^{*1/2}}{\gamma \, n_b^{1/2} \, f_o^{1/2} \, \sigma_y^{*1/2} \; \sigma_z}$$

$$= (10^{-17}) \, \frac{\mathscr{L}^{1/2} \, \sigma_y^{*\,1/2}}{E \, n_b^{1/2} \, f_o^{1/2} \, \sigma_x^{*1/2} \, \sigma_z} , \tag{15}$$

which, together with Eq. (12) gives finally

$$(4 \times 10^{-53}) \, \frac{\mathscr{L}^{3/2} \; E \, n_{ir}}{f_o^{\;3/2} \, \sigma_z^2 \, n_b^{\;3/2}} \left(\frac{\sigma_y^*}{\sigma_x^*} \right)^{3/2} \leq 1 \; . \tag{16}$$

Now f_o is just $(c/2\pi R)$, where R is the gross radius of the ring which must be larger than ρ_o but not greatly larger. Its actual value will depend on requirements for free space along the orbit for experimental areas, radiofrequency systems, etc. We estimate

$$R = 1.5 \; \rho_o$$

which implies, according to Eq. (11), that

$$R = 0.75 \; E^2$$

and

$$f_o = 64 \times 10^6 E^{-2} \qquad (17)$$

As noted above the beam dimensions σ_x^* and σ_y^* at the interaction regions do not vary greatly among PEP, PETRA, and LEP. Equation (16) argues that the ratio (σ_y^*/σ_x^*) should be made as small as possible to maximize luminosity, but we do not believe it is feasible to reduce this ratio much below its value in LEP of 0.06. The reason for this belief is that, in a practical machine, the minimum attainable orbit distortions together with the sextupole systems necessary to correct chromaticity produce non-zero vertical dispersion in the bending magnets where quantum excitation drives vertical oscillations. Hence we take

$$\frac{\sigma_y^*}{\sigma_x^*} = 0.06$$

as in the LEP design. With this choice, and using Eq. (17) we get the approximate scaling law,

$$(10^{-66}) \frac{\mathscr{L}^{3/2} E^4 n_{ir}}{\sigma_z^2 n_b^{3/2}} \leq 1 \qquad (18)$$

The reader is reminded that energies are in GeV, distances are in meters and $10^{32} cm^{-2} s^{-1} = 10^{36} m^{-2} s^{-1}$.

Choosing LEP-like parameters

$$n_b = \frac{n_{ir}}{2} = 4$$

$$\sigma_z = 3 \times 10^{-2} m$$

$$\mathscr{L} = 10^{36} m^{-2} s^{-1}$$

we find the energy restricted to

$$E \lesssim 200 \text{ GeV}$$

In order to push machines of this type to higher energies, either the bunch length or the number of bunches stored would have to be increased dramatically.

At constant luminosity E varies as $\sigma_z^{1/2}$. To increase the maximum energy by a factor of two, the bunch length, and with it presumably the rf wavelength, would have to be increased by a factor of four. The corresponding radiofrequency is about 90 MHz.

To accomplish the same result by increasing the number of bunches, which could be done either by building two separate rings or by providing many special beam-separation sections in a single ring, the number of bunches would have to be increased by a factor of $2^{8/3}$ to about 24 with collisions taking place in only eight interaction regions.

Finally, we add a remark on costs. The peak luminosity of a storage ring is given by

$$\mathscr{L} = \frac{3}{8\pi r_e^2 mc^2} \frac{P \rho_0}{\gamma^3 \beta_y^*} \qquad (19)$$

where P is the power delivered to each beam. According to Eq. (11) $\rho_o \sim \gamma^2$, so for constant luminosity, the power must vary as γ and the circumference as γ^2. The rf cavity length also varies as γ^2 because the voltage gradient is limited and the total voltage varies as γ^2. Consequently, we expect the cost of a storage ring to increase almost in proportion to the square of the energy, and this expectation is borne out by recent cost optimizations of large e^+-e^- storage-ring designs (Richter, 1976).

Colliding Linac Beams

In an electron storage ring with the customary distribution of radiation damping among the oscillation modes, the longitudinal damping time may be interpreted as the time in which each particle would radiate away all of its kinetic energy if it were to continue radiating at the same rate. It follows that the rf system re-supplies to each beam its entire kinetic energy each damping time (n_d/f_o). In LEP this damping time is 5.8 milliseconds and, following the dictates of Eq. (11), it scales as γ to higher-energy storage rings, so the damping time for a 350-GeV storage ring would be about 30 milliseconds.

Now consider two linacs aimed at one another so that their output beams collide. Suppose, for comparison, that their energies are the same as that of a given storage ring and that they accelerate, in each pulse, the same number of particles as is stored in each beam of the storage ring. Then if their pulse repetition period is equal to a damping time of the storage ring, they will have to deliver no more average power to the beams than does the rf system of the storage ring. In terms of rf power, the linacs are at no disadvantage relative to the storage ring provided the efficiency of rf power usage is equivalent. In order to produce the same luminosity, however, the lateral area of the linac bunches at the interaction point must be much smaller than it is in the storage ring. For example, if we compare the linac system with all its particles in one bunch -- that is an rf bunch, not a pulse train -- with a storage ring having n_b stored bunches in each beam, the interaciton areas must stand in the ratio

$$\frac{A_L}{A_s} = \frac{n_b}{n_d} \tag{20}$$

to obtain the same luminosity, where A_L is the interaction area of the linac bunch and A_s is the interaction area of the storage ring bunches. This relation follows from the relation

$$\mathscr{L} = \frac{f_r N^2}{A_L} = \frac{f_o N^2}{n_d A_L} = \frac{f_o N^2}{n_b A_s} \tag{21}$$

in which f_r is the repetition frequency of the linacs and $A_s = 4\pi \sigma_x^* \sigma_y^*$. What Eq. (20) tells us is that (A_L/A_s) is a small number. For orientation we may compare LEP with a pair of 70-GeV linacs. In this case $n_b = 4$ and $n_d = 70$, so $(A_L/A_s) = 0.05$.

We conclude from these comparisons that colliding linac beams may compete favorably with storage rings at the energies of interest if sufficiently high phase-space densities can be attained in them. Our next goal will be to determine the phase-space densities required in the general case, to assess how severe the beam-beam disruption will be and to estimate the effects of beam-strahlung.

We begin by restating Eq. (21) in the form

$$\mathscr{L} = \frac{f_r N^2}{4\pi \sigma_y^{*2} R}, \tag{22}$$

where $R = (\sigma_x^*/\sigma_y^*)$ the aspect ratio of the beam cross section at the interaction point. We are still considering a single linac bunch in each pulse. The average beam power of each linac is

$$P = f_r N \gamma mc^2 \tag{23}$$

Next we define a "disruption parameter",

$$D = \frac{\sigma_z}{F} = \frac{2r_e \sigma_z N}{\gamma \sigma_y^{*2} (1+R)} \tag{24}$$

This parameter may be understood through Fig. 1. If the focal length F is long compared to the bunch length, D << 1 and the trajectories of incident particles are little affected by the beam-beam interaction. If, on the other hand, F is short compared to σ_z, disruption is severe and the focal length is meaningless.

We also define the beam-strahlung energy loss δ in units of the beam energy.

$$\delta = \frac{\Delta U}{E} = \frac{2r_e}{3} \cdot \frac{\gamma^3 D^2 \sigma_y^{*2}}{\sigma_z^3} \tag{25}$$

From Eqs. (22), (23), and (24) we find immediately that

$$\mathscr{L} = \frac{DP}{8\pi\, mc^2\, r_e\, \sigma_z} \left(\frac{1+R}{R}\right) \tag{26}$$

independent of γ and independent of δ! However, we must also obey the following laws at the same time:

$$f_r = \frac{4r_e^2}{3mc^2} \frac{\gamma\, DP}{\delta\, \sigma_z^2\, (1+R)} \tag{27}$$

$$N = \frac{3}{4r_e^2} \frac{\delta\, \sigma_z^2\, (1+R)}{\gamma^2\, D} \tag{28}$$

$$\sigma_y^{*2} = \frac{3}{2r_e} \frac{\delta\, \sigma_z^3}{\gamma^3\, D^2} . \tag{29}$$

We must pick our way among these relations to find a satisfactory set of parameters. For example, having chosen D, P, σ_z, and R to give the desired luminosity, we must choose δ, probably on experimental grounds.

The smaller we choose it, the higher is the repetition frequency and the smaller are N and σ_y^*. Furthermore, the energy dependencies of the last three equations dictate that

$$f_r \sim \gamma$$

$$N \sim \gamma^{-2}$$

$$\sigma_y^{*2} \sim \gamma^{-3}$$

The beam cross section becomes very small at very high energy.

Figure 2 shows Eq. (27) and Eq. (28) plotted as functions of energy for both a flat beam and a round beam. Also plotted is the normalized emittance of the beam

$$\phi = \gamma \, \sigma_y^{*2} / \beta_y^*$$

We have assumed $\beta_y^* = \sigma_z$, approximately the optimum value it can take. The normalized emittance of a machine is independent of beam energy. For the SLAC linac the values measured as averages over many S-band bunches are $\phi^- = 6.4 \times 10^{-4}$ mc-m for electrons and $\phi^+ = 6 \times 10^{-3}$ mc-m for positrons. For single bunches the values may be smaller.

We close by putting forth the following example of colliding linac beams followed by some remarks on this technique. We choose

D	=	1	(upper limit?)
σ_z	=	5×10^{-3} m	(S-band)
R	=	1	(round beams)
δ	=	10^{-2}	
$\mathscr{L}$	=	$10^{36}\ \mathrm{m}^{-2}\ \mathrm{s}^{-1}$	$(= 10^{32}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1})$

with these choices

$$P = 14 \times 10^6 \text{ W} \quad \text{(each beam)}$$

$$f_r = 3.7 \times 10^{-3} \gamma \text{ s}^{-1}$$

$$N = 4.7 \times 10^{22} \gamma^{-2}$$

$$\sigma^{*2} = 6.6 \times 10^5 \gamma^{-3} \text{ m}^2$$

For $E = 350$ GeV, $\gamma = 6.8 \times 10^5$

$$f_r = 2500 \text{ s}^{-1}$$

$$N = 1.0 \times 10^{11}$$

$$\sigma^{*2} = 2 \times 10^{-12} \text{ m}^2 = 2 \text{ μm}^2$$

For this example the average beam current from each linac would be 40 microamperes.

These parameters appear quite challenging in the light of current linac technology. For comparison the SLAC linac can now produce an electron beam with more than 10^9 electrons in one S-band bunch. Nevertheless, suggestions have been made to get around some of the difficulties. At CERN (Amaldi, 1976) and at Novosibirsk (Balakin, Budker and Skrinski, 1978) studies have been made to regenerate positrons from the spent beams and to "cool" them in intermediate storage rings to decrease the spot size. Amaldi also suggested recovering energy from the spent beam.

The cost of a colliding-linac-beam system may be expected to increase approximately linearly with energy, because the length of the machines will increase in that way. Since, as we have remarked above, the costs of storage rings will vary approximately as the square of the energy, linac systems, should they prove feasible, may become economically quite attractive at sufficiently high energies.

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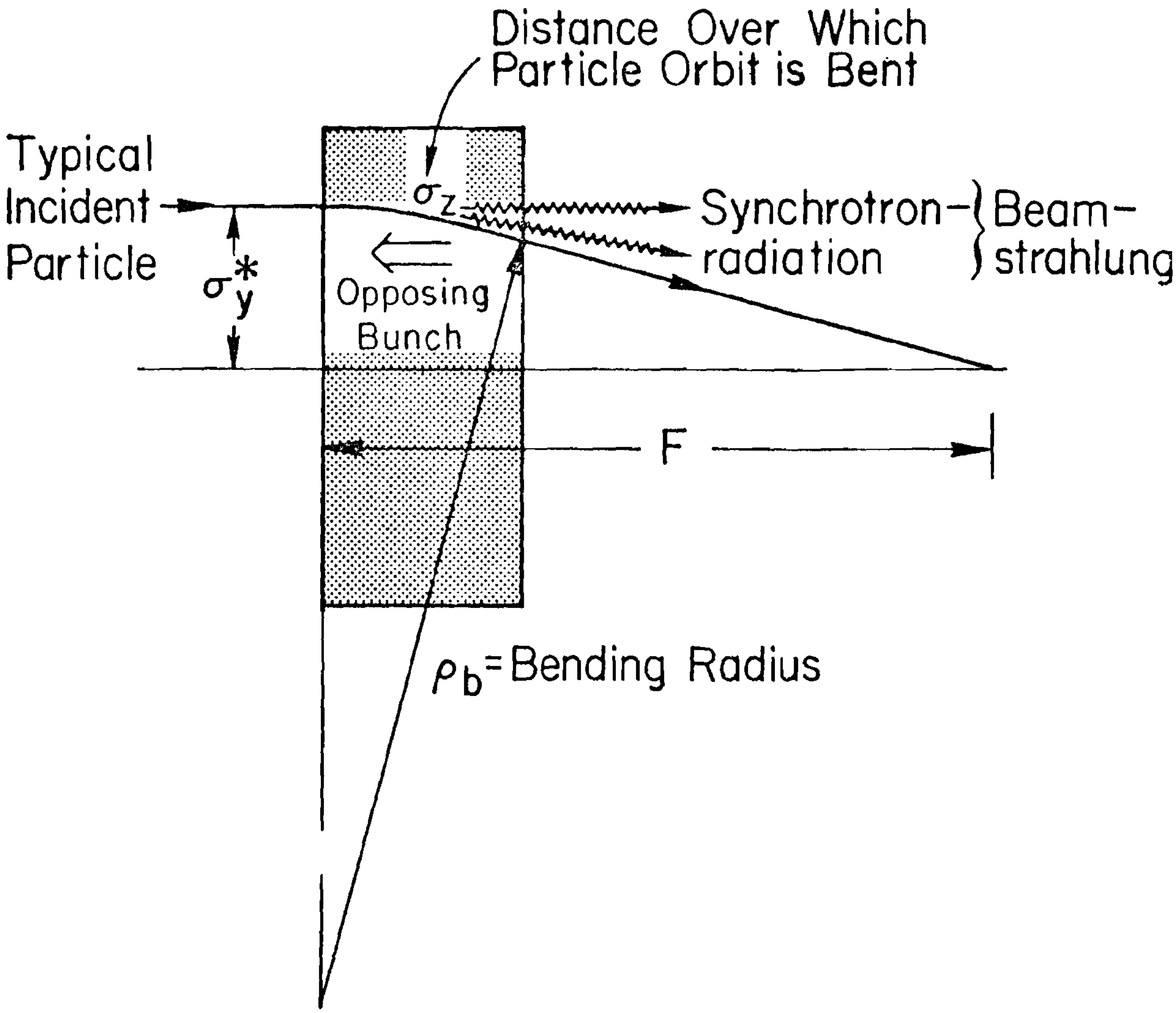

Fig. 1. Typical particle incident on an opposing bunch.

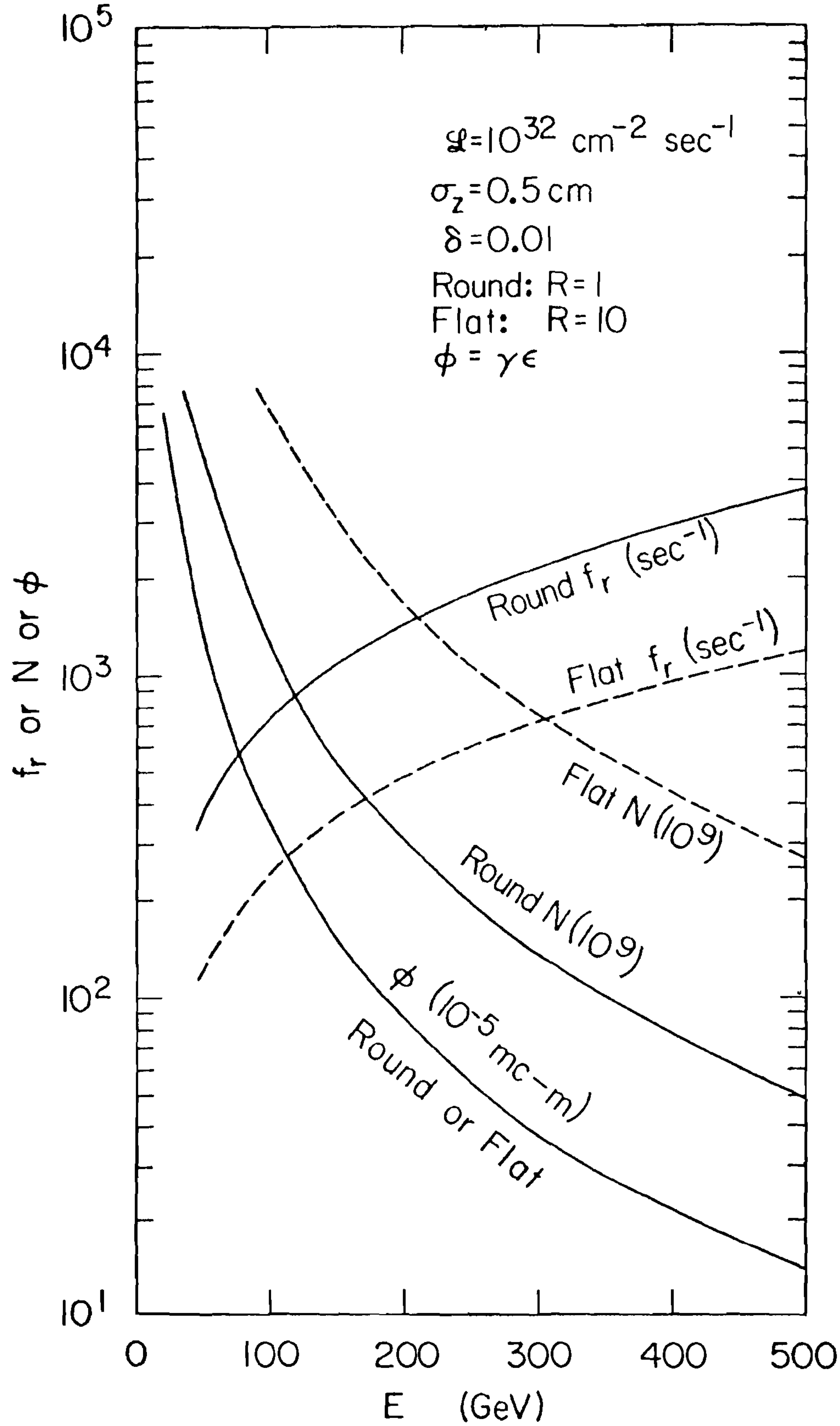

Fig. 2. Colliding linac beams. Parameters for constant luminosity.