Both proton and electron rings start with the same simple lattice, an alternation of focusing and defocusing lenses with strength $p = 1/f$ and separation $t$. The matrix from the center of a focusing lens to the center of a defocusing lens (half-cell)

$$
\begin{pmatrix}
1 - \frac{1}{2}pt & t \\
-\frac{t}{2} & 1 + \frac{1}{2}pt
\end{pmatrix}
$$

can be equated to

$$
\begin{pmatrix}
(\beta_0 \beta_1)^{\frac{1}{2}} \cos \phi & (\beta_0 \beta_1)^{\frac{1}{2}} \sin \phi \\
(\beta_0 \beta_1)^{\frac{1}{2}} \sin \phi & (\beta_0 \beta_1)^{\frac{1}{2}} \cos \phi
\end{pmatrix},
$$

where $\beta$ and $\phi$ are the usual $\alpha$-$\gamma$ functions. $\beta_0$ in this case is $\beta_{\text{max}}$.

Multiplying the principal diagonal terms one finds the basic relation $pt/2 = \sin \phi$. The choice for $\phi$ does not significantly affect $\beta_0$ and therefore we choose $\phi = 45^\circ$ ($90^\circ$ per period) which is very desirable for manipulation and corrections. With this choice:

$$
\beta_0 = (2 + \sqrt{2})t
$$

$$
\beta_1 = (2 - \sqrt{2})t.
$$

For off-momentum closed orbits one finds, with the aid of the diagram (Fig. 1), $\eta_0 = (2 + \sqrt{2})t\theta$, $\eta_1 = (2 - \sqrt{2})t\theta$, where $\theta$ is the bend angle in length $t$. Thus to determine the principal orbit parameters we need only to choose the half-cell length $t$. At this point proton and electron designs diverge.

**Proton Designs** are dominated by maximum magnetic fields and gradients. Usually the overall size of the ring is tentatively chosen by consideration of cost, site, etc. and approximately 80% of $t$ must be bending magnets to reach design energy. The required quadrupole power $p$, at maximum gradient, varies as $t/l$ and the fraction of $t$ for the quads varies as $t^2/l^2$. As a result one finds a rather firm minimum value for the cell half-length, and we choose this value because it gives directly the minimum value for $\beta_{\text{max}}$. 

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This design method appears very simple if compared to the design justifications for previous machine proposals which used (in different ways) such things as alignment, orbit distortions, acceptances etc. However, one finds, to a surprising accuracy, that \( \beta_{\text{max}} \) varies as \( \sqrt{E} \), which is just the above method with a constant fraction of \( \beta \) devoted to focusing.

For larger machines one is forced to weaker focusing—large \( \beta \)—and to a magnet design dominated by nonlinear errors. For storage rings, field quality is particularly important because in addition to field and space charge nonlinearities one must also add the strong nonlinear part of the beam-beam interaction.

The following numbers "squeeze" towards smaller \( \beta \):

- Cell length \( l = 29 \) m
- Quad power \( p = 0.0488/m \), length 3.25 m at 500 kgauss/m.
- \( \beta_{\text{max}} = 100 \) m
- Radius = 1005 m, bending \( r = 750 \) m.

For a gaussian beam width, define emittance \( \epsilon = \sigma^2 / \beta \), \( \sigma \) = std. dev.,

\[
\epsilon = 0.2 \times 10^{-8} \text{ m (single turn into booster)}
\]

Beam width \( \pm 3\sigma \) = 2.7 mm at \( \beta_{\text{max}} \),

Electron designs are dominated by synchrotron radiation considerations which included damping times, beam size, and storage times in addition to the major expense of rf power. The radiated power per turn is \( P = 0.0885 \frac{E^4}{r} \text{ MeV/turn} \) where \( E \) is the energy in GeV and \( r \) is the bending radius in meters. The usual scaling to higher energies requires that the radius increase at least as \( E^2 \) in which case the total rf power also increases as \( E^2 \) and the bending field drops with \( 1/E \). Obviously the rf power becomes the dominant cost item.

The emittance of the beam (as defined above) is given by \( \epsilon = 1.47 \times 10^{-6} \frac{E^2}{r} \) (or \( \beta_{\text{max}} \))

(provided we have not spoiled the damping). The lattice function \( f(l) \) is very close to \( 20^2 l \) where \( \theta \) is the bend angle per half-cell, or \( l/R \). The emittance increases as \( l^3 \). We can use smaller \( l \) for electrons than for protons because the quads are not gradient limited. Assuming 2 ft as a minimum quad length one finds for 20-GeV electrons,

- \( l = 12 \) m \( \beta_{\text{max}} = 44 \) m
- Quads 0.6 m at 131 kgauss/m
- Bends 9 m total at 889 gauss
- or \( r = 750 \) m for \( R = 1000 \) m

then \( \epsilon = 0.27 \times 10^{-8} \) m

for a total beam width \( (\pm 3\sigma) \) of 4 mm at \( \beta_{\text{max}} \).

One must note that the loss (by nonlinear effects) of the tails of the distribution reduces the storage time because these electrons are promptly replaced by others!

Is a Combined Proton or Electron Lattice practical? Consider two alternates: storing the second proton beam in an electron lattice and storing the electron beam in a proton lattice,
1. Raising the fields in the electron lattice one finds the quads are the limit: 50-GeV protons for ordinary and 100-GeV for superconducting quads. Not satisfactory.

2. Electrons in a proton lattice would be 24 mm wide with $\epsilon = 4 \times 10^{-8}$ m. The very good part of the magnetic fields would have to be 9 times wider to achieve electron storage than for protons! Even if we managed this without making the magnets 9 times bigger, we would be faced with an interaction region design dominated by the electron $\beta$. A reduction of $\beta$ by 40 is clearly not possible and the luminosity would suffer.

Alternatively one could reduce the electron beam size by dropping the energy to 7 GeV, in which case it would be little better than the electron target.

The conclusion is that for these energies combined lattice use does not provide a practical solution to the "third beam in the interaction region" problem.

FIG. 1