I. Introduction

This is a partial description of a colliding beam system employing the NAL Main Ring (MR) together with a small (R -100 m) superconducting proton storage ring (SR), located tangent to a MR straight section, and operating at fields between 2.2 T and 5 T. The 0.4 A MR design beam, intersecting for one meter with a 10 A beam in the storage ring yields a luminosity of 2 x 10^{31} cm^{-2}sec^{-1} at 30 GeV x 300 GeV when modest low β insertions reduce the beam sizes to 1.5 mm x 3.0 mm. The luminosity can possibly be increased by a factor of about three for periods of an hour or more by rebunching the SR beam with reasonable rf voltages.

Since the storage ring can be filled at any desired energy up to its maximum, one can employ the by now conventional momentum stacking method. On the other hand, since the beam can be bunched, one can also test the less well-understood process of accelerating high current beams in superconducting devices, a process which is important in long-range planning at NAL and elsewhere.

The physics to be done with such a device has been discussed extensively elsewhere, however it seems clear that strong interaction physics in this energy range can be well studied; that V mesons of mass less than 100 GeV would be detected if they exist; and that the luminosity and energy are probably too low for other weak interaction experiments.

II. Storage-Ring Lattice

The choice of v value is dictated by consideration of resonances driven by the superperiod structure. A very good
choice seems to be between 8.5 and 9 (in common with the AGS and ISR). Between these values, none of the nonlinear resonances through fifth order are driven by threefold periodicity. The resonance \( v = 9 \) is driven as a third, fourth and fifth order resonance. However, if the superperiod and insertions are symmetric, then the third and fifth order structure terms are zero (since \( \frac{27}{3} \) and \( \frac{45}{3} \) are odd) and \( v = 9 \) is driven in the fourth order, necessitating compensating octupoles symmetrically disposed in the superperiod. Then all other nonlinear resonances except \( 4v = 36 \) are driven only by imperfections and threefold periodicity seems safe. For twofold periodicity, \( 4v = 36 \) and \( 5v = 44 \) are driven by symmetric superperiod while \( 4v = 34, 3v = 26 \) and \( 2v = 18 \) are driven by asymmetric superperiods. For sixfold periodicity \( 4v = 36 \) is again driven by symmetric period; no other resonances are driven. (Note: when we discuss a resonance \( n\nu = N \), we include all resonances in the form \( k_1\nu_x + k_2\nu_y = N \) where \(|k_1| + |k_2| = n\). Difference resonances, where either \( k_1 \) or \( k_2 \) is negative are generally not harmful unless there is a great disparity between the \( x \) and \( y \) emittances.)

We do not yet know the value of the phase shifts in the long-straight sections, but it is unlikely to be more than one or two \( \pi \) units. Then we would like to have about \( 7.5 \times 2\pi \) phase shift in a number of normal cells which is divisible by three. Since \( \pi/2 \) phase shift per cell is about optimum for beam measurement and correction the choice of 30 cells is obvious. If each long-straight section (LSM) is 56 meters, with an orbit length of \( 2\pi \times 100 \text{ m} \) and three LSM the length of each cell is then \( L = 15.94 \text{ m} \). Let us reserve 12 m for bending magnets and assume 80% or 9.6 m to be effective in which case \( \rho = \frac{9.6}{15.94} \times 76.13 \text{ m} = 45.85 \text{ m} \) (76.13 = 100 m - \( \frac{150 \text{ m}}{2\pi} \) is the average radius of the curved sections). If we use straight magnets and require the sagitta to be of the order of 5 mm, then the length of each magnet should be smaller than \( \sqrt{8 \times 57.3 \times 5 \times 10^{-3}} \text{ m} \approx 1.5 \text{ m} \). Then there should be eight magnets.
per period each 1.2 m (= 1 \text{ m}) long with a field of \( B = \frac{(B_p)}{\rho} = 102 \text{Tm/45.85 m} = 2.22 \text{T} \) at 30 GeV.

For a symmetrical FODO lattice (this can be refined later)

\[
\cos \mu = 1 - \frac{1}{8} \left( L \frac{B'_1 \epsilon_1}{B_p} \right)^2.
\]

If \( \mu = \pi/2 \) and \( B' = 31.7 \text{T/m} \), corresponding to 2.22 T at 70 mm coil radius, then \( \epsilon_1 = 0.57 \text{ m} \). To obtain this effective length a physical length of 1 m is needed. Thus 14 m is used for bends and quads, the rest for needed beam monitors, trim magnets, etc. The magnetic field can be raised to above 5 T yielding a maximum energy of about 70 GeV in the SR.

The experimental insertions are the subject of SS-73/234 and SS-73/258. Solutions appear to be in hand which allow continuous collinear crossing over a wide range of momenta during MR ramp. Although in an elementary stage, the design of straight-section insertions already allows one half of the 50 m space free of quadrupoles, while achieving appropriate SR \( \beta \)-functions in the intersection regions. The more modest reduction in the MR \( \beta \)-functions poses no serious design problems.

The experience at the ISR and measurements at Culham by McCracken of ion desorption from cold surfaces would seem to dictate a warm vacuum surface, baked at elevated temperature (preferably above 400°C) and cleaned by ion bombardment. Estimates by H. Halama and J. Bittner at BNL as well as by the ISR Group, would lead to the conclusion that the chamber aperture should be about 50 - 80 mm diameter for pumps at each 1.5 m length to conduct 10 A of beam. A careful reestimate should be made for the case in hand, for if 50 mm is sufficient, the magnets might well be simple extensions of the 1 m models recently tested by the BNL superconductivity group.
III. Transfer and Stacking

The most straightforward method of filling the small ring is to inject several booster batches into the MR, accelerate to the stacking energy, extract and inject single turn into the SR with the beam still bunched, and to decelerate the beam into a stack. The performance of the system will depend primarily upon (1) the phase space density of the beam and (2) the care with which the above processes are carried out. Here we will attempt to evaluate the performance on the basis of our best estimate of what the phase space densities will be, and for an operating energy of 30 GeV against 300 GeV.

At present the linac emittance is about $10^{-3}$ m, to get 0.4 A in the MR requires four turns from the linac in the horizontal plane, one in the vertical. At the present, single turn operation in the booster shows a factor of 2 increase in transverse phase space area at 8 GeV in the MR. To account for the dilution, we should then take $80 \times 10^{-4}$ m in the horizontal and $20 \times 10^{-4}$ m in the vertical for the 200-MeV emittances which we scale to higher energies. These numbers are in fair agreement with the booster operation. Present measurements of debunching and matching at 8 GeV in the MR indicate a longitudinal phase space area of .03 eV-sec per bunch. This is somewhat less than the design minimum booster bucket area of .04 eV-sec, while it is more than the linac phase space corresponding to the normal $\frac{\Delta \rho}{\rho} = \pm 0.2\%$. The difference can be ascribed to space charge effect in the 200-MeV transport line and to problems in the booster. A debuncher will probably be required to correct this problem, but for our purposes it would be safest to take .04 eV-sec as the longitudinal phase space area of one bunch.

The mechanics of extraction from the MR and injection into the SR, transverse matching, etc. are essentially no different from those for the ISR. A closed kicker which opens to let the beam move radially after injection would appear to be very similar to that in use in the ISR.
Longitudinal matching, however, will take more care. Let us define $W$, $\phi$ where $W = \frac{AE}{2\pi \text{rf}}$ and $\phi$ is the phase of a particle with respect to the rf wave. Then for a matched ellipse in the $W-\phi$ space define

$$Z = \frac{W_{\text{max}}}{\phi_{\text{max}}} = \frac{1}{(2\pi)^2} \sqrt{\frac{V_\text{r}(\cos \phi_\text{s})E c}{f_\text{rf}^2 R \eta}}$$

where $V_\text{T}$ is total volts/turn

$$\phi_\text{s} = \text{synchronous phase angle} = \sin^{-1}\left(\frac{\text{demand energy gain/turn}}{\text{rf volts/turn}}\right)$$

$E$ = total energy

$f_\text{rf}$ = rf frequency/8

$R$ = radius

$\eta = \frac{1}{\gamma} - \frac{1}{\gamma_E}$

$\gamma_E = \text{transition energy} = \frac{\text{mc}^2}{E}$

Also $\nu_\text{s}$, the phase oscillation wave number (# oscillations per turn) is given by

$$\nu_\text{s} = \frac{1}{2} \sqrt{\frac{V_\text{r}(\cos \phi_\text{s})\eta R f_\text{rf}}{cE}}$$

For completeness, in a bucket the maximum stable value of $W$ is $W_M = 2\pi$, and the area of a bucket is $A = 16\pi \alpha_3 (\sin \phi_\text{s})$ where $Z_0$ is the value of $Z$ when $\sin \phi_\text{s} = 0$, and $\alpha_3$ is a function which is 1 when $\sin \phi_\text{s} = 0$, is 0 when $\sin \phi_\text{s} = \pm 1$, and can be looked up in tables.*

*See MURA 105 or CERN/MPS-SI/Int DL/70/4
In the case at hand, the product \( R_n \) is almost the same in the MR and the SR at 30 GeV, so proper matching requires similar rf voltages in the two rings at transfer. In order to avoid duplicating the 1 MV MR rf system in the SR it is advisable to operate the MR onto a 30-GeV flattop and adiabatically reduce the MR voltage. A further step would be to reduce the MR ramp to 30 GeV/sec so that less (1.2 MV) rf voltage is required to accelerate (the advisability of this step is in question). Let us suppose that one MR type cavity is available in the SR, so that \( V_{SR} = 200 \text{ kV} \). Then at transfer \( V_{MR} = V_{SR} \frac{(R_n)_{MR}}{(R_n)_{SR}} = 172 \text{ kV} \).

The time for this process is determined by following the adiabatic law:

\[
V = \frac{V_0}{(1 + \xi f_{so} t)^2}
\]

where

- \( V_0 \) = initial voltage
- \( f_{so} \) = initial phase oscillation frequency = \( v_{so} \)/period of revolution
- \( \xi \) is a number determined by experience \( \leq 4^* \).

Here \( f_{so} = 130 \text{ Hz} \) so \( t \geq 3.5 \text{ msec} \).

In the storage ring the beam must be decelerated into the stack with the bucket full, so as not to dilute the stacked beam. Let us assume that we take one second for this process, and that the stack is 2 cm average radius from the injection radius. Then the energy change is

\[
\Delta E = Apc = \frac{Ax}{x_p}pc = \frac{1.62}{3.3} \frac{m}{E} \times 31 \text{ GeV} = .475 \text{ GeV}
\]

\( (x_p = \frac{R}{v^2} = 1.3 \text{ m}) \quad (T = \frac{2\pi x 100}{3x 10^3} \text{ usec} = 2.09 \text{ usec}) \)

and the required voltage per turn is

\[
V \sin \phi_s = .475 \frac{\text{GV}}{\text{sec}} \times 2.09 \times 10^{-6} \frac{\text{sec}}{\text{turn}} = 992 \text{ V/turn}.
\]

The required bucket area is .04 eV-sec.

\[
A = 16a_3 \frac{(\sin \phi_s)^2}{(2\pi)^2} \sqrt{\frac{E_{n} cY}{\frac{3}{2} R_n}} = .04 \text{ eV-sec}.
\]

This has the solution \( \sin \phi_s = .540 \) and \( a_3 = .302 \), so that the

*K.R. Symon - MURA Report*
required voltage during stacking is \( V = 1837 \text{ V/turn} \). Then the SR voltage must be reduced from the capture voltage to about 2 kV. The minimum time to do this is given by

\[
f_{50} = 549 \text{ Hz} \quad \text{and} \quad t_{\text{min}} = \frac{1}{f_{50}} \left[ \sqrt{\frac{200}{1.837}} - 1 \right] = 4.3 \text{ msec.}
\]

Then the SR voltage can be leisurely reduced to 1800 V/turn, arriving at this value somewhat prior to entering the stack. The actual choice of transfer voltage is dictated by hardware considerations. Clearly the SR problem is easier if the voltage range is smaller. This puts the burden on the MR, where lower voltage may be obtained by shifting the relative phases of cavities. On the other hand, if one wants to rebunch the SR stacked beam to improve luminosity, at least several MR type cavities are required.

Now each injection into the SR will have the following properties. The betatron width at the "average" lattice point \((8 - \frac{R}{v} = 11.4 \text{ m}, x_p - \frac{R}{v^2} = 1.3 \text{ m})\) will be \((r, \gamma_{80} \approx 2 \times 10^{-6} \text{ m})\)

\[
w = 2x = 2 \sqrt{\frac{8}{r}} = 2 \sqrt{11.4 \times 1.6 \times 10^{-6}} = 8.75 \text{ mm}
\]

\[
h = 2y = 4.37 \text{ mm}
\]

while the momentum width of a single stack (unbunched) corresponds to

\[
W_{\text{max}} = \frac{\Delta p}{p} |_{\text{debunched}} = \frac{x_p f_{50}}{p c} = (1.3 \text{ m}) (1.2 \times 10^7 \text{ keV})(0.04 \text{ eV-sec}) = 31 \times 10^9 \text{ eV}
\]

\[= .085 \text{ mm.}
\]

To reach 10 A, we should use 25 MR stacks so the momentum width is 2.1 mm, at most. Stacking efficiencies of 80-90% are typical so that the actual momentum width should not be more than 3 mm.

IV. Luminosities, Etc.

For collinear interaction regions the luminosity is given by

\[
L = (2.6 \times 10^{31} / \text{cm}^2 \text{ sec}) \frac{I_1 I_2 \lambda}{\hbar n}
\]
where $I_1, I_2$ are the beam currents in Amp, $l$ is the interaction length in meters, $w$ and $h$ are the width and height of the larger beam in mm.

Let us suppose that at $30 \times 300$ the interaction region has $w = 3$ mm, $h = 1.5$ mm, $l = 1$ m, $I_1 = 0.2$ A, $I_2 = 10$ A, then $L = 2.3 \times 10^{11}$ cm$^2$ sec. This requires the MR $\beta$-function to be reduced from its normal 70 m to 13 m in both planes and the SR beta functions to be reduced to 1.34 m in each plane, using our "design" emittances. Further it will be useful to set $x_p = 0$ in the SR interaction region. Suppose we wish to rebunch the SR beam to improve luminosity. Let $\Delta \phi_m = \frac{\pi}{3}$ so that the bunches are 2 m long out of the 6 m wavelength. Now $A = 25 \times 0.04 = 1$ eV-sec and $Z = \frac{A}{\pi \sin^2} = 0.290$ eV-sec. Then

$$V = Z^2 (2\pi) \hbar \frac{\rho_n H_n}{E_{\text{c}}} = 2.25 \text{ MV}.$$ 

This is somewhat formidable. One could partially bunch and then turn on a higher frequency cavity, say 102 MHz where shunt impedances are higher, and bunching is stronger.