We have considered some features of neutrino physics utilizing neutrinos produced by 1000 GeV protons. We feel that this is important and exciting physics which, among others, makes it very desirable to continue raising the energy of the NAL accelerator, up to a 1000 GeV or possibly even higher. Useful intensities for a 1000 GeV accelerator would be of the order of $10^{12}$ to $10^{13}$ protons per second on the average, such that experiments using a total of $10^{19}$ protons would be feasible.

1. The Neutrino Spectrum

The neutrino spectrum produced by 1000 GeV protons has been calculated by P. Nezrick\textsuperscript{1} for a wideband horn focused beam. An 800 meter decay path and a 600 meter shield length has been assumed, according to the 1973 NAL Summer Study design for a 1000 GeV neutrino beam.\textsuperscript{2} This particular calculation was performed using the CKP particle production model. Since the Hagedorn-Ranft model seems to agree better with the particle production data from the CERN ISR, the spectrum has been extrapolated to the meson production predicted for 1000 GeV protons by the Hagedorn-Ranft model. This extrapolation was done somewhat crudely without the use of a computer and therefore has introduced some uncertainty into the spectrum, especially at the high end of the energy range. However, it should be good enough for our present purpose.

The 1000-GeV spectrum thus obtained is shown in Fig. 1. For comparison, the spectra for 200-GeV and 500-GeV protons, calculated for the existing beam with 400 m decay path and a 1000 m shield, are also shown on the figure. It is apparent that a whole new energy region with neutrino
energies from 300 to 600 GeV would be opened up by the use of 1000 GeV protons.

2. Total Event Rate

We have calculated the total number of neutrino events using the \( \nu \) spectrum of Fig. 1 and a total cross section of \( 0.8 \times 10^{-38} \text{ E} \text{ cm}^2 / \text{nucleon} \). The numbers have been normalized for \( 10^{19} \) protons on the neutrino target and a 20 ton detector (the 15 ft bubble chamber filled with liquid neon, for example). The total number of events produced as a function of the \( \nu \) energy under these conditions is shown in Fig. 2. There are \( \sim 5000 \) events per 10 GeV bin near 600 GeV. Thus even in a fairly modest size detector (20 tons) appreciable numbers of events are produced even at the highest \( \nu \) energies.

3. Deep Inelastic \( \gamma \) Scattering

The study of inelastic \( \gamma \) scattering will most likely continue to be of great interest up to the highest energies available. We have assumed scale invariance, i.e. a linearly rising total cross section, to estimate the yield of events as a function of \( E_\nu \) (Fig. 2). To estimate the numbers of events at the highest \( q^2 \), we use the values of the structure functions obtained in the recent Gargamelle experiment at CERN.\(^3\)

The inelastic process

\[ \nu_\mu + N \rightarrow \mu^- + \text{hadrons} \quad (1) \]

can be described by three independent variables \( E_\nu \), \( q^2 \), and \( \nu \), where \( q^2 = (P_\nu - P_\mu)^2 \) and \( \nu = E_\nu - E_\mu \).

In the region where \( q^2 \) and \( \nu \) are both large compared to the nucleon mass, the differential cross section can be written, in the notation of Bjorken and Paschos.\(^4\) as

\[ \frac{d^2\sigma}{dq^2 d\nu} = \frac{\alpha^2}{2\pi E_\nu} W_2(q^2, \nu)[1 + \frac{\nu}{E_\nu} L(q^2, \nu) - \frac{\nu}{E_\nu} R(q^2, \nu)] \quad (2) \]

The three structure functions, \( W_2 \), \( L \), and \( R \), are in general functions of \( q^2 \) and \( \nu \). They can be expressed in
terms of $\sigma_R$, $\sigma_L$, and $\sigma_S$, the cross sections for right-handed, left-handed, and scalar currents (or virtual intermediate bosons) as

$$\nu W_2(q^2, \nu) = \frac{1}{2\pi} \frac{q^2}{1+q^2/\nu} (1- \frac{q^2}{2m\nu})(\sigma_R + \sigma_L + 2\sigma_S)$$

$$R(q^2, \nu) = \sigma_R / (\sigma_R + \sigma_L + 2\sigma_S)$$

$$L(q^2, \nu) = \sigma_L / (\sigma_R + \sigma_L + 2\sigma_S)$$.

In other notations, the sum $(L+R)$ is proportional to $W_1$ or $F_2$, and the difference $(L-R)$ is proportional to $W_3$ or $F_3$.

It is convenient to introduce the dimensionless variables

$$x = \frac{q^2}{2m\nu}$$

$$y = \frac{\nu}{E}$$.

In terms of these variables, the hypothesis of scale invariance can be stated simply that the structure functions $\nu W_2$, $R$, and $L$ are functions of $x$ only. The differential cross section can then be written as

$$\frac{d^2\sigma}{dx dy} = \frac{G^2 mE}{\pi} \nu W_2[(1-y) + yL - y(1-y)R]$$.

From the recent CERN results on the anti-neutrino to neutrino cross section ratio, $\sigma_{\bar{\nu}}/\sigma_\nu = 0.377 \pm 0.023$ and the $x$ and $y$ distributions, the structure functions can be estimated to be

$$\nu W_2(x) = (1.1 \pm 0.2) (1-x^2)^3$$

$$R \approx 0.05$$

$$L \approx 0.95$$

Assuming that these structure functions are roughly right at high energies, we can estimate the $q^2$ and $\nu$ distributions of the $\nu$ events. For example, taking a band of neutrino energies $E_\nu = 600 \text{ GeV} \pm 10\%$, we obtain the distribution shown in Fig. 3, for the experimental conditions discussed in Sec. 2. We see that there are meaningful numbers of events up to the kinematic limits in $q^2$ and $\nu$. If we integrate over $E_\nu$, i.e. take all events, we obtain
the \( q^2 \) distribution shown in Fig. 4. There are over 100 events per bin of 10 GeV\(^2\) near a \( q^2 \) of 600 GeV\(^2\); thus detailed studies of the structure functions can be done up to such \( q^2 \) values.

4. Search for the Intermediate Boson W

a. The most satisfactory way to search for the \( W \) is to produce it directly via the reaction

\[ \nu_\mu + N \rightarrow N + \mu^- + W^+ \]  

(4)

and observe its decay into \( \mu^+ + \nu_\mu, e^+ + \nu_e \), or hadrons. The cross section for the \( W \) production process (4) as a function of the neutrino energy and the \( W \) mass has been recalculated many times since the original calculation by Lee and Yang and is not subject to serious controversy. Folding these cross sections into the spectrum of Fig. 1, and integrating over the neutrino energy give the \( W \) production rates as a function of the \( W \) mass shown in Fig. 5. These rates have been normalized for the experimental conditions discussed in Sec. 2. Assuming that the detector is sensitive to most of the \( W \) decay modes (as a neon-filled bubble chamber would be, for example), we see that we expect about 50 events up to \( m_W = 24 \) GeV. This is a substantial extension of the range of \( W \) masses that can be reached by this process.

b. Indirect \( W \) search. The existence of the intermediate boson could manifest itself indirectly because of its effect on the cross section for inelastic neutrino scattering process (1). In the differential cross section for this process, Eqs. (2) and (3), the coupling constant \( g^2 \) would be modified to

\[ g^2 = \frac{2(g^2/m_W^2)}{(1+q^2/m_W^2)^2} \]

(5)

where \( g \) is the \( W-Z-\nu \) coupling constant and \( m_W \) is the \( W \) mass. For example, if scale invariance holds, the total cross section rises linearly with \( E_\nu \), as can be seen by integrating
the differential cross section of Eq. (3) over $x$ and $y$. Such a linear rise would be damped by the $W$ propagator of Eq. (5). This effect is quantitatively displayed in Fig. 6 for various $W$ masses. The two error bars shown on the linearly rising ($m_W = \infty$) curve represent the statistical error expected if the number of events shown in Fig. 2 were binned in 50 GeV bins. The effect of a 37 GeV $W$ is not so striking, especially if we keep in mind that even if scale invariance holds the slope of the linearly rising cross section is not known a priori.

However, the procedure discussed above of looking at the total cross section as a function of $E_\nu$ (Fig. 6) is not the most sensitive way to do such an analysis since the effect of the $W$, Eq. (5), is a function of $q^2$ and not of $E_\nu$. At any given $E_\nu$, the average $q^2$ is quite small. From the definitions $x = q^2/2m_W$ and $y = \nu/E_\nu$, we see that

$$q^2 = 2m_x y E_\nu$$  \hspace{1cm} (5)

To calculate the curves on Fig. 6, we used the CERN data to estimate that

$$x_{ave} \sim 1/4$$
$$y_{ave} \sim 1/2$$

and therefore

$$q^2_{ave} \sim 1/4 E_\nu.$$  

The analysis is much more sensitive if it is done as a function of $x$ and $y$. To illustrate the sensitivity, consider the analysis for one particular region of $x$ and $y$. Select events with $x = 0.9 \pm 0.1$ and $y = 0.9 \pm 0.1$. This region should contain $\sim 90,000$ events for the experimental conditions discussed in Sec. 2. The distribution of these events in $E_\nu$ is shown in Fig. 7. The average $q^2$ for these events is

$$q^2 = 2m_x y E_\nu \approx 1.6 E_\nu.$$  

The effect of the $W$ propagator for various masses is shown in Fig. 8. The error bars represent the expected
statistical errors for a run as discussed in Sec. 2. To make the sensitivity to the W more apparent to the eye, we replot the cross section as a function of $E^2_v$.

Since

$$\frac{d\sigma}{d(E^2_v)} \approx \frac{1}{2E_v} \frac{d\sigma}{dE_v}$$

a linearly rising cross section, $d\sigma/dE_v \propto E_v$ becomes $d\sigma/dE^2_v = \text{constant}$, or a horizontal line as shown on Fig. 9. The curves for $m_W = 37$ and 74 GeV are also shown with their expected statistical errors. The intercepts of the cross section curve with the $E^2_v = 20 \times 10^4$ GeV$^2$ line is indicated for W masses from 20 to 90 GeV on Fig. 9. If the only errors were statistical, the effects of a W even above 74 GeV would be clearly distinguishable. One would actually do a simultaneous fit to all of the x-y regions, which should be even more sensitive and would provide some consistency checks.

The limiting considerations are going to be the systematic errors, especially those due to the uncertainty in the shape of the $\nu$ energy spectrum. However, there is a possibility that the shape of the spectrum can be learned internally from the data. If scale invariance holds, then the cross section for events with small $x$ and $y$ should be linear in $E_\nu$ independently of the existence of a W, and can be used to check the shape of the spectrum. (For example, at $x = y = 0.05$, where there are many events, $q^2 = 2m_\nu E_\nu = 0.005 E_\nu$, and therefore the effect of the W propagator is negligible.)

5. Purely Leptonic Processes

The detection and detailed study of purely leptonic processes is extremely important. Some typical processes of interest are

$$\nu_\mu + e^- \to \mu^- + \nu_e$$  \hspace{1cm} \text{(7)}
$$\nu_e + e^- \to \nu_e + e^-$$  \hspace{1cm} \text{(8)}
Up to the present time, these processes have not been studied because of their extremely low cross sections. However, their cross sections are expected to rise with the incident neutrino energy. For this reason, very high energy neutrinos will be necessary. With 1000 GeV incident protons, substantial numbers of these events should be obtainable.

The estimates for the numbers of these events under the experimental conditions discussed in Sec. 2 are listed in Table I. For processes (7) and (8), we have used the cross section predicted by the V-A theory, \( \sigma = 1.5 \times 10^{-41} \frac{E}{\text{cm}^2/\text{electron}} \). We also assumed the \( \nu_e \) flux to be 1/200 of the \( \nu_\mu \) flux. Reaction (9) can proceed only if neutral currents exist. We used the Weinberg model to calculate the cross section, which varies from \( 10^{-42} \frac{E}{\text{cm}^2} \) to \( 10^{-41} \frac{E}{\text{cm}^2} \), depending on the value of the mixing angle. The cross section for reactions (10) and (11) have been taken from theoretical calculations based on the V-A theory and \( \mu-e \) universality. The numbers of events of this reaction as a function of \( E \) are shown in Fig. 10.

REFERENCES

1. F.A. Nezrick, private communication.
### Table I

Cross Sections and Event Rates

**1000 GeV Protons, $10^{19}$ Protons on Target, 20 ton Detector**

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Cross Section</th>
<th>Total Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_e + N \rightarrow e^- + \text{hadrons}$</td>
<td>$0.8 \times 10^{-38} \ E_{\nu}$</td>
<td>$10^5$</td>
</tr>
<tr>
<td>$\nu_e + n \rightarrow e^- + p$</td>
<td>$0.75 \times 10^{-38}$</td>
<td>$10^3$</td>
</tr>
<tr>
<td>$\mu^- + e^- \rightarrow \mu^- + \nu_e$</td>
<td>$1.5 \times 10^{-41} \ E_{\nu}$</td>
<td>$20,000$</td>
</tr>
<tr>
<td>$\nu_e + e^- \rightarrow \nu_e + e^-$</td>
<td>$1.5 \times 10^{-41} \ E_{\nu}$</td>
<td>$100$</td>
</tr>
<tr>
<td>$\nu_e + e^- \rightarrow \nu_e + e^-$</td>
<td>$(10^{-42} - 10^{-41}) \ E_{\nu}$</td>
<td>$1400-14,000$</td>
</tr>
<tr>
<td>$\nu_e + N_e \rightarrow e^+ + \nu_e + \nu_e + \nu_e$</td>
<td>$\sim 10^{-42}$</td>
<td>$140$</td>
</tr>
<tr>
<td>$\nu_e + N_e \rightarrow e^+ + \nu_e + e^+ + \nu_e$</td>
<td>$\sim 10^{-42}$</td>
<td>$360$</td>
</tr>
</tbody>
</table>

*The cross section for this process, which is allowed only if neutral currents exist, have been calculated using the Weinberg model.*
1000 GeV SPECTRUM:
800m DECAY PATH
600m SHIELD

200 AND 500 GeV SPECTRA:
EXISTING $\nu$ BEAM

$\nu$/GeV/m$^2$/10$^{13}$ INTERACTING PROTON

E$_{\nu}$ IN GeV

FIG. 1
TOTAL EVENT RATE, ASSUMING $\sigma_{TOT} = 0.8 \times E_\nu \times 10^{38}$ cm$^2$

$10^{19}$ PROTONS ON TARGET
20 TON DETECTOR

EVENTS / 10 GeV

$E_\nu$ IN GeV

FIG. 2
$E_\nu = 600 \pm 10\%$  
52,000 EVENTS

$10^{19}$ PROTONS ON TARGET  
20 TON DETECTOR

FIG. 3
$q^2$ DISTRIBUTION, ALL EVENTS

$10^{19}$ PROTONS ON TARGET
20 TON DETECTOR

FIG. 4
$\nu_\mu + N \rightarrow N + \mu^- + W^*$, ALL W DECAY MODES

$10^{19}$ PROTONS ON TARGET
20 TON DETECTOR

$W$'s PRODUCED / $10^{19}$ PROTONS

$M_W$ IN GeV

FIG. 5
$\sigma_{\text{TOT}} \text{ vs } E_\nu$, ALL EVENTS

$\sigma_{\text{TOT}}$

$5 \times 10^{-36} \text{ cm}^2$

$E_\nu \text{ in GeV}$

$M_W = \infty$

$M_W = 37$

$M_W = 20$

FIG. 6
FIG. 7

$E\nu$ (GeV)

$10^5$

$10^4$

$10^3$

$10^2$

$10$

$0$

$200$

$400$

$600$

$10^5$

$10^4$

$10^3$

$10^2$

$10$

$0$

$200$

$400$

$600$

$E\nu$ (GeV)

$10^5$

$10^4$

$10^3$

$10^2$

$10$

$0$

$200$

$400$

$600$

$E\nu$ (GeV)

$10^5$

$10^4$

$10^3$

$10^2$

$10$

$0$

$200$

$400$

$600$

$y = 0.9 \pm 0.1$

$y = 0.9 \pm 0.1$

$y = 0.9 \pm 0.1$

$y = 0.9 \pm 0.1$

$y = 0.9 \pm 0.1$

$y = 0.9 \pm 0.1$

$y = 0.9 \pm 0.1$

$y = 0.9 \pm 0.1$

$y = 0.9 \pm 0.1$

$y = 0.9 \pm 0.1$

$y = 0.9 \pm 0.1$
TOTAL CROSS SECTION AT FIXED $x$ AND $y$

$x = 0.9 \pm 0.1$

$y = 0.9 \pm 0.1$

FIG. 8
TOTAL CROSS SECTION vs $E_\nu^2$ FOR $x=0.9\pm0.1$, $y=0.9\pm0.1$
$\nu_\mu + Ne \rightarrow Ne + \mu^- + \mu^+ + \nu_\mu$

$v_\mu + Ne \rightarrow Ne + \mu^- + e^+ + \nu_e$

$15$ GeV $W^+$

8400 EVENTS

$\mu^- e^+ \nu_e$, 300 EVENTS

$\mu^- \mu^+ \nu_\mu$, 140 EVENTS

EVENTS / 10 GeV

$E_\nu$, GeV

FIG.10