MAGIC -- A THIN LENS AND BENDS MATCHING PROGRAM

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The design of the intersection region for the NAL main ring and proton storage ring (30-70 GeV) has been carried out with the aid of a computer code "MAGIC" specially written for magnet lattice insertion using thin-lens magnets. This computer code enables the user to fit the values of the \( \beta, \alpha, \eta, \eta' \) and \( \psi \)-functions at both ends of an insertion and the values of the transport matrix elements. The fitting is done by using VMM \(^2\), a least square fitting code from ANL. Since MAGIC is a special purpose code and since it uses thin lens magnets, the computation time and the rate of convergence are faster than those of a general purpose code such as TRANSPORT \(^3\).

The method of computation and its application in designing the insertions for the intersection region are presented in this report.

Method of Computation

Consider an insertion composed of a number of quadrupole magnets, bending magnets and drift lengths. The transport matrix for either horizontal or vertical motion for the insertion is given by

\[
T = M_n M_{n-1} \cdots M_2 M_1
\]

(1)

where \( M_k \) is the transport matrix for the \( k^{th} \) element in the insertion and \( n \) is the total number of elements.

For a focusing quadrupole magnet

\[
M_k = \begin{pmatrix}
1 & 0 & 0 \\
-x_k & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

(2)

where \( x_k = 1/\text{focal length} \); for a bending magnet

\[
M_k = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & x_k \\
0 & 0 & 1
\end{pmatrix}
\]

(3)

where \( x_k = \text{bend angle} \); for a drift space

\[
M_k = \begin{pmatrix}
1 & x_k & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

(4)

where \( x_k = \text{drift length} \).
The values of the \( \beta, \sigma, \eta, \eta', \) and \( \psi \) -function at the exit of the insertion are given by:

\[
\begin{pmatrix}
\beta_2 \\
\sigma_2 \\
\gamma_2 \\
1
\end{pmatrix} =
\begin{pmatrix}
T_{11}^2 & -2T_{11}T_{12} & T_{12}^2 \\
-T_{11}T_{21} & T_{12} & -2T_{12}T_{21} \\
T_{21}^2 & -2T_{21}T_{22} & T_{22}^2 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
\beta_1 \\
\sigma_1 \\
\gamma_1 \\
1
\end{pmatrix}
\] (5)

\[
\begin{pmatrix}
\eta_2 \\
\eta'_2 \\
1
\end{pmatrix} =
\begin{pmatrix}
T_{11} & T_{12} & T_{13} \\
T_{21} & T_{22} & T_{23} \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
\eta_1 \\
\eta'_1 \\
1
\end{pmatrix}
\] (6)

where

\[
\psi_2 = \tan^{-1} \left( \frac{T_{12}}{\beta_1 T_{11} - \sigma_1 T_{12}} \right) - \psi_1
\] (7)

\[
\gamma = \left( 1 + \frac{\sigma_2}{\beta} \right)^{1/2}
\] (8)

and the values of these functions at the entrance (with subscript 1) equal to the values desired.

Let \( f_i \) be the value of the \( i \)-th function and \( \bar{f}_i \) be the corresponding desired value with \( i = 1, 3, \ldots, 21 \) refer to \( \beta_2, \sigma_2, \eta_2, \eta'_2, \psi_2, T_{11}, T_{12}, T_{13}, T_{21}, T_{22}, T_{23} \) for the \( x \)-motion, and \( i = 2, 4, \ldots, 22 \) refer to these functions for the \( y \)-motion.

Consider the function

\[
F = \sum (f_i - \bar{f}_i)^2
\] (9)

with summation over the indexes of those functions whose values are to be fitted. The fitting is done via a least square minimization of \( F \). In particular a solution is obtained if the minimum value of \( F \) is zero, i.e., \( f_i = \bar{f}_i \) for all the desired functions. If the minimum value of \( F \) is not zero we fail to find a solution.

For this minimization procedure we need to compute the derivatives of \( F \) with respect to each parameter

\[
\frac{\partial F}{\partial x_k} = \sum 2 (f_i - \bar{f}_i) \frac{\partial f_i}{\partial x_k}
\] (10)
From the differentiation of Eqs. (5) to (7) we can express \( \frac{\partial f}{\partial x} \) in terms of \( T_{ij} \) and \((D_k T)_{ij}\), where
\[
D_k T = M_n M_{n-1} \cdots (DM_k) \cdots M_1. \tag{11}
\]

\( DM_k \) is the derivative of the \( M_k \) matrix with respect to \( x_k \). It may be noted that the \( DM_k \) matrix has only one nonzero element. In addition, we need a guess solution \( (x_{ko})'\)'s. Constraints may be imposed upon the values of system parameters by the conditions
\[
\sum d_k x_k = \sum c_k x_{ko}, \tag{12}
\]

where \( c_k \)'s are constants whose values may be specified by the user. For example, \( c_k = \delta (k - \beta) \) keeps \( x_p = x_p^0 \); \( c_k = 1 \) for all drift spaces and \( c_k = 0 \) for the other elements keeps the length of the insertion fixed.

**Insertions for the Intersection Region**

The low-\( \beta \) insertion for the main ring utilizing the available free space of 50 m in the long-straight section is shown in Fig. 1. The insertion, which is anti-symmetric in its focusing actions with respect to the midpoint of the insertion region, gives the midpoint \( \beta \), or \( \beta^\circ \), a value of 7 m. The corresponding tune shift \( \Delta \nu \), which is the same for both planes in the case of an anti-symmetric insertion, is 0.32. The insertion quadrupoles, whose focusing strengths are defined as

\[
k = \frac{(\text{field gradient}) \times (\text{length})}{(\beta_p \text{ of particle})},
\]

are physically resonable with conventional magnets. The resulting momentum dispersion function in the region is also shown in Fig. 1. The midpoint \( \eta \), or \( \eta^\circ \), is now 0.64 m. Since it is desirable to match the beam sizes at the interaction point, the value of the \( \beta^\circ \) for the proton storage ring is to be determined by the beam energies involved. Figure 2 shows an insertion for \( \beta^\circ = 1 \) m corresponding to an interaction of, say, 30 GeV \( \times \) 210 GeV. The insertion, which is also anti-symmetric, has a length of 56 m (3-1/2 cells) and quadrupoles used are physically resonable with superconducting magnets.

The vertical crossing insertion is accomplished using a pair of septum magnets together with a series of superconducting dipoles. The insertion has a crossing angle of 6.67 mrad and a clear space of 15 m for the experimental apparatus. The detailed arrangements for the interaction region are shown in Fig. 3 and the relevant parameters are given in Table 1. The total insertion length in this case for the proton storage ring is 88 m (5-1/2 cells). Additional space is needed if low (or zero) dispersion at the interaction point is desired. Since any colliding beam experiment using the main ring will be run parasitically, the insertions described here fulfill the basic requirement that they constitute a minimum interference with the normal main ring operations.
### Table 1. Parameters for the Insertion Magnets

#### Main Ring

**Conventional Quadrupoles:** $B' = 140 \text{ kG/m at 210 GeV}$  
Vertical Dimension = 22 cm

<table>
<thead>
<tr>
<th>Length (m)</th>
<th>Distance from Interaction Point (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>QM1 1.92</td>
<td>13</td>
</tr>
<tr>
<td>QM2 6.24</td>
<td>17.35</td>
</tr>
<tr>
<td>QM3 1.80</td>
<td>22.75</td>
</tr>
</tbody>
</table>

#### Storage Ring

**Superconducting Quadrupoles:** $B' = 354 \text{ kG/m at 30 GeV}$  
Vertical Dimension = 30 cm

<table>
<thead>
<tr>
<th>Length (m)</th>
<th>Distance from Interaction Point (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1 0.80</td>
<td>17</td>
</tr>
<tr>
<td>Q2 0.82</td>
<td>19.17</td>
</tr>
<tr>
<td>Q3 1.24</td>
<td>28</td>
</tr>
<tr>
<td>Q5 0.5</td>
<td>36 and 44 (normal quad)</td>
</tr>
</tbody>
</table>

**Septum Magnet:**  
$B = 4.94 \text{ kG at 30 GeV}$  
Vertical Dimension = 22 cm

<table>
<thead>
<tr>
<th>Length (m)</th>
<th>Distance from Interaction Point (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1 4.4</td>
<td>9.7</td>
</tr>
</tbody>
</table>

**Superconducting Dipoles:**  
$B = 17.45 \text{ kG at 30 GeV}$  
Vertical Dimension = 30 cm

<table>
<thead>
<tr>
<th>Length (m)</th>
<th>Distance from Interaction Point (m)</th>
</tr>
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<tbody>
<tr>
<td>H1 2</td>
<td>22</td>
</tr>
<tr>
<td>V2 1.63</td>
<td>25.03</td>
</tr>
<tr>
<td>V3 3.58</td>
<td>32</td>
</tr>
<tr>
<td>V4 3.58</td>
<td>40</td>
</tr>
</tbody>
</table>
References


LOW-β INSERTION FOR THE MAIN RING

FIG. 1
LOW-\(\beta\) INSERTION FOR THE PROTON STORAGE RING

FIG. 2
ELEVATION VIEW OF THE CROSSING INSERTION

FIG. 3

θ₁ = 6.67 MRAD
θ₂ = 28.4 MRAD
θ₃ = 62.5 MRAD
θ₄ = 35.0 MRAD